Some differential equations and their solutions used in intro physics.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\frac{d^2\psi}{dx^2} + \left[k^2\right] \psi = 0$$

$$\frac{df}{dt} + \frac{1}{\tau} f = 0$$

$$\frac{dN}{dt} + \lambda N = 0$$

$$\frac{dQ}{dt} + \frac{Q}{[RC]} = 0$$

Aside on solution of differential equation

Schrödinger Equation for free particle

$$\frac{p^2}{2m}\ \psi(x) = E\ \psi(x)$$

$$\frac{p^2}{2m}\ \psi - E\psi=0$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - E\psi=0$$

$$\frac{d^2\psi}{dx^2} + \left[\frac{2mE}{\hbar^2}\right]\psi=0$$

$$k=\sqrt{\frac{2mE}{\hbar^2}}=\frac{2\pi}{\lambda}$$

$$\frac{d^2\psi}{dx^2} + \left[k^2\right]\psi=0$$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

Newton's Equation for harmonic oscillator

$$F=ma=-kx$$

$$ma+kx=0$$

$$a+[\frac{k}{m}]x=0$$

$$\frac{d^2x}{dt^2}+[\frac{k}{m}]x=0 \qquad \omega=\sqrt{\frac{k}{m}}$$

$$\frac{d^2x}{dt^2}+\omega^2\ x=0$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

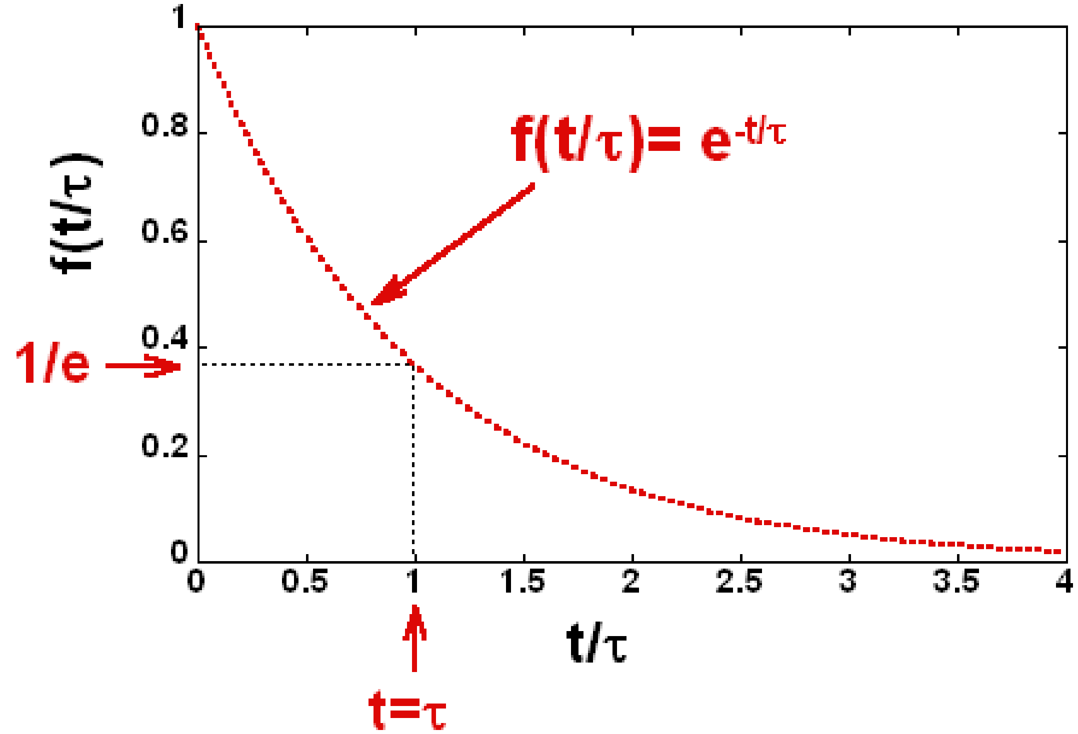
or $x(t) = A\sin(\omega t + \delta)$

Exponential Function

$$f(t) = e^{-\frac{t}{\tau}}$$

τ = time constant

$$f(t=\tau) = e^{-1} = \frac{1}{e} = \frac{1}{2.732}$$



$$\frac{df}{dt} = -\frac{1}{\tau} f$$

$$\frac{df}{dt} + \frac{1}{\tau} f = 0$$

or

$$\frac{\Delta f}{\Delta t} = -\frac{1}{\tau} f$$

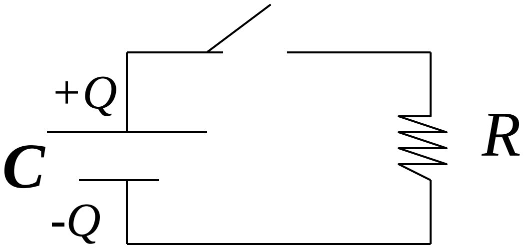
$$\frac{df}{f} = -\frac{1}{\tau} dt \qquad \int \frac{df}{f} = -\frac{1}{\tau} \int dt$$

$$\ln(f) = -\frac{t}{\tau} \qquad f(t) = e^{-\frac{t}{\tau}}$$

Note: gen. soln. $f(t) = f_0 \ e^{-\frac{t}{\tau}}$ (f_0= constant)

Time varying electrical current Example: Capacitor discharge

t=0 close switch



$0=V_c+V_R$

t=0 Q=Q_0
$0=\frac{Q}{C}+IR$

$Q\to 0$, $t=\infty$

$I=\frac{dQ}{dt}$

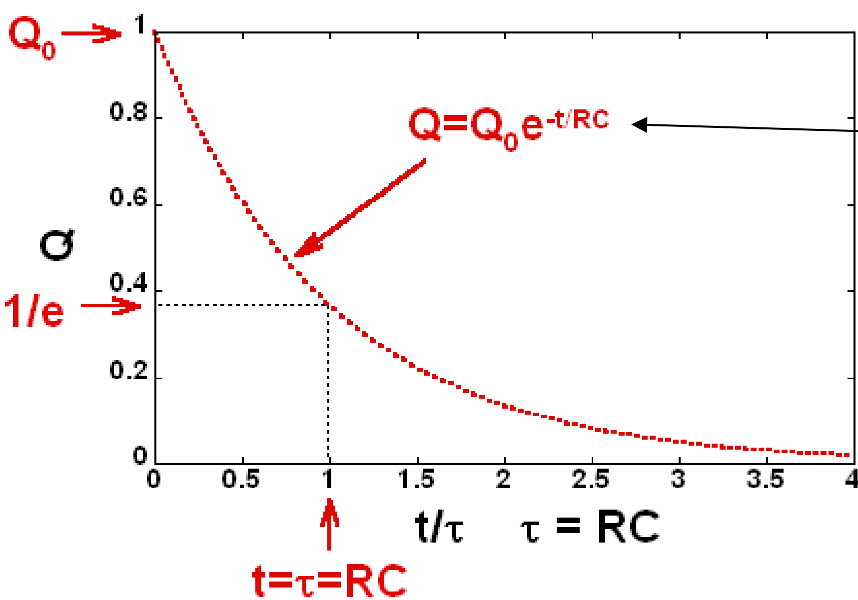
$$\frac{dQ}{dt}+\frac{Q}{[RC]}=0$$

τ=RC=time constant

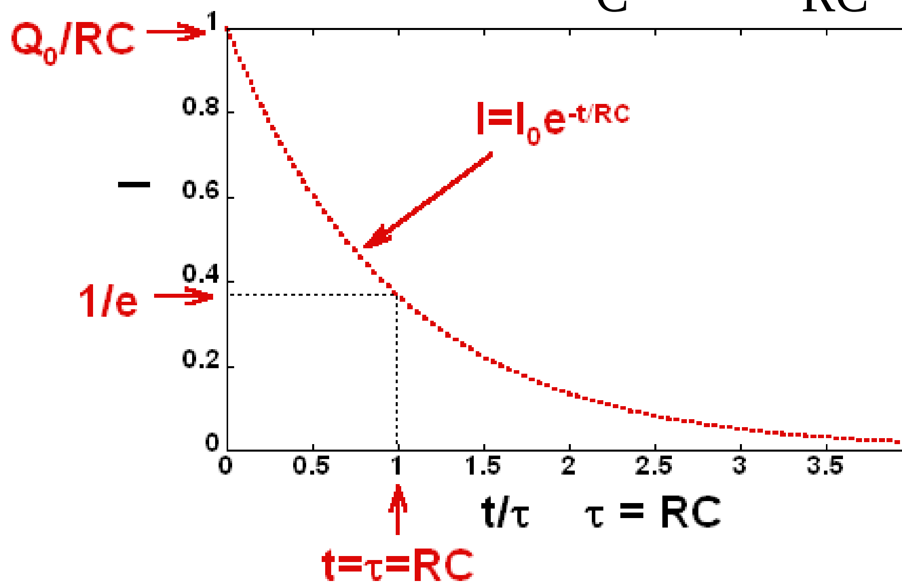
$$Q=[?]\ e^{-\frac{t}{Rc}}$$

$$t=0 \quad Q=Q_0 \Rightarrow [?]=Q_0$$

$$\therefore Q=Q_0e^{-\frac{t}{RC}}$$

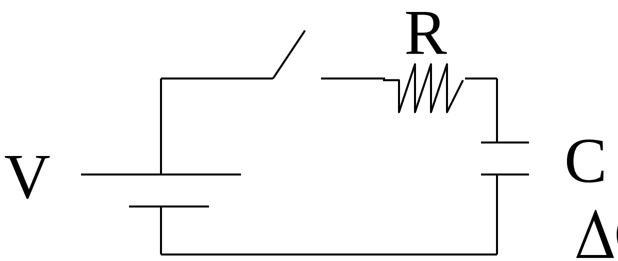


$$\text{Recall } IR=\frac{Q}{C}\to I=-\frac{Q}{RC}$$



t=0 close switch.

Capacitor Charging



$$-V + IR + \frac{Q}{C} = 0$$

Again: $I = \frac{\Delta Q}{\Delta t}$

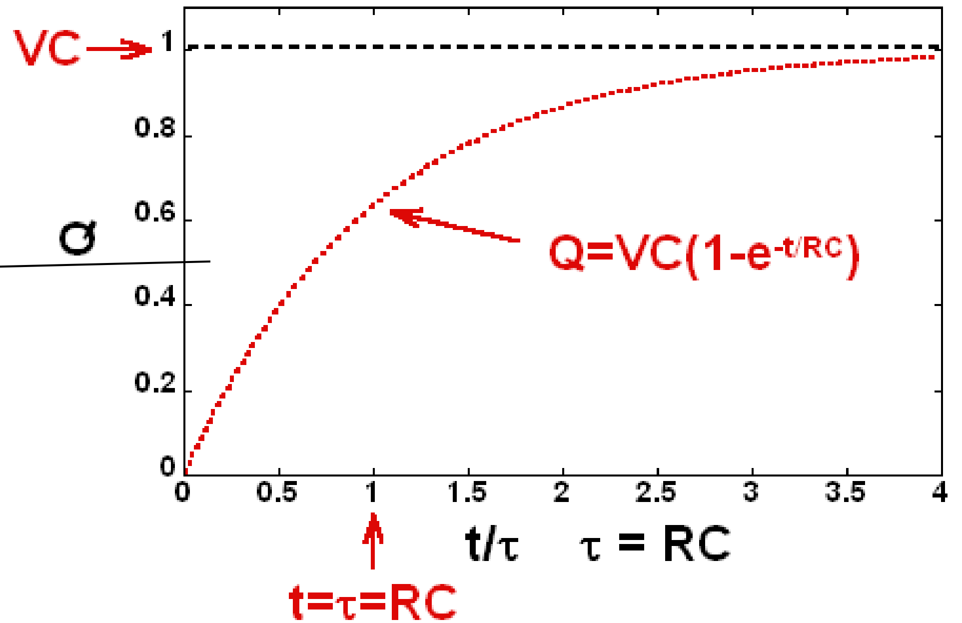
$$\frac{\Delta Q}{\Delta t} R + \frac{Q}{C} = V \Rightarrow \frac{\Delta Q}{\Delta t} + \frac{Q}{RC} = \frac{V}{R} \Rightarrow \frac{dQ}{dt} + \frac{Q}{RC} = \frac{V}{R}$$

Very similar to before but not identical

t=0 Q=0: t=∞ I=0

ie. $Q_o = VC$

$$Q = VC\,(1 - e^{-\frac{t}{RC}})$$



$$I = -\frac{Q}{RC} + \frac{V}{R} = -\frac{V}{R}(1 - e^{-\frac{t}{RC}}) + \frac{V}{R}$$

$$I = \frac{V}{R} e^{-\frac{t}{RC}}$$

<u>**Radioactive decay**</u>: a stochastic statistical process

-a given nuclei decays randomly and independently of others

-constant statistical decay rate (probability per unit time)

{short time compared to time when ~ nothing left)

⇒1) the more nuclei the more decays

2) the longer the time the more decays.

1) ⇒ N radioactive nuclei at time t , then ΔN~N

2) ⇒ ΔN ~Δt

$$\Rightarrow \quad \Delta N = -\lambda N \, \Delta t$$

$$\Rightarrow \quad dN = -\lambda N \, dt$$

$$\Rightarrow \quad \frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \quad \frac{dN}{dt} + \lambda N = 0$$

Have seen this before !!

13-14a

Poisson process: every object has a fixed probability of decaying in a given time [Remember the Fr. mathematician Siméon Denis Poisson (1781 – 1840) who was wrong about " Poissions' bright spot".]

Exponential Function **remember**

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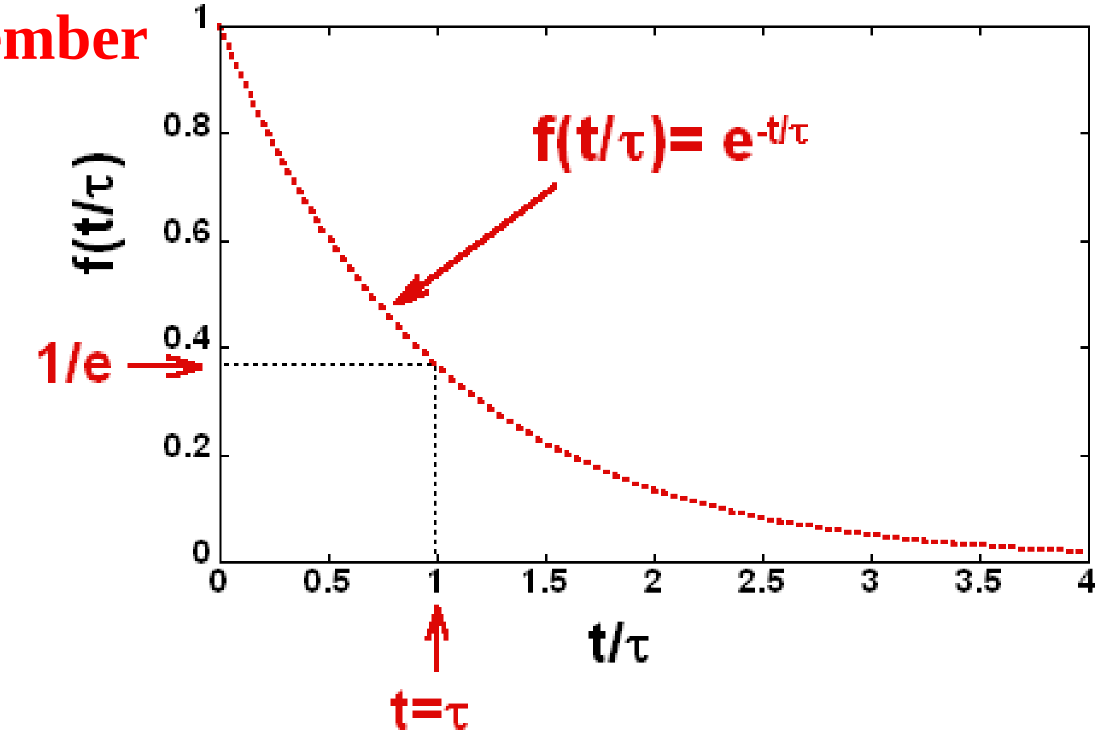

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