

Temperature effects in the band structure of topological insulators

Bartomeu Monserrat^{1,2,*} and David Vanderbilt¹

¹*Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA*

²*TCM Group, Cavendish Laboratory, University of Cambridge,
J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom*

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We study the effects of temperature on the band structure of the Bi_2Se_3 family of topological insulators using first-principles methods. Increasing temperature drives these materials towards the normal state, with similar contributions from thermal expansion and from electron-phonon coupling. The band gap changes with temperature reach 0.3 eV at 600 K, of similar size to the changes caused by electron correlation. Our results suggest that temperature-induced topological phase transitions should be observable near critical points of other external parameters.

Topological insulators and related materials have consolidated as a new field of condensed matter of both fundamental and applied interest. Potentially novel applications have motivated the search for new materials exhibiting topological properties. In this quest, composition [1], pressure [2, 3], strain [4, 5], and electromagnetic fields [6–8] have all been used to control topological order. By contrast, temperature has so far exclusively played a passive role, for example constraining the quantum anomalous Hall effect to the millikelvin regime [9], and thus limiting potential applications of the associated dissipationless currents. Only recently have Saha and co-workers proposed that temperature can also be exploited when studying topological materials, for example by using phonon linewidths to identify band inversions [10].

The effects of temperature on topological insulators can be divided into two areas. The first concerns the electronic occupation of states, leading to questions such as the proper definition of topological invariants, which must then be based on the density matrix rather than the ground state wave function [11–14]. The second relates to the renormalization of the single-particle bands, which could lead to temperature-induced band inversions [15, 16], and it has been argued on the basis of simple models that temperature should favor topological phases.

First-principles calculations have proved central in predicting new topological materials and their behaviour, many times leading to subsequent experimental observation [17, 18]. The study of temperature effects on topological materials would equally benefit from first-principles calculations, but none have been available. Fully first-principles calculations of the temperature dependence of band structures have only recently become possible, as first and second order terms in the electron-phonon interaction contribute similarly and must be included on an equal footing [19, 20]. Because of such difficulties, to date these calculations have only been performed on simple semiconductors and insulators [21–25]. To use these techniques for topological materials, they need to be extended to include the effects of spin-orbit coupling in the calculation of the electron-phonon interaction.

In this work, we study the effects of temperature on topological insulators with first-principles methods. Using the Bi_2Se_3 family of topological insulators, we demonstrate the feasibility of such calculations and unravel the importance of temperature when studying topological order. Our calculations show that both thermal expansion and electron-phonon coupling make similar contributions to the temperature dependence of band structures, and that, at variance with predictions from model studies [15, 16], increased temperature tends to suppress topological order in these materials.

We consider the Bi_2Se_3 family of compounds for our first-principles study [17]. Bi_2Te_3 , Bi_2Se_3 , and Sb_2Te_3 crystallize in the rhombohedral $R\bar{3}m$ space group, with five atoms in the primitive cell. They form a layered structure, in which groups of five layers are tightly bound (so-called quintuple layers), and these are then weakly bound to each other. Sb_2Se_3 has $Pnma$ symmetry in its ground state. Metastable structures of these compounds have been synthesized [26], and in particular the $R\bar{3}m$ structure of Sb_2Se_3 has been experimentally studied [3]. In this work, we consider the $R\bar{3}m$ structure for all compounds.

Our calculations are based on density functional theory (DFT) using the VASP package [27–30]. We use the PBE functional [31] and the projector augmented-wave method [32, 33] with an energy cut-off of 600 eV and a \mathbf{k} -point grid of size $12 \times 12 \times 12$ for the primitive cells and commensurate grids for the supercells. All calculations are performed with spin-orbit coupling unless otherwise stated.

The PBE functional overestimates static volumes and thermal expansion compared to experiment. For example, in Bi_2Se_3 the experimental hexagonal c axis at 10 K is 28.48 Å [34], compared with the PBE c axis of 30.21 Å. Therefore, we use experimental volumes when available. For Bi_2Se_3 and Sb_2Te_3 , experimental data is only available up to about 250 K [34], and we fit a Bose factor to this data to extrapolate to higher temperatures, which is justified in this case as the linear asymptotic regime is already reached at the highest temperatures for which data is available [35]. For Bi_2Te_3 , data is available up

to 600 K [36]. No data is available for the rhombohedral structure of Sb_2Se_3 , so in this case we use the structure relaxed using the PBE functional. The atomic coordinates of all atoms are relaxed until the forces are smaller than 10^{-3} eV/Å.

The phonon and electron-phonon coupling calculations are performed using the finite displacement method in conjunction with the recently developed nondiagonal supercell approach [37]. Finite displacement methods allow us to straight-forwardly incorporate the effects of the spin-orbit interaction on electron-phonon coupling which, as far as we are aware, has never been attempted in this context. The reported results correspond to a vibrational Brillouin zone sampling using grid sizes of $4 \times 4 \times 4$, as calculations for Bi_2Se_3 using grids of size $8 \times 8 \times 8$ show that the electron-phonon induced band gap corrections are converged to better than 8 meV at all temperatures considered. The matrix of force constants is constructed by considering small positive and negative symmetry-inequivalent atomic displacements of amplitude 0.005 Å. The dynamical matrix, obtained by a Fourier transformation, is diagonalized to calculate the vibrational frequencies $\omega_{\mathbf{q}\nu}$ at each phonon wave vector \mathbf{q} and branch ν . The band gap at temperature T is then calculated as [19–25]

$$E_g(T) = E_g^{\text{static}} + \frac{1}{N_{\mathbf{q}}} \sum_{\mathbf{q},\nu} \frac{a_{\mathbf{q}\nu}}{\omega_{\mathbf{q}\nu}} \left[\frac{1}{2} + n_B(\omega_{\mathbf{q}\nu}, T) \right], \quad (1)$$

where $a_{\mathbf{q}\nu}$ are the electron-phonon matrix elements averaged over $N_{\mathbf{q}}$ \mathbf{q} -points, and n_B is a Bose-Einstein factor. The electron-phonon matrix elements are determined using finite displacements along the normal modes of vibration.

The topological nature of the Bi_2Se_3 family of compounds is determined by the parity of the occupied bands at the Γ point, where band inversion occurs in the presence of spin-orbit coupling [17]. Therefore, in order to investigate the effects of temperature on the topological order, we focus on the temperature dependence of the band gap at Γ , as shown in Fig. 1.

Bi_2Te_3 , Bi_2Se_3 , and Sb_2Te_3 are topological insulators, and their gaps close with increasing temperature. Using Bi_2Se_3 as an example, we observe that the gap closure is driven by both thermal expansion and electron-phonon coupling. Thermal expansion alone (orange dashed line) closes the band gap by almost 0.1 eV at 600 K. Electron-phonon coupling alone (blue dashed-dotted line) causes a change in the band gap of 0.1 eV at 600 K. The asymptotic linear dependence of the gap change with temperature is reached at around 100 K, determined by the low-energy vibrations that are a consequence of the heavy atoms in the compounds under study. Thermal expansion and electron-phonon coupling make similar contributions, and therefore both must be included (red solid line). To date, the vast majority of first-principles calcu-

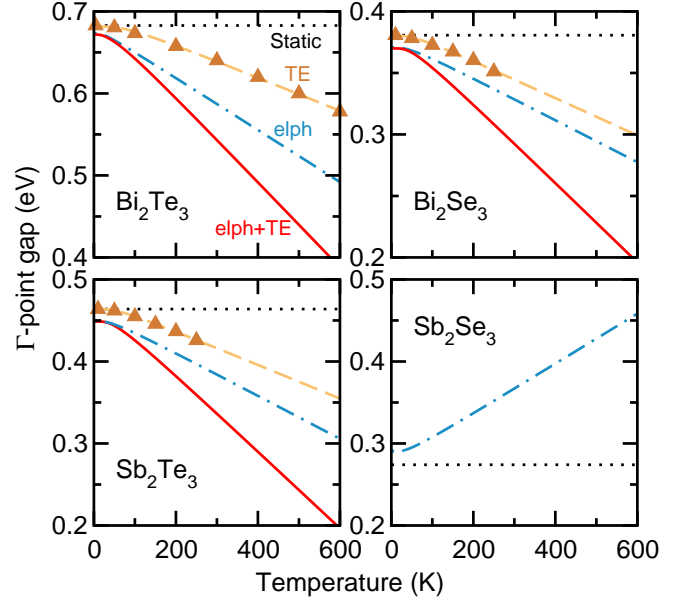


FIG. 1. Temperature dependence of the Γ -point band gaps of the Bi_2Se_3 family of compounds. The black dotted lines correspond to the static lattice band gaps; the orange dashed and blue dashed-dotted lines show the result of including only thermal expansion or only electron-phonon coupling effects, respectively; and the red solid curves give the total temperature dependence.

lations of the temperature dependence of the band gap have neglected the effects of thermal expansion. This may be justifiable for simple materials made of light atoms, such as diamond and silicon, that exhibit weak thermal expansion. By contrast, topological insulators are composed of heavy elements, for which thermal expansion is important as shown in Fig. 1.

Not surprisingly, for Sb_2Se_3 , which is already a normal insulator with an uninverted gap at low temperature, the same electron-phonon effects lead instead to a band gap opening with increasing temperature. Thermal expansion also tends to increase the band gap, but we have not included this contribution as there is no experimental data available for this system.

The sizes of the band gap shifts are similar to those observed and calculated in many semiconductors and insulators. The only reported phonon-induced gap shifts in topological insulators we are aware of are those of Kim and Jhi, who theoretically studied the change in the band gap induced by exciting selected phonon modes in a series of IV-VI semiconductors and found shifts on the 0.1 eV scale [38]. In the Bi_2Se_3 family of compounds, zero-point contributions are small, in the range 0.01-0.02 eV in all systems, but thermal changes are larger, reaching 0.2-0.3 eV at 600 K. The GW approximation has been shown to correct the DFT gaps by about 0.2-0.3 eV in the Bi_2Se_3 family of compounds [39], to lead to qualitative changes in the shape of the bands near Γ [40], and more generally

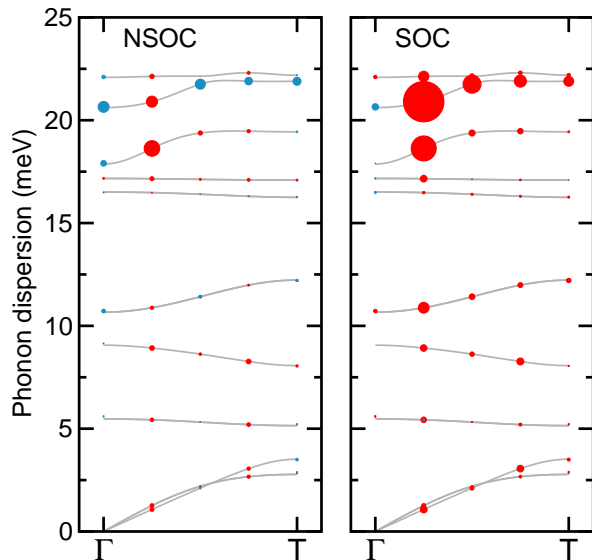


FIG. 2. Phonon dispersion of Bi_2Se_3 with mode resolved electron-phonon coupling strength without (left) and with (right) spin-orbit coupling. The area of the circles are proportional to the strength of electron-phonon coupling, and the sign of the coupling is positive (gap opening) for blue circles and negative (gap closing) for red circles.

to question the validity of some predictions of topological insulators based on semilocal DFT [41]. Our results show that the effects of temperature modify the band structure to a similar extent, and therefore should also be included for an accurate description of topological insulators.

We next investigate the microscopic origin of the electron-phonon coupling strength, taking Bi_2Se_3 as an example. In Fig. 2 we show the phonon dispersion of Bi_2Se_3 along the high-symmetry line from Γ at $\mathbf{q} = (0,0,0)$ to T at $\mathbf{q} = (0.5,0.5,0.5)$, chosen because electron-phonon coupling is strongest along this line. The mode-resolved strength of electron-phonon coupling to the direct band gap at the electronic Γ point is indicated by the filled circles, and shown both without and with spin-orbit coupling. The inclusion of the spin-orbit interaction has two effects. The first is to reverse the gap-opening induced by electron-phonon coupling to gap closing upon band inversion. The second is to modify the strength of electron-phonon coupling beyond the mere exchange of band extrema. For example, the gap closing induced by the modes at $\mathbf{q} = (0.25,0.25,0.25)$ with spin-orbit coupling is larger than the opening of the gap induced by the same modes when spin-orbit coupling is not included. For Bi_2Te_3 and Sb_2Te_3 , similar behaviour is observed, but for Sb_2Se_3 , the inclusion of spin-orbit coupling does not induce a band inversion, and therefore the sign of the band gap correction does not reverse.

A detailed analysis of the couplings reveals that the modes that couple most strongly to the electronic states are those that involve atomic vibrations along the c -axis

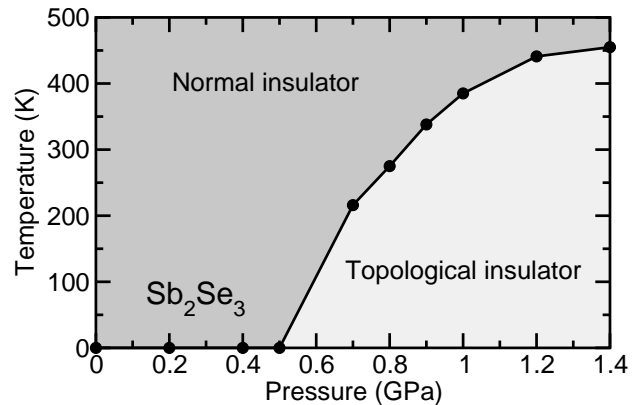


FIG. 3. Pressure-temperature phase diagram of Sb_2Se_3 .

that change the interlayer distance. It is also the change in length of the c -axis that drives the band gap change due to thermal expansion. This can be explained by the nature of the states at the band extrema of the Γ point, which mainly arise from the hybridized p orbitals of the constituent atoms. The crystal field splits the p_z from the $p_{x,y}$ orbitals, which remain degenerate, and it is the p_z that form the band extrema and undergo band inversion when the spin-orbit interaction is included [17]. Changing the interlayer distance, either by atomic vibrations or by lattice expansion, modifies the states at the gap edges, and thus drives the observed temperature dependence.

The inclusion of temperature in our calculations allows us to construct the pressure-temperature phase diagram of Sb_2Se_3 shown in Fig. 3. Sb_2Se_3 is a normal insulator under ambient pressure, but with increasing external pressure it undergoes a phase transition to a topological insulator at 2.5 GPa [3]. First-principles calculations based on the PBE functional give a transition pressure of 1.0 GPa [42]. Our own calculations using PBE give a similar transition pressure of 0.5 GPa, and we also perform HSE calculations [43, 44] which increase the transition pressure to 2.5 GPa, in better agreement with experiment. Due to the computational cost of HSE calculations, we perform the finite temperature calculations using the PBE functional instead, and we expect that our results will be qualitatively correct. Figure 3 shows that, with increasing temperature and for pressures above 0.5 GPa, a band inversion occurs, inducing a transition from a topological to a normal phase. Experimental observation of a temperature-induced topological phase transition should be simplest at pressures just above the zero-temperature transition pressure. More generally, temperature-induced topological phase transitions should be observable near critical values of other external parameters driving a topological phase transition. The pressure-temperature phase diagrams of Bi_2Te_3 , Bi_2Se_3 , and Sb_2Te_3 are qualitatively similar to that of Sb_2Se_3 . These materials, however, are

deep inside the topological part of the phase diagram at ambient pressure so that unrealistically high temperatures would be needed to induce a transition to the normal state (see Fig. 1).

Our results show that in the Bi_2Se_3 family of compounds, increasing temperature favors the normal phase. This is at odds with the theoretical prediction of Refs. [15, 16], where it was argued that higher temperature should favor topological phases. The focus there is on electron-phonon coupling only, but we have shown that thermal expansion is equally important. Even if only electron-phonon coupling is considered, our calculations still show that temperature does not favor topological phases in the Bi_2Se_3 family of compounds. To understand the discrepancy, we note several possibilities. First, the analysis of Refs. [15, 16] only includes the so-called Fan term, which arises from treating the first-order phonon-induced change in the Hamiltonian at second order in perturbation theory. However, the Debye-Waller term, which arises from the second-order change in the Hamiltonian treated at first order in perturbation theory, has been shown to be equally important for the calculation of the temperature dependence of band gaps [21]. Our finite displacement approach includes both terms. Second, the analysis of Refs. [15, 16] assumes that the two bands that participate in the band inversion are well separated in energy from the rest of the bands. This ensures that, for sufficiently small gaps, the dominant contribution comes from intraband transitions, which have a gap-opening tendency in topological insulators. In the systems we have studied, the bands at Γ involved in the band inversion are not well separated in energy from other bands. For example, the conduction band minimum of Bi_2Te_3 and Sb_2Te_3 is not even at Γ , where band inversion occurs. Therefore, we do not expect that the analysis of Refs. [15, 16] applies in our case. Nonetheless, there is no reason why temperature could not favor topological order in some other materials that obey the assumptions laid out in Refs. [15, 16], and it would certainly be interesting to identify some such examples.

In summary, we have shown that thermal expansion and electron-phonon coupling drive the temperature dependence of the band structure of topological insulators. Increasing temperature favors the normal state in the Bi_2Se_3 family of compounds, and induces a topological phase transition in pressurized Sb_2Se_3 . Temperature-induced changes to the band gaps reach 0.3 eV at 600 K, and are of similar size to the changes induced by electron correlation in these materials. Open questions remain, such as the effects of thermally induced changes in electron occupation, and the possible importance of non-adiabatic phenomena, especially close to band inversions.

Our work shows that first-principles calculations of the effects of temperature on topological materials are feasible, and that temperature can have important effects on the band structure and topological order of these mate-

rials. It should facilitate future work, for example in the search for a room temperature Chern insulator.

Note added. Antonius and Louie recently reported first-principles calculations of the temperature dependence of the band structure of the topological insulator $\text{BiTl}(\text{S}_{1-\delta}\text{Se}_\delta)_2$ [45].

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* monserrat@physics.rutgers.edu

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