

Ab initio calculations of BaTiO₃ and PbTiO₃ (001) and (011) surface structures

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We present and discuss the results of calculations of surface relaxations and rumplings for the (001) and (011) surfaces of BaTiO₃ and PbTiO₃, using a hybrid B3PW description of exchange and correlation. On the (001) surfaces, we consider both AO (A = Ba or Pb) and TiO₂ terminations. In the former case, the surface AO layer is found to relax inward for both materials, while outward relaxations of all atoms in the second layer are found at both kinds of (001) terminations and for both materials. The surface relaxation energies of BaO and TiO₂ terminations on BaTiO₃ (001) are found to be comparable, as are those of PbO and TiO₂ on PbTiO₃ (001), although in both cases the relaxation energy is slightly larger for the TiO₂ termination. As for the (011) surfaces, we consider three types of surfaces, terminating on a TiO layer, a Ba or Pb layer, or an O layer. Here, the relaxation energies are much larger for the TiO-terminated than for the Ba or Pb-terminated surfaces. The relaxed surface energy for the O-terminated surface is about the same as the corresponding average of the TiO and Pb-terminated surfaces on PbTiO₃, but much less than the average of the TiO and Ba-terminated surfaces on BaTiO₃. We predict a considerable increase of the Ti-O chemical bond covalency near the BaTiO₃ and PbTiO₃ (011) surface as compared to both the bulk and the (001) surface.

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I. INTRODUCTION

Thin films of ABO₃ perovskite ferroelectrics play an important role in numerous microelectronic, catalytic, and other high-technology applications, and are frequently used as substrates for growth of other materials such as cuprate superconductors.^{1,2} Therefore, it is not surprising that a large number of *ab initio* quantum mechanical calculations,^{3,4,5,6,7,8,9,10,11,12,13} as well as several classical shell-model (SM) studies,^{14,15,16} have dealt with the atomic and electronic structure of the (001) surface of BaTiO₃, PbTiO₃, and SrTiO₃ crystals. In order to study the dependence of the surface relaxation properties on the exchange-correlation functionals and the type of basis (localized vs. plane-wave) used in the calculations, a detailed comparative study of SrTiO₃ (001) surfaces based on ten different quantum-mechanical techniques^{17,18} was recently performed.

Due to intensive development and progressive miniaturization of electronic devices, the surface structure as well as the electronic properties of the ABO₃ perovskite thin films have been extensively studied experimentally during the last years. The SrTiO₃ (001) surface structure has been analyzed by means of low-energy electron diffraction (LEED),¹⁹ reflection high-energy electron diffraction (RHEED),²⁰ X-ray photoelectron spectroscopy (XPS), ultraviolet electron spectroscopy (UPS), medium-energy ion scattering (MEIS),²¹ and surface X-ray diffraction (SXRD).²² Nevertheless it is important to note that the LEED¹⁹ and RHEED²⁰ experiments contradict each other in the sign (contraction or expansion) of the interplanar distance between top metal atom and the second crystal layer for the SrO-terminated SrTiO₃ (001) surface. The most recent experimental studies

on the SrTiO₃ surfaces include a combination of XPS, LEED, and time-of-flight scattering and recoil spectrometry (TOF-SARS),²³ as well as metastable impact electron spectroscopy (MIES).²⁴ In these recent studies, well-resolved 1×1 LEED patterns were obtained for the TiO₂-terminated SrTiO₃ (001) surface. Simulations of the TOF-SARS azimuthal scans indicate that the O atoms are situated 0.1 Å above the Ti layer (surface plane) in the case of the TiO₂-terminated SrTiO₃ (001) surface.

While the (001) surfaces of SrTiO₃, BaTiO₃ and PbTiO₃ have been extensively studied, much less is known about the (011) surfaces. The scarcity of information about these surfaces is likely due to the polar character of the (011) orientation. (011) terminations of SrTiO₃ have frequently been observed, but efforts towards the precise characterization of their atomic-scale structure and corresponding electronic properties has only begun in the last decade, specifically using atomic-force microscopy,²⁵ scanning tunneling microscopy (STM), Auger spectroscopy, and low-energy electron-diffraction (LEED)²⁶ methods.

To the best of our knowledge, very few *ab initio* studies of perovskite (011) surfaces exist. The first *ab initio* study of the electronic and atomic structures of several (1×1) terminations of the (011) polar orientation of the SrTiO₃ surface was performed by Bottin *et al.*²⁷ One year later Heifets *et al.*²⁸ performed very comprehensive *ab initio* Hartree-Fock calculations for four possible terminations (TiO, Sr, and two kinds of O terminations) of the SrTiO₃ (011) surface. Recently Heifets *et al.*²⁹ performed *ab initio* density-functional calculations of the atomic structure and charge redistribution for different terminations of the BaZrO₃ (011) surfaces. However, despite the high technological potential of BaTiO₃ and PbTiO₃, we are unaware of any previous *ab initio* cal-

culations performed for the BaTiO₃ and PbTiO₃ (011) surfaces. In this study, therefore, we have investigated the (011) as well as the (001) surfaces of BaTiO₃ and PbTiO₃, with an emphasis on the effect of the surface relaxation and rumpling, surface energies, and the charge redistributions and changes in bond strength that occur at the surface.

II. PRELIMINARIES

A. Computational method

We carry out first-principles calculations in the framework of density-functional theory (DFT) using the CRYSTAL computer code.³⁰ Unlike the plane-wave codes employed in many previous studies,^{31,32} CRYSTAL uses localized Gaussian-type basis sets. In our calculations, we adopted the basis sets developed for BaTiO₃ and PbTiO₃ in Ref. [33]. Our calculations were performed using the hybrid exchange-correlation B3PW functional involving a mixture of non-local Fock exact exchange, LDA exchange, and Becke's gradient corrected exchange functional,³⁴ combined with the non-local gradient corrected correlation potential of Perdew and Wang.^{35,36,37} We chose the hybrid B3PW functional for our current study because it yields excellent results for the SrTiO₃, BaTiO₃, and PbTiO₃ bulk lattice constant and bulk modulus.^{17,33}

The reciprocal-space integration was performed by sampling the Brillouin zone with an 8×8×8 Pack-Monkhorst mesh,³⁸ which provides a balanced summation in direct and reciprocal spaces. To achieve high accuracy, large enough tolerances of 7, 8, 7, 7, and 14 were chosen for the dimensionless Coulomb overlap, Coulomb penetration, exchange overlap, first exchange pseudo-overlap, and second exchange pseudo-overlap parameters, respectively.³⁰

An advantage of the CRYSTAL code is that it treats isolated 2D slabs, without any artificial periodicity in the z direction perpendicular to the surface, as commonly employed in most previous surface band-structure calculations (e.g., Ref. [8]). In the present *ab initio* investigation, we have studied several isolated periodic two-dimensional slabs of cubic BaTiO₃ and PbTiO₃ crystals containing 7 planes of atoms.

B. Surface geometries

The BaTiO₃ and PbTiO₃ (001) surfaces were modeled using symmetric (with respect to the mirror plane) slabs consisting of seven alternating TiO₂ and BaO or PbO layers, respectively. One of these slabs was terminated by BaO planes for the BaTiO₃ crystal (or PbO planes for PbTiO₃) and consisted of a supercell containing 17 atoms. The second slab was terminated by TiO₂ planes for both materials and consisted of a supercell containing

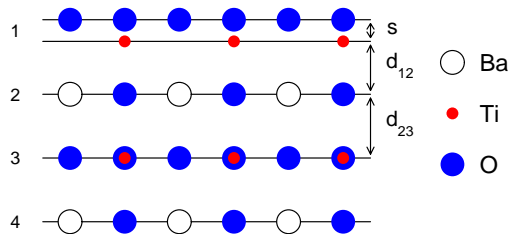


FIG. 1: (Color online.) Side view of a TiO₂-terminated BaTiO₃ (001) surface with the definitions of the surface rumpling s and the near-surface interplanar distances d_{12} and d_{23} , respectively.

18 atoms. These slabs are non-stoichiometric, with unit cell formulae Ba₄Ti₃O₁₀ or Pb₄Ti₃O₁₀, and Ba₃Ti₄O₁₁ or Pb₃Ti₄O₁₁ for BaTiO₃ and PbTiO₃ perovskites, respectively. These two (BaO or PbO and TiO₂) terminations are the only two possible flat and dense (001) surfaces for the BaTiO₃ or PbTiO₃ perovskite lattice structure. The sequence of layers at the TiO₂-terminated (001) surface of BaTiO₃ is illustrated in Fig. 1.

Unlike the (001) cleavage of BaTiO₃ or PbTiO₃, which naturally gives rise to non-polar BaO (or PbO) and TiO₂ terminations, a naive cleavage of BaTiO₃ or PbTiO₃ to create (011) surfaces leads to the formation of polar surfaces. For example, the stacking of the BaTiO₃ crystal along the (011) direction consists of alternating planes of O₂ and BaTiO units having nominal charges of $-4e$ and $+4e$ respectively, assuming O²⁻, Ti⁴⁺, and Ba²⁺ constituents. (Henceforth we shall use BaTiO₃ for presentation purposes, but everything that is said will apply equally to the PbTiO₃ case.) Thus, a simple cleavage leads to O₂-terminated and BaTiO-terminated (011) surfaces that are *polar* and have nominal surface charges of $-2e$ and $+2e$ per surface cell respectively. These are shown as the top and bottom surfaces in Fig. 2(a) respectively. If uncompensated, the surface charge would lead to an infinite electrostatic cleavage energy. In reality, the polar surfaces would probably become metallic in order to remain neutral, but in view of the large electronic gaps in the perovskites, such metallic surfaces would presumably be unfavorable. Thus, we may expect rather generally that such polar crystal terminations are relatively unstable in this class of materials.³

On the other hand, if the cleavage occurs in such a way as to leave a half layer of O₂ units on each surface, we obtain the non-polar surface structure shown in Fig. 2(b). Every other surface O atom has been removed, and the remaining O atoms occupy the same sites as in the bulk structure. We shall refer to this as the “O-terminated” (011) surface, in distinction to the “O₂-terminated” polar surface already discussed in Fig. 2(a). The non-polar nature of the O-terminated surface can be confirmed by noting that the 7-layer 15-atom Ba₃Ti₃O₉ slab shown in Fig. 2(b), which has two O-terminated surfaces, is neutral. It is also possible to make non-polar TiO-terminated and Ba-terminated surfaces, as shown in Figs. 2(c) and

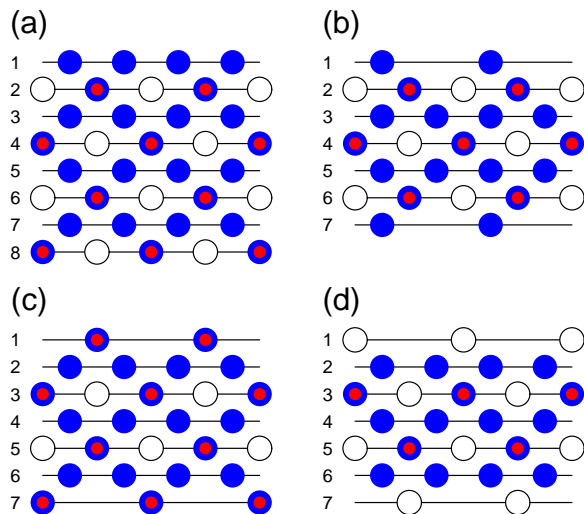


FIG. 2: (Color online.) Side views of slab geometries used to study BaTiO_3 (011) surfaces. (a) Stoichiometric 8-layer slab with O_2 -terminated and BaTiO -terminated surfaces at top and bottom respectively. (b) 7-layer slab with O -terminated surfaces. (c) 7-layer slab with TiO -terminated surfaces. (d) 7-layer slab with Ba -terminated surfaces.

(d), respectively. This is accomplished by splitting a BaTiO layer during cleavage, instead of splitting an O_2 layer. For the TiO and Ba -terminated surfaces, we use 7-layer slabs having composition $\text{Ba}_2\text{Ti}_4\text{O}_{10}$ (16 atoms) and $\text{Ba}_4\text{Ti}_2\text{O}_8$ (14 atoms) as shown in Fig. 2(c-d), respectively. These are again neutral, showing that the surfaces are non-polar (even though they no longer have precisely the bulk BaTiO_3 stoichiometry).

III. RESULTS OF CALCULATIONS

A. BaTiO_3 and PbTiO_3 (001) surface structure

In the present calculations of the BaTiO_3 and PbTiO_3 (001) surface atomic structure, we allowed the atoms located in the two outermost surface layers to relax along the z -axis (the forces along the x and y -axes are zero by symmetry). Here we use the term “layer” to refer to a BaO , PbO , or TiO_2 plane, so that there are two layers per stacked unit cell. For example, on the BaO or PbO -terminated surfaces, the top layer is BaO or PbO and the second layer is TiO_2 ; displacements of the third-layer atoms were found to be negligibly small in our calculations and thus were neglected.

The calculated atomic displacements for the TiO_2 and BaO -terminated (001) surfaces of BaTiO_3 , and for the TiO_2 and PbO -terminated (001) surfaces of PbTiO_3 , are presented in Table I. For BaTiO_3 (001), comparisons are also provided with the surface atomic displacements obtained by Padilla and Vanderbilt⁴ using plane-wave DFT methods in the local-density approximation (LDA), and by Heifets *et al.* using a classical shell-model (SM)

approach.¹⁶ Similarly, for PbTiO_3 (001), Table I shows comparisons with the plane-wave LDA calculations of Meyer and Vanderbilt.⁵ The relaxation of the surface metal atoms in both the BaTiO_3 and PbTiO_3 surfaces is much larger than that of the oxygen ions, leading to a considerable surface rumpling, which we quantify via a parameter s defined as the relative displacement of the oxygen with respect to the metal atom in a given layer. The surface rumpling and relative displacements of three near-surface planes are presented in Table II. According to our calculations, atoms of the first surface layer relax inwards (i.e., towards the bulk) for BaO and PbO terminations of both materials. Our calculations are in a qualitative agreement with the *ab initio* calculations performed by Padilla and Vanderbilt⁴ for BaTiO_3 , and by Meyer and Vanderbilt⁵ for PbTiO_3 . However, the predictions of the SM calculation disagree with the first-principles calculations; the SM predicts that the first-layer oxygen ions relax outward on the BaO -terminated BaTiO_3 (001) surface,¹⁶ rather than inwards. However, the magnitudes of the displacements are relatively small (-0.63% of the lattice constant a_0 in this study and 1.00% of a_0 in the SM calculations)¹⁶ which may be close to the error bar of the classical shell model. Outward relaxations of all atoms in the second layer are found at both (001) terminations of the BaTiO_3 and PbTiO_3 surfaces. From Table I, we can conclude that the magnitudes of the atomic displacements calculated using different *ab initio* methods and using the classical shell model are in a reasonable agreement.

In order to compare the calculated surface structures further with experimental results, the surface rumpling s and the changes in interlayer distances Δd_{12} and Δd_{23} , as defined in Fig. 1, are presented in Table II. Our calculations of the interlayer distances are based on the positions of relaxed metal ions, which are known to be much stronger electron scatters than the oxygen ions.¹⁹

For BaTiO_3 (001), the rumpling of TiO_2 -terminated surface is predicted to exceed that of BaO -terminated surface by a factor of two. This finding is in line with the values of surface rumpling reported by Padilla and Vanderbilt.⁴ In contrast, PbTiO_3 demonstrates practically the same rumpling for both terminations. From Table II one can see that qualitative agreement between all theoretical methods is obtained. In particular, the relaxed (001) surface structure shows a reduction of interlayer distance Δd_{12} and an expansion of Δd_{23} according to all *ab initio* and shell-model results.

As for experimental confirmation of these results, we are unfortunately unaware of experimental measurements of Δd_{12} and Δd_{23} for the BaTiO_3 and PbTiO_3 (001) surfaces. Moreover, for the case of the SrO -terminated SrTiO_3 (001) surface, existing LEED¹⁹ and RHEED²⁰ experiments actually contradict each other regarding the sign of Δd_{12} . In view of the absence of clear experimental determinations of these parameters, therefore, the first-principles calculations are a particularly important tool for understanding the surface properties.

TABLE I: Vertical atomic relaxations (in percent of bulk lattice constant) for BaTiO₃ and PbTiO₃ (001) surfaces. Positive sign corresponds to outward atomic displacement. ‘SM’ indicates shell-model calculation of Ref. [16]; ‘LDA’ are previous calculations of Ref. [4] and [5] for BaTiO₃ and PbTiO₃ respectively.

| BaTiO ₃ (001) surface relaxations | | | | | | PbTiO ₃ (001) surface relaxations | | | | |
|--|-------|-----|------------|-------|-------|--|-------|-----|------------|-------|
| Termination | Layer | Ion | This study | SM | LDA | Termination | Layer | Ion | This study | LDA |
| BaO | 1 | Ba | -1.99 | -3.72 | -2.79 | PbO | 1 | Pb | -3.82 | -4.36 |
| | | O | -0.63 | 1.00 | -1.40 | | | O | -0.31 | -0.46 |
| | 2 | Ti | 1.74 | 1.25 | 0.92 | | 2 | Ti | 3.07 | 2.39 |
| | | O | 1.40 | 0.76 | 0.48 | | | O | 2.30 | 1.21 |
| | 3 | Ba | | -0.51 | 0.53 | | 3 | Pb | | -1.37 |
| | | O | | 0.16 | 0.26 | | | O | | -0.20 |
| TiO ₂ | 1 | Ti | -3.08 | -2.72 | -3.89 | TiO ₂ | 1 | Ti | -2.81 | -3.40 |
| | | O | -0.35 | -0.94 | -1.63 | | | O | 0.31 | -0.34 |
| | 2 | Ba | 2.51 | 2.19 | 1.31 | | 2 | Pb | 5.32 | 4.53 |
| | | O | 0.38 | -0.17 | -0.62 | | | O | 1.28 | 0.43 |
| | 3 | Ti | | -0.33 | -0.75 | | 3 | Ti | | -0.92 |
| | | O | | -0.01 | -0.35 | | | O | | -0.27 |

B. BaTiO₃ and PbTiO₃ (011) surface structure

To our knowledge, we have performed the first *ab initio* calculations for BaTiO₃ and PbTiO₃ (011) surfaces. We have studied the TiO₂-terminated, BaO or PbO-terminated, and O-terminated surfaces illustrated in Fig. 2(c), (d), and (b), respectively. The computed surface atomic relaxations are reported in Table III.

Focusing first on the BaTiO₃ surfaces, we find that the Ti ions in the outermost layer of the TiO-terminated surface move inwards (towards the bulk) by $0.0786a_0$, whereas the O ions in the outermost layer move outwards by a $0.0261a_0$. The Ba atoms in the top layer of the Ba-terminated surface of Fig. 2(d) and the O atoms in the outermost layer of O-terminated surface of Fig. 2(b) move inwards by $0.0867a_0$ and $0.0540a_0$, respectively. The agreement between our *ab initio* B3PW and the classical SM calculations is satisfactory for all three of these surface terminations. In particular, the directions of the displacements of first and second-layer atoms coincide for all three terminations. This indicates that classical SM calculations with a proper parameterization can serve as a useful initial approximation for modeling the atomic

structure in perovskite thin films.

Turning now to our results for the PbTiO₃ (011) surfaces, we find that all metal atoms in the outermost layer move inwards irrespective of the termination. Surface oxygen atoms are displaced outwards for the TiO-terminated surface, while oxygen atoms move inwards in the O-terminated surface. The displacement patterns of atoms in the outermost surface layers are similar to those of the BaTiO₃ (011) surfaces, as well as classical shell model results for BaTiO₃.¹⁶ For example, the atomic displacement magnitudes of Ti and oxygen atoms in the TiO-terminated PbTiO₃ (011) surface are $-0.0813a_0$ and $0.033a_0$ respectively. The Pb atom is displaced inwards by $0.1194a_0$ for the Pb-terminated surface, similar to the corresponding BaTiO₃ case. Overall, Table III shows similar displacement patterns for the BaTiO₃ and PbTiO₃ (011) surfaces, as well as qualitatively similar results for both *ab initio* and classical shell-model descriptions.

C. BaTiO₃ and PbTiO₃ (001) and (011) surface energies

In the present work, we define the unrelaxed surface energy of a given surface termination X to be one half of the energy needed to cleave the crystal rigidly into an unrelaxed surface X and an unrelaxed surface with the complementary termination X' . For BaTiO₃, for example, the unrelaxed surface energies of the complementary BaO and TiO₂-terminated (001) surfaces are equal, as are those of the TiO and Ba-terminated (011) surfaces (and similarly for PbTiO₃). The relaxed surface energy is defined to be the energy of the unrelaxed surface plus the (negative) surface relaxation energy. These definitions are chosen for consistency with Refs. [17,28]. Unlike the authors of Refs. [27,29], we have made no effort to introduce chemical potentials here, so the results must be used with caution when addressing questions of the

TABLE II: Calculated surface rumpling s and interlayer displacements Δd_{ij} (in percent of bulk lattice constant) for near-surface planes on the BaO or PbO and TiO₂-terminated (001) surfaces of BaTiO₃ and PbTiO₃.

| | BaO or PbO term. | | | TiO ₂ termination | | |
|--------------------------|------------------|-----------------|-----------------|------------------------------|-----------------|-----------------|
| | s | Δd_{12} | Δd_{23} | s | Δd_{12} | Δd_{23} |
| BaTiO ₃ (001) | | | | | | |
| This study | 1.37 | -3.74 | 1.74 | 2.73 | -5.59 | 2.51 |
| LDA, Ref.4 | 1.39 | -3.71 | 0.39 | 2.26 | -5.20 | 2.06 |
| SM, Ref.16 | 4.72 | -4.97 | 1.76 | 1.78 | -4.91 | 2.52 |
| PbTiO ₃ (001) | | | | | | |
| This study | 3.51 | -6.89 | 3.07 | 3.12 | -8.13 | 5.32 |
| LDA, Ref.5 | 3.9 | -6.75 | 3.76 | 3.06 | -7.93 | 5.45 |

TABLE III: Calculated surface relaxations of BaTiO₃ and PbTiO₃ (011) surfaces (in percent of the lattice constant) for the three surface terminations. ‘SM’ indicates comparative results from the shell-model calculation of Ref. [16].

| BaTiO ₃ (011) surface | | | | | | PbTiO ₃ (011) surface | | | |
|----------------------------------|-----|------------|------------|-----------------|-----------------|----------------------------------|-----|------------|------------|
| Layer | Ion | Δz | Δy | Δz (SM) | Δz (SM) | Layer | Ion | Δz | Δy |
| TiO-terminated | | | | | | TiO-terminated | | | |
| 1 | Ti | -7.86 | | -6.93 | | 1 | Ti | -8.13 | |
| 1 | O | 2.61 | | 6.45 | | 1 | O | 3.30 | |
| 2 | O | -1.02 | | -1.66 | | 2 | O | -0.41 | |
| 3 | Ba | -0.88 | | -3.85 | | 3 | Pb | -2.54 | |
| 3 | O | | | -2.40 | | 3 | O | -4.07 | |
| 3 | Ti | | | 1.59 | | 3 | Ti | 0.30 | |
| Ba-terminated | | | | | | Pb-terminated | | | |
| 1 | Ba | -8.67 | | -13.49 | | 1 | Pb | -11.94 | |
| 2 | O | 0.80 | | 2.80 | | 2 | O | -0.61 | |
| 3 | Ti | 0.16 | | -1.20 | | 3 | Ti | 1.78 | |
| 3 | O | -0.43 | | -2.94 | | 3 | O | 1.67 | |
| 3 | Ba | | | 2.52 | | 3 | Pb | 1.52 | |
| O-terminated | | | | | | O-terminated | | | |
| 1 | O | -5.40 | -1.67 | -11.16 | -6.70 | 1 | O | -7.37 | -0.07 |
| 2 | Ti | -0.15 | -6.38 | -1.83 | -5.33 | 2 | Ti | 0.20 | -2.54 |
| 2 | Ba | 1.54 | -1.27 | 4.84 | -2.21 | 2 | Pb | 0.18 | -7.50 |
| 2 | O | 1.95 | 2.97 | 4.54 | 5.90 | 2 | O | 0.51 | 2.19 |
| 3 | O | 0.90 | 4.49 | 6.52 | 5.58 | 3 | O | -0.41 | 3.30 |

relative stability of surfaces with different stoichiometry.

With these definitions, and using the 7-layer slab geometries specified in Sec. II B, the energy of the unrelaxed BaTiO₃ (001) surface is

$$E_{\text{surf}}^{\text{unr}}(X) = \frac{1}{4}[E_{\text{slab}}^{\text{unr}}(\text{BaO}) + E_{\text{slab}}^{\text{unr}}(\text{TiO}_2) - 7E_{\text{bulk}}] \quad (1)$$

where $X = \text{BaO}$ or TiO_2 specifies the termination, $E_{\text{slab}}^{\text{unr}}(\text{BaO})$ and $E_{\text{slab}}^{\text{unr}}(\text{TiO}_2)$ are the unrelaxed BaO and TiO₂-terminated slab energies, E_{bulk} is energy per bulk BaTiO₃ unit cell, and the factor of four comes from the fact that four surfaces are created by the two cleavages needed to make the two slabs. The relaxation energy for each termination can be computed from the corresponding slab alone using

$$\Delta E_{\text{surf}}^{\text{rel}}(X) = \frac{1}{2}[E_{\text{slab}}(X) - E_{\text{slab}}^{\text{unr}}(X)], \quad (2)$$

where $E_{\text{slab}}(X)$ is a slab energy after relaxation. The relaxed surface energy is then

$$E_{\text{surf}}(X) = E_{\text{surf}}^{\text{unr}}(X) + \Delta E_{\text{surf}}^{\text{rel}}(X). \quad (3)$$

Similarly, for the BaTiO₃ (011) case, a cleavage on a bulk BaTiO plane gives rise to the complementary TiO and Ba-terminated surfaces shown in Fig. 2(c) and (d) respectively. Thus,

$$E_{\text{surf}}^{\text{unr}}(X) = \frac{1}{4}[E_{\text{slab}}^{\text{unr}}(\text{Ba}) + E_{\text{slab}}^{\text{unr}}(\text{TiO}) - 6E_{\text{bulk}}], \quad (4)$$

where the energy is the same for $X = \text{TiO}$ or Ba , $E_{\text{slab}}^{\text{unrel}}(\text{Ba})$ and $E_{\text{slab}}^{\text{unrel}}(\text{TiO})$ are energies of the unrelaxed slabs. Relaxation energies can again be computed independently for each slab in a manner similar to Eq. (2).

Finally, the (011) surface can also be cleaved to give two identical self-complementary O-terminated surfaces of the kind shown in Fig. 2(b). In this case the 7-layer slab has the stoichiometry of three bulk unit cells, so the relaxed surface energy of the O-terminated (011) surface is

$$E_{\text{surf}}(\text{O}) = \frac{1}{2}[E_{\text{slab}}(\text{O}) - 3E_{\text{bulk}}], \quad (5)$$

where $E_{\text{slab}}(\text{O})$ is the relaxed energy of the slab having two O-terminated surfaces. Everything said here about BaTiO₃ surfaces applies in exactly the same way to the corresponding PbTiO₃ surfaces.

The calculated surface energies of the relaxed BaTiO₃ (001) and (011) surfaces are presented in Table IV. In BaTiO₃, the relaxation energies of the TiO₂ and BaO-

TABLE IV: Calculated surface energies for BaTiO₃ and PbTiO₃ (001) and (011) surfaces (in eV per surface cell). ‘SM’ indicates comparative results from the shell-model calculation of Ref. [16].

| Surface | Termination | E_{surf} | E_{surf} (SM) |
|--------------------------|------------------|-------------------|------------------------|
| BaTiO ₃ (001) | TiO ₂ | 1.07 | 1.40 |
| | BaO | 1.19 | 1.45 |
| | | | |
| BaTiO ₃ (011) | TiO | 2.04 | 2.35 |
| | Ba | 3.24 | 4.14 |
| | O | 1.72 | 1.81 |
| PbTiO ₃ (001) | TiO ₂ | 0.74 | |
| | PbO | 0.83 | |
| PbTiO ₃ (011) | TiO | 1.36 | |
| | Pb | 2.03 | |
| | O | 1.72 | |

TABLE V: Calculated magnitudes of atomic displacements D (in Å), effective atomic charges Q (in e), and bond populations P between metal-oxygen nearest neighbors (in $10^{-3}e$) for the BaTiO₃ and PbTiO₃ (001) surfaces.

| Layer | Property | Ion | BaTiO ₃ (001) surface | | | Ion | PbTiO ₃ (001) surface | | |
|-------|----------|-----|----------------------------------|-----|----------------|-----|----------------------------------|-----|----------------|
| | | | TiO ₂ -terminated | Ion | BaO-terminated | | TiO ₂ -terminated | Ion | PbO-terminated |
| 1 | D | Ti | -0.123 | Ba | -0.080 | Ti | -0.111 | Pb | -0.150 |
| | Q | | 2.307 | | 1.752 | | 2.279 | | 1.276 |
| | P | | 126 | | -30 | | 114 | | 54 |
| | D | O | -0.014 | O | -0.025 | O | 0.012 | O | -0.012 |
| | Q | | -1.280 | | -1.473 | | -1.184 | | -1.128 |
| | P | | -38 | | 80 | | 44 | | 106 |
| 2 | D | Ba | 0.101 | Ti | 0.070 | Pb | 0.209 | Ti | 0.121 |
| | Q | | 1.767 | | 2.379 | | 1.275 | | 2.331 |
| | P | | -30 | | 88 | | 8 | | 80 |
| | D | O | 0.015 | O | 0.056 | O | 0.050 | O | 0.091 |
| | Q | | -1.343 | | -1.418 | | -1.167 | | -1.258 |
| | P | | 90 | | -30 | | 80 | | 6 |
| 3 | Q | Ti | 2.365 | Ba | 1.803 | Ti | 2.335 | Pb | 1.358 |
| | P | | 104 | | -36 | | 108 | | 24 |
| | Q | O | -1.371 | O | -1.417 | O | -1.207 | O | -1.259 |
| | P | | -34 | | 98 | | 18 | | 96 |
| Bulk | Q | Ba | 1.797 | Ba | 1.797 | Pb | 1.354 | Pb | 1.354 |
| | P | | -34 | | -34 | | 16 | | 16 |
| | Q | O | -1.388 | O | -1.388 | O | -1.232 | O | -1.232 |
| | P | | 98 | | 98 | | 98 | | 98 |
| | Q | Ti | 2.367 | Ti | 2.367 | Ti | 2.341 | Ti | 2.341 |
| | | | | | | | | | |

terminated surfaces (-0.23 and -0.11 eV respectively) are comparable, leading to rather similar surface energies. On the (011) surfaces, however, the relaxation energies vary more strongly with termination. For example, we find a relaxation energy of -2.13 eV for the TiO-terminated surface, much larger than -0.93 eV for the Ba-terminated surface. The relaxation energy of -1.15 eV for O-terminated surface gives rise to a relaxed energy of the O-terminated surface (1.72 eV) that is much lower than the average of the TiO and Ba-terminated surfaces (2.64 eV), indicating that it takes much less energy to cleave on an O₂ plane than on a BaTiO plane. The shell-model results of Ref. [16] for the BaTiO₃ surfaces are given shown for comparison; the results are qualitatively similar, but there are some significant quantitative differences, especially for the Ba-terminated (011) surface.

The corresponding results are also given for the (001) and (011) surfaces of PbTiO₃ in Table IV. The results for the (001) surfaces are similar to those for BaTiO₃, although the relaxed surface energies are somewhat lower. For the case of the (011) surfaces, however, we find a different pattern than for BaTiO₃. We find a very large relaxation energy of -1.75 eV for the TiO-terminated surface, compared with -1.08 eV for the Pb-terminated surface and -1.12 eV for the O-terminated surface. The average energy of the TiO and Pb-terminated surfaces is now 1.69 eV, to be compared with 1.72 eV for the O-terminated surface, indicating that the cleavage on a Pb-TiO or an O₂ plane has almost exactly the same energy cost.

D. BaTiO₃ and PbTiO₃ (001) and (011) surface charge distribution and chemical bonding

To characterize the chemical bonding and covalency effects, we used a standard Mulliken population analysis for the effective static atomic charges Q and other local properties of the electronic structure as described, for example, in Ref. [39,40]. The results are presented in Table V. Our calculated effective charges for bulk PbTiO₃ are $+1.354e$ for the Pb atom, $+2.341e$ for the Ti atom, and $-1.232e$ for the O atom. The bond population describing the chemical bonding is $+98me$ between Ti and O atoms, $+16me$ between Pb and O atoms, and $+2me$ between Pb and Ti atoms. Our calculated effective charges for bulk BaTiO₃ are $+1.797e$ for the Ba atom, $+2.367e$ for the Ti atom, and $-1.388e$ for the O atom indicate a high degree of BaTiO₃ chemical bond covalency. The bond population between Ti and O atoms in BaTiO₃ bulk is exactly the same as in PbTiO₃, while that between Ba and Ti is slightly negative, suggesting a repulsive interaction between these atoms in the bulk of the BaTiO₃ crystal.

For the TiO₂-terminated BaTiO₃ and PbTiO₃ (001) surfaces, the major effect observed here is a strengthening of the Ti-O chemical bond near the BaTiO₃ and PbTiO₃ (001) surfaces, which was already pronounced for the both materials in the bulk. Note that the Ti and O effective charges for bulk BaTiO₃ and PbTiO₃ are much smaller than those expected in an ionic model ($+4e$, and $-2e$), and that the Ti-O chemical bonds in bulk BaTiO₃ and PbTiO₃ are fairly heavily populated

TABLE VI: The A - B bond populations P (in $10^{-3}e$) and interatomic distances R (in Å) on (011) surfaces of BaTiO_3 and PbTiO_3 . Symbols I-IV denote the number of each plane enumerated from the surface. The nearest-neighbor Ti-O distance is 2.004 Å and 1.968 Å in bulk BaTiO_3 and PbTiO_3 , respectively.

| BaTiO ₃ (011) surface | | | | PbTiO ₃ (011) surface | | | |
|----------------------------------|---------|-----|-------|----------------------------------|---------|-----|-------|
| Atom A | Atom B | P | R | Atom A | Atom B | P | R |
| TiO-terminated | | | | TiO-terminated | | | |
| Ti(I) | O(I) | 130 | 2.047 | Ti(I) | O(I) | 132 | 2.019 |
| | O(II) | 198 | 1.784 | | O(II) | 196 | 1.766 |
| O(II) | Ti(III) | 112 | 2.009 | O(II) | Ti(III) | 120 | 1.948 |
| | Ba(III) | -24 | 2.808 | | Pb(III) | 24 | 2.826 |
| | O(III) | -26 | 2.837 | | O(III) | -20 | 2.857 |
| Ti(III) | Ba(III) | -2 | 3.471 | Ti(III) | Pb(III) | 2 | 3.410 |
| | O(III) | 118 | 2.004 | | O(III) | 108 | 1.975 |
| | O(IV) | 96 | 2.004 | | O(IV) | 88 | 1.976 |
| Ba(III) | O(III) | -32 | 2.834 | Pb(III) | O(III) | 20 | 2.783 |
| | O(IV) | -38 | 2.816 | | O(IV) | 8 | 2.734 |
| O(III) | O(IV) | -30 | 2.834 | O(III) | O(IV) | -36 | 2.706 |
| Ba-terminated | | | | Pb-terminated | | | |
| Ba(I) | O(II) | -38 | 2.664 | Pb(I) | O(II) | 126 | 2.589 |
| O(II) | Ba(III) | -36 | 2.850 | O(II) | Pb(III) | 24 | 2.742 |
| | Ti(III) | 84 | 2.022 | | Ti(III) | 74 | 1.902 |
| | O(III) | -38 | 2.859 | | O(III) | -46 | 2.739 |
| Ba(III) | O(III) | -36 | 2.834 | Pb(III) | O(III) | -10 | 2.783 |
| | O(IV) | -36 | 2.834 | | O(IV) | 36 | 2.813 |
| Ti(III) | O(III) | 76 | 2.004 | Ti(III) | O(III) | 62 | 1.968 |
| | Ba(III) | -2 | 3.471 | | Pb(III) | 0 | 3.408 |
| | O(IV) | 98 | 2.008 | | O(IV) | 92 | 2.018 |
| O(III) | O(IV) | -46 | 2.825 | O(III) | O(IV) | -46 | 2.816 |
| O-terminated | | | | O-terminated | | | |
| O(I) | Ba(II) | -26 | 2.697 | O(I) | Pb(II) | 50 | 2.503 |
| | Ti(II) | 168 | 1.722 | | Ti(II) | 128 | 1.694 |
| | O(II) | -24 | 2.801 | | O(II) | -26 | 2.689 |
| Ba(II) | O(II) | -40 | 2.664 | Pb(II) | O(II) | 78 | 2.574 |
| | Ti(II) | -2 | 3.306 | | Ti(II) | 4 | 3.094 |
| Ti(II) | O(II) | 82 | 2.040 | Ti(II) | O(II) | 92 | 1.977 |
| | O(III) | 112 | 1.689 | | O(III) | 126 | 1.831 |
| O(II) | O(III) | -12 | 2.825 | O(II) | O(III) | -24 | 2.779 |
| Ba(II) | O(III) | -10 | 2.968 | Pb(II) | O(III) | 26 | 2.716 |
| O(III) | O(IV) | -14 | 2.945 | O(III) | O(IV) | -42 | 2.842 |
| | Ti(IV) | 60 | 2.159 | | Ti(IV) | 60 | 2.051 |
| | Ba(IV) | -24 | 2.767 | | Pb(IV) | -2 | 2.712 |

for both materials.

The Ti-O bond population for the TiO_2 -terminated BaTiO_3 and PbTiO_3 (001) surfaces are +126 me and +114 me respectively, which is about 20% larger than the relevant value in the bulk. In contrast, the Pb-O bond population of +54 me is small for the PbO-terminated PbTiO_3 (001) surface, and the Ba-O bond population of -30 me is even negative for the BaO-terminated BaTiO_3 (001) surface, indicating a repulsive character. The effect of the difference in the chemical bonding is also well seen from the Pb and Ba effective charges in the first surface layer, which are close to the formal ionic charge of +2 e only in the case of the BaTiO_3 crystal.

The interatomic bond populations for three possible

TABLE VII: Calculated Mulliken atomic charges Q (in e) and changes in atomic charges ΔQ with respect to the bulk charges (in e) on (011) surfaces of BaTiO_3 and PbTiO_3 . for three terminations. The Mulliken charges are 2.341 e for Ti, -1.232 e for O, and 1.354 e for Pb in bulk PbTiO_3 , and 2.367 e for Ti, -1.388 e for O, and 1.797 e for Ba in bulk BaTiO_3 .

| BaTiO ₃ (011) surface | | | PbTiO ₃ (011) surface | | |
|----------------------------------|--------|------------|----------------------------------|--------|------------|
| Atom | Q | ΔQ | Atom | Q | ΔQ |
| TiO-terminated | | | TiO-terminated | | |
| Ti(I) | 2.216 | -0.151 | Ti(I) | 2.212 | -0.129 |
| O(I) | -1.316 | 0.072 | O(I) | -1.261 | -0.029 |
| O(II) | -1.155 | 0.233 | O(II) | -1.057 | 0.175 |
| Ba(III) | 1.757 | -0.04 | Pb(III) | 1.253 | -0.101 |
| Ti(III) | 2.353 | -0.014 | Ti(III) | 2.328 | -0.013 |
| O(III) | -1.299 | 0.089 | O(III) | -1.18 | 0.052 |
| O(IV) | -1.402 | -0.014 | O(IV) | -1.239 | -0.007 |
| Ba-terminated | | | Pb-terminated | | |
| Ba(I) | 1.636 | -0.161 | Pb(I) | 1.122 | -0.232 |
| O(II) | -1.483 | -0.095 | O(II) | -1.140 | 0.092 |
| Ba(III) | 1.799 | 0.002 | Pb(III) | 1.340 | -0.014 |
| Ti(III) | 2.368 | 0.001 | Ti(III) | 2.343 | 0.002 |
| O(III) | -1.446 | -0.058 | O(III) | -1.277 | -0.045 |
| O(IV) | -1.392 | -0.004 | O(IV) | -1.247 | -0.015 |
| O-terminated | | | O-terminated | | |
| O(I) | -1.158 | 0.23 | O(I) | -1.011 | 0.221 |
| Ba(II) | 1.766 | -0.031 | Pb(II) | 1.257 | -0.097 |
| Ti(II) | 2.213 | -0.154 | Ti(II) | 2.237 | -0.104 |
| O(II) | -1.452 | -0.064 | O(II) | -1.261 | -0.029 |
| O(III) | -1.317 | 0.071 | O(III) | -1.215 | 0.017 |
| Ba(IV) | 1.792 | -0.005 | Pb(IV) | 1.355 | 0.001 |
| Ti(IV) | 2.317 | -0.05 | Ti(IV) | 2.317 | -0.024 |
| O(IV) | -1.407 | -0.019 | O(IV) | -1.233 | -0.001 |

BaTiO_3 and PbTiO_3 (011) surface terminations are given in Table VI. The major effect observed here is a strong increase of the Ti-O chemical bonding near the BaTiO_3 and PbTiO_3 (011) surface as compared to already large bonding near the (001) surface (+126 me and +114 me, respectively) and in the bulk (+98 me). For the O-terminated (011) surface the O(I)-Ti(II) bond population is as large as +168 me for BaTiO_3 and +128 me for PbTiO_3 , i.e., considerably larger than in the bulk and on the (001) surface.

Our calculations demonstrate that for the TiO-terminated BaTiO_3 and PbTiO_3 (011) surfaces, the Ti-O bond populations are larger in the direction perpendicular to the surface (+198 me for BaTiO_3 and +196 me for PbTiO_3) than in plane (+130 me for BaTiO_3 and +132 me for PbTiO_3). The Ti-O bond populations for the TiO-terminated PbTiO_3 (011) surface in the direction perpendicular to the surface is twice as large as the Ti-O bond population in PbTiO_3 bulk.

In Table VII we present the calculated Mulliken effective charges Q , and their changes ΔQ with respect to the bulk values, near the surface. We analyzed the charge redistribution between different layers in slabs with all three BaTiO_3 and PbTiO_3 (011) surface terminations. The charge of the surface Ti atoms in the TiO-terminated

BaTiO₃ and PbTiO₃ (001) surface is reduced by $0.151e$ and $0.129e$, respectively. Metal atoms in the third layer lose much less charge. Except in the central layer (and, in the case of PbTiO₃, in the subsurface layer), the O ions also reduce their charges, becoming less negative. The largest charge change is observed for BaTiO₃ and PbTiO₃ subsurface O atoms ($+0.233e$ and $+0.175e$, respectively). This gives a large positive change of $+0.466e$ and $+0.350e$ in the charge for each BaTiO₃ and PbTiO₃ subsurface layer.

On the Ba-terminated and Pb-terminated BaTiO₃ and PbTiO₃ (011) surface, negative changes in the charge are observed for all atoms except for Ba and Ti in the BaTiO₃ third layer, Ti atom in the PbTiO₃ third layer, and subsurface oxygen atom in PbTiO₃. The largest charge changes are at the surface Ba and Pb ions. It is interesting to notice that, due to the tiny difference in the chemical bonding between BaTiO₃ and PbTiO₃ perovskites, the charge change for the BaTiO₃ subsurface O ion ($-0.095e$) and PbTiO₃ subsurface O ion ($+0.092e$) have practically the same magnitude, but opposite signs.

For the O-terminated BaTiO₃ and PbTiO₃ (011) surfaces, the largest calculated changes in the charge are observed for the BaTiO₃ and PbTiO₃ surface O atom ($+0.230e$ and $+0.221e$, respectively). The change of the total charge in the second layer is negative and almost equal for both materials. For the BaTiO₃ crystal, this reduction by $0.249e$ comes mostly from Ti atom ($-0.154e$). In the PbTiO₃ crystal, the reduction by $0.230e$ appears mostly due to a decrease of the Ti atom charge by $0.104e$, as well as a reduction of the Pb atom charge by $0.097e$.

IV. CONCLUSIONS

In summary, motivated by the scarcity of experimental investigations of the BaTiO₃ and PbTiO₃ surfaces and the contradictory experimental results obtained for the related SrTiO₃ surface,^{19,20} we have carried out predictive electronic structure calculation to investigate the surface atomic and electronic structure of the BaTiO₃ and PbTiO₃ (001) and (011) surfaces. Using a hybrid B3PW approach, we have calculated the surface relax-

ation of the two possible terminations (TiO₂ and BaO or PbO) of the BaTiO₃ and PbTiO₃ (001) surfaces, and three possible terminations (TiO, Ba or Pb, and O) of the BaTiO₃ and PbTiO₃ (011) surfaces. The data obtained for the surface structures are in a good agreement with previous LDA calculations of Padilla and Vanderbilt,⁴ the LDA plane-wave calculations of Meyer *et al.*,⁵ and in fair agreement with the shell-model calculations of Heifets *et al.*¹⁶

According to our calculations, atoms of the first surface layer relax inwards for BaO and PbO terminated (001) surfaces of both materials. Outward relaxations of all atoms in the second layer are found at both terminations of BaTiO₃ and PbTiO₃ (001) surfaces. In BaTiO₃, the rumpling of the TiO₂-terminated (001) surface is predicted to exceed that of the BaO-terminated (001) surface by a factor of two. In contrast, PbTiO₃ exhibits practically the same rumplings for both (TiO₂ and PbO) terminations. Our calculated surface energies show that the TiO₂-terminated (001) surface is slightly more stable for both materials than the BaO or PbO-terminated (001) surface. The O-terminated BaTiO₃ and TiO-terminated PbTiO₃ (011) surfaces have surface energies close to that of the (001) surface. Our calculations suggest that the most unfavorable (011) surfaces are the Ba or Pb-terminated ones for both the BaTiO₃ and PbTiO₃ cases. We found that relaxation of the BaTiO₃ and PbTiO₃ surfaces for is considerably stronger for all three (011) terminations than for the (001) surfaces. The atomic displacements in the third plane from the surface for the three terminations of BaTiO₃ and PbTiO₃ (011) surfaces are still large. Finally, our *ab initio* calculations indicate a considerable increase of Ti-O bond covalency near the BaTiO₃ and PbTiO₃ (011) surface relative to BaTiO₃ and PbTiO₃ bulk, much larger than for the (001) surface.

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