Physics 417: Problem Set 5

(*DUE ON THURSDAY 10/17 – NOTE NEW DAY OF THE WEEK*)

Problem 1: Griffiths 2.10

Problem 2: Griffiths 2.12

Problem 3: Coherent states of the harmonic oscillator

In the previous problem, you should have found that the energy eigenstates of the harmonic oscillators do not look anything like a classical oscillator, in particular their average position is always zero. In this problem, we will show that the wavepacket that describes a classical oscillator is the coherent state that we studied in Problem Set 2. Recall that the coherent state is given by (for any complex λ):

$$|\lambda\rangle = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-|\lambda|^2/2}}{(n!)^{1/2}} |n\rangle \tag{1}$$

where $|n\rangle$ are the energy eigenstates of the harmonic oscillator, and the coefficients square to the Poisson distribution. (Note that I'm changing the notation from problem set 2, sorry.) Recall also that the coherent state satisfies the eigenvalue equation

$$a_-|\lambda\rangle = \lambda|\lambda\rangle \tag{2}$$

Using these facts...

(a) Find the time-evolved coherent state $|\lambda, t\rangle$. Show that it remains coherent up to an overall phase factor: $|\lambda, t\rangle = e^{-\frac{1}{2}i\omega t}|\lambda(t)\rangle$. What is $\lambda(t)$?

(b) Compute the expectation value of X in the time-evolved coherent state, i.e. $x(t) \equiv \langle\lambda, t|X|\lambda, t\rangle$. (*Hint: you will also need eq [2.69] from the book.*) Show that $x(t)$ describes a classical oscillator with the correct frequency. What is the amplitude of oscillation?

Problem 4: Quantum Fabry-Perot Interferometer

(a) Read through section 2.5 of Griffiths.

(b) Find the transmission probability T for a *double-delta function barrier*:

$$V(x) = \alpha(\delta(x+a) + \delta(x-a)) \qquad (\alpha > 0) \tag{3}$$

Express your result in terms of the rescaled dimensionless variables $\tilde{\alpha} \equiv \frac{ma\alpha}{\hbar^2}$ and $\tilde{k} \equiv \frac{4ak}{\tilde{\alpha}}$. (The final answer is reasonable, but the intermediate steps can get messy – use Maple/Mathematica!)

(c) Make plots of T vs. $\tilde{k}$ for $\tilde{\alpha} \ll 1$ and $\tilde{\alpha} \gg 1$. Do you notice any qualitatively different behavior in these two regimes?

(d) In the latter case ($\tilde{\alpha} \gg 1$), derive an approximate formula valid at small $\tilde{k}$ for the values of $\tilde{k}$ at which the transmission probability becomes one. (This is an example of the phenomenon of resonant tunneling.)