Physics 417: Problem Set 1 (*Due in class Wednesday 9/18*)

Problem 1: Blackbody radiation

Recall the Planck blackbody spectrum: $\rho(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$.

(a) What is the frequency ν at which the spectrum peaks as a function of the blackbody temperature T? (An approximate numerical formula is okay.)

(b) At what ranges of temperatures does the blackbody peak in the visible spectrum (which you can take to be the range of wavelengths $400 - 700$ nm)?

Problem 2: Atoms

(a) Estimate the lifetime of hydrogen in the classical nuclear model, i.e. an electron orbiting a proton bound by the Coulomb force. (Hint: use the Larmor formula for the power radiated by an accelerating charge, $\frac{dE}{dt} = \frac{2}{3}\frac{q^2 a^2}{c^3}$.)

(b) Estimate the size of an atom using Avogadro's number, the molar mass and the typical density of matter (say ~ 10 g/mol and ~ 1 g/cm^3).

(c) In class we neglected the gravitational force between the proton and electron in describing the nuclear model. Justify why this is correct.

Problem 3: Magnetic moments

(a) Consider a solid spherical ball of mass m rotating about the z axis with a charge q uniformly distributed on its surface. Show that the magnetic moment $\vec{\mu}$ is related to the angular momentum $\vec{L}$ by

$$\vec{\mu} = \frac{5q}{6m}\vec{L} \tag{1}$$

(b) Find the analogous relation for a solid spherical ball of mass M with charge Q uniformly distributed throughout the ball. What is the proportionality factor in this case?

(c) Convert your answers in (a) and (b) into g-factors. Do either of them exceed the g-factor for the electron spin, i.e. $g \approx 2$? Comment on what it would take to achieve $g = 2$ while maintaining spherical symmetry.

Problem 4: Spin states

In class, we will show that the spin operators S_x, S_y, S_z can be represented as

$$S_x = \frac{\hbar}{2}(|-\rangle\langle+| + |+\rangle\langle-|), \quad S_y = \frac{i\hbar}{2}(|-\rangle\langle+| - |+\rangle\langle-|), \quad S_z = \frac{\hbar}{2}(|+\rangle\langle+| - |-\rangle\langle-|) \quad (2)$$

(a) In the basis where $|+\rangle \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|-\rangle \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, find the matrix representation of $S_{x,y,z}$.

(b) Show that $[S_i, S_j] = i\epsilon_{ijk}\hbar S_k$ and $\{S_i, S_j\} = \frac{\hbar^2}{2}\delta_{ij}$, using both the operator representation (2) and the matrix representation.

Problem 5: More on states and operators

Suppose I have a 3-state Hilbert space spanned by the orthonormal states $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$, and an operator A that acts as:

$$A|\alpha\rangle = \frac{1}{\sqrt{2}}|\beta\rangle, \qquad A|\beta\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |\gamma\rangle), \qquad A|\gamma\rangle = \frac{1}{\sqrt{2}}|\beta\rangle \quad (3)$$

(a) Find a matrix representation of this operator A.

(b) What are the eigenvalues and the normalized eigenkets of A? Can you guess a physical system which is described by all this?

Problem 6: GRE quickies

I realize that many of you will have to take the GRE this semester. I would like to help you study for the quantum mechanics portion of the test, so periodically I will give you HW problems taken from past GRE exams. They are not intended to take very long. (On the actual test you have about 3 hours to solve about 100 problems.)

(a) (from 1996) When alpha particles are directed onto atoms in a thin metal foil, some make very close collisions with the nuclei of the atoms and are scattered at large angles. If an alpha particle with an initial kinetic energy of 5 MeV happens to be scattered through an angle of 180°, what is the distance of closest approach to the scattering nucleus? (Assume that the metal foil is made of silver, with $Z = 50$.)

(b) (from 2001)

$$\begin{aligned} |\psi_1\rangle &= 5|1\rangle - 3|2\rangle + 2|3\rangle \\ |\psi_2\rangle &= |1\rangle - 5|2\rangle + x|3\rangle \end{aligned} \quad (4)$$

The states $|1\rangle$, $|2\rangle$, $|3\rangle$ are orthonormal. For what values of x are the states $|\psi_1\rangle$, $|\psi_2\rangle$ given above orthogonal?

(c) (from 2001): The state $|\psi\rangle = \frac{1}{\sqrt{6}}|-1\rangle + \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{3}}|2\rangle$ is a linear combination of three orthonormal eigenstates of the operator A corresponding to eigenvalues -1, 1 and 2. What is the expectation value of A in this state?