# Einstein's Special Theory of Relativity

**Postulate 1: The laws of physics have the same form in all inertial reference frames.**

**Postulate 2: Light propagates through empty space with a definite speed (c) independent of the speed of the source or of the observer.**

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**Modifications to Newtonian physics as speed approaches c.**

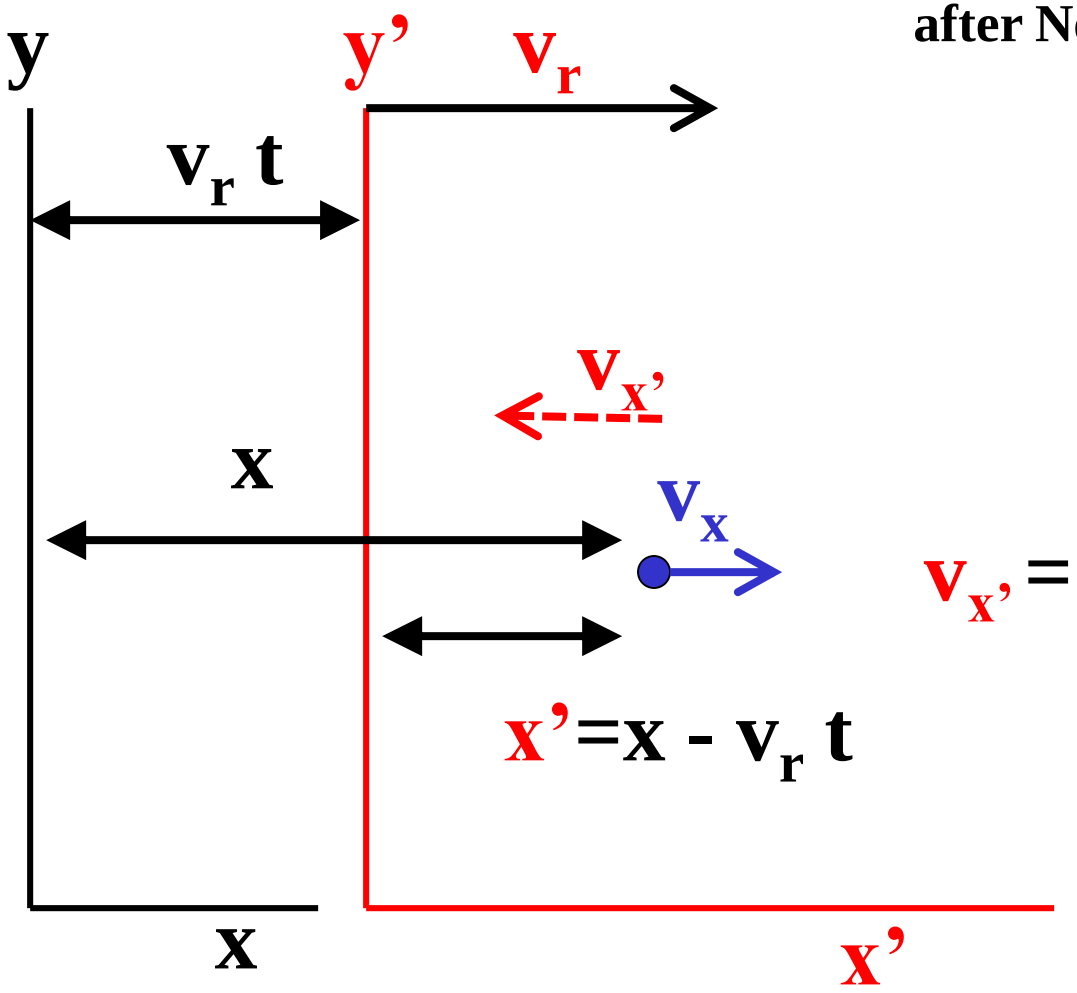
| Newton | Einstein |
|---|---|
| $E = \frac{1}{2}mv^2$ | $E = m^*c^2$ <br> $KE = m^*c^2 - m_0c^2$ <br> $m^* = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ |
| $p = mv$ | $p = m^*v$ |
| $L = L_0$ | $L = L_0\sqrt{1 - v^2/c^2}$ |
| $\Delta t = \Delta t_0$ | $\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$ |

$$E = \frac{p^2}{2m}$$

$$E^2 = p^2c^2 + m_0^2c^4$$

# Inertial reference frame ( constant v, a=0, F=ma is ok)

after Newton & Galileo




$y' = y$

$v_{y'} = v_y$

$v_{x'} = v_x - v_r$

Viewed from moving reference frame

Initial and final "conditions" (x, v) change

Problem transforms but physics the same ( F=ma same !!!)

original reference frame

object dropped

$v_i = 0$

accelerated downward motion

g

$y = y_i - gt^2/2$

$v_y = -gt$

moving reference frame

object projected upward

$v'_i = -v_r$

g

$y' = y_i' + v_r t - gt^2/2$

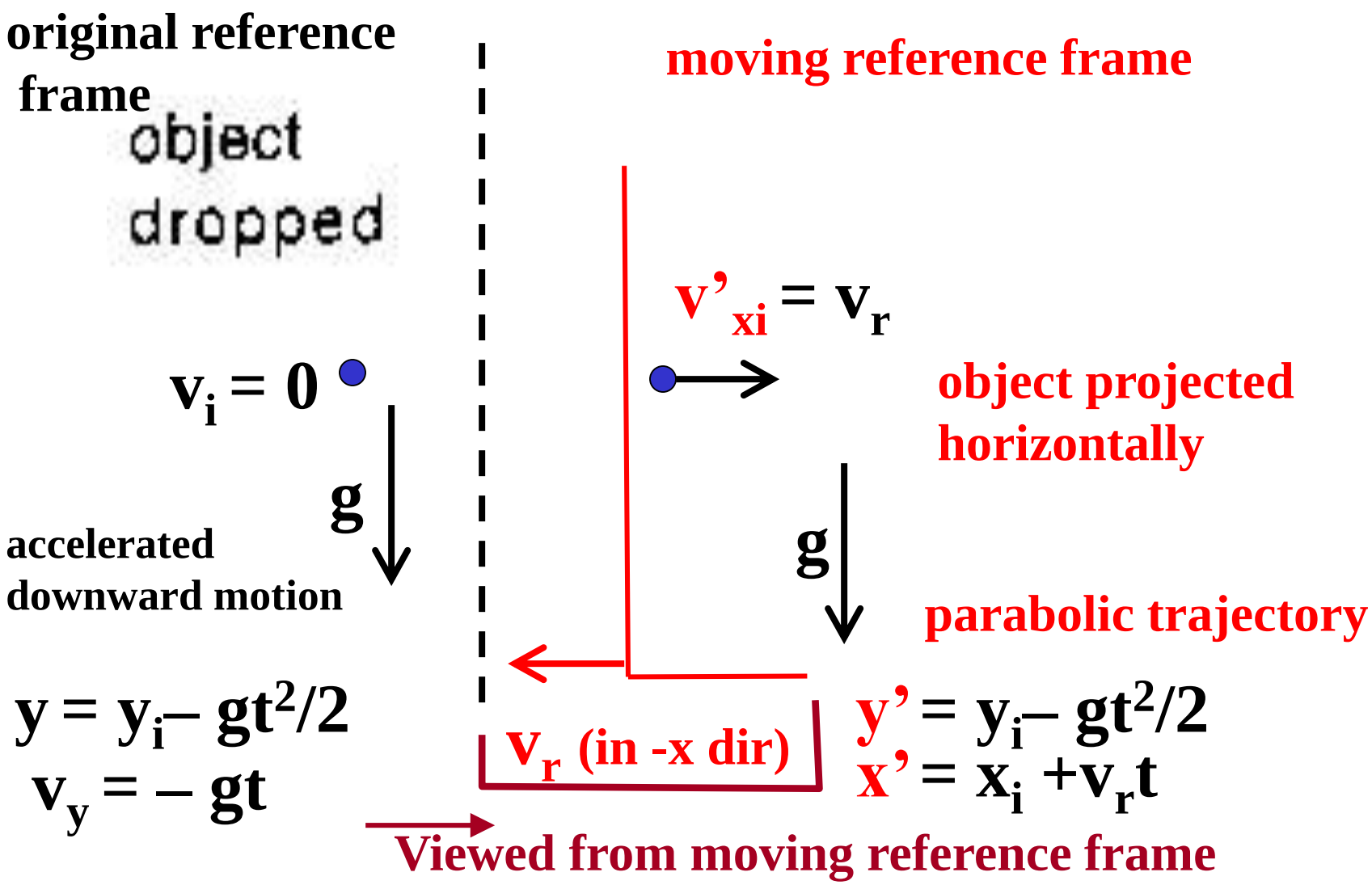
$v_y' = v_r - gt$

$v_r$ (in – y dir)

upward motion stop downward

Viewed from moving reference frame

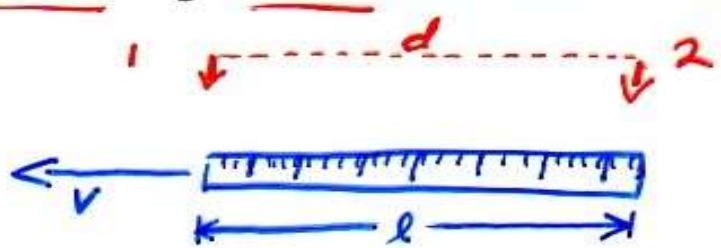
Problem transforms but physics the same ( F=ma same !!!)




**Problem transforms but physics the same ( F=ma same !!!)**

Illustration- measure length of moving ruler

- 1-flashes light to 2 when the end of the ruler is opposite to him

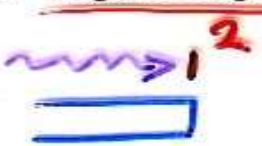



- adjust positions until 2 sees other end opposite to him at the same time he gets light signal from 1

What really happens

t=0 1-sends signal

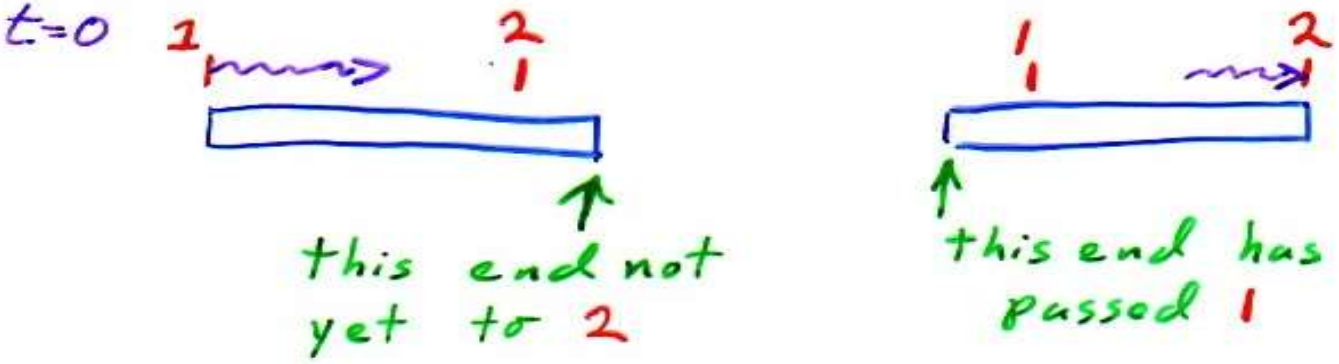

time-t later 2 gets signal & sees end




It took light time-t to travel distance d

d= ct distance = velocity X time

∴ ruler must have moved in time t



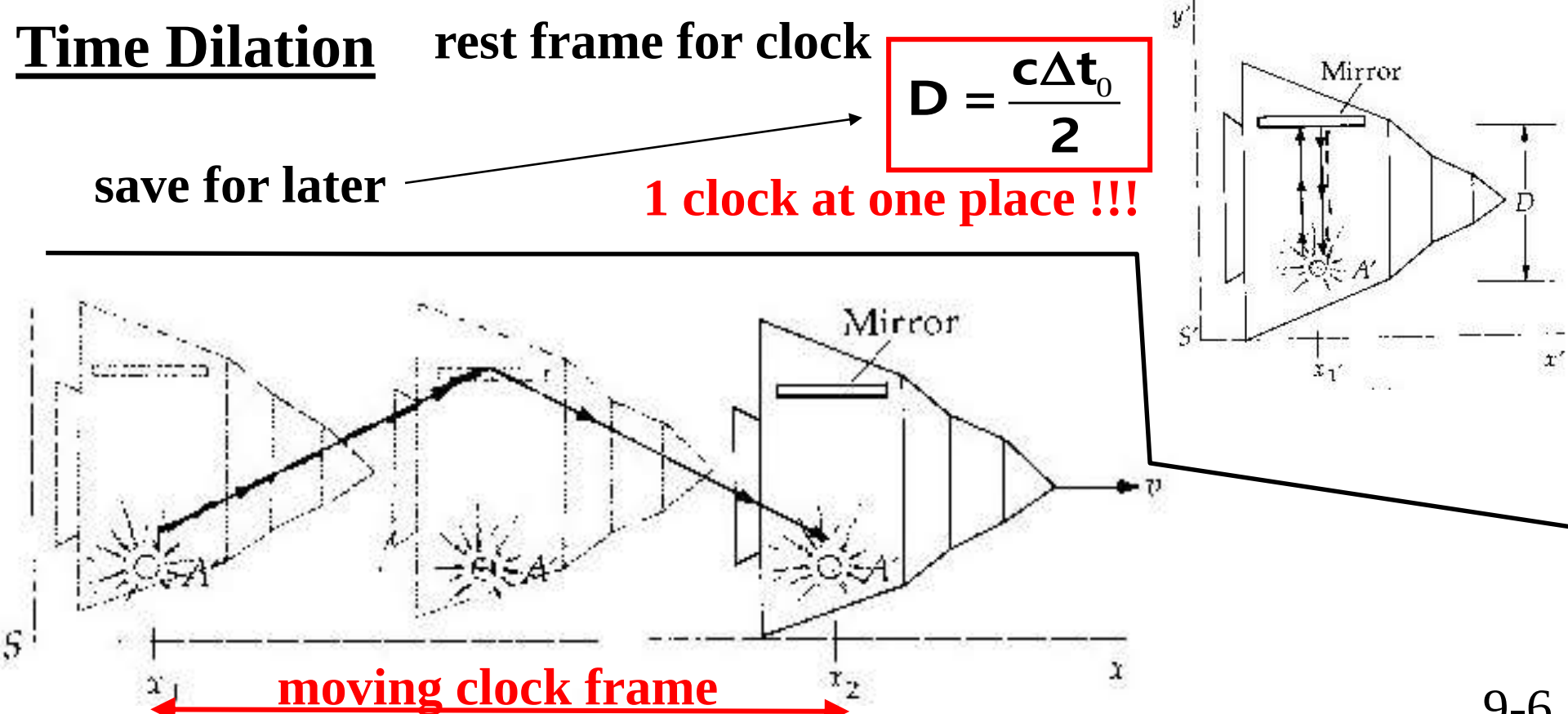
# Time Dilation

rest frame for clock

$$D = \frac{c\Delta t_0}{2}$$

save for later

1 clock at one place !!!




moving clock frame

clock 1
record time

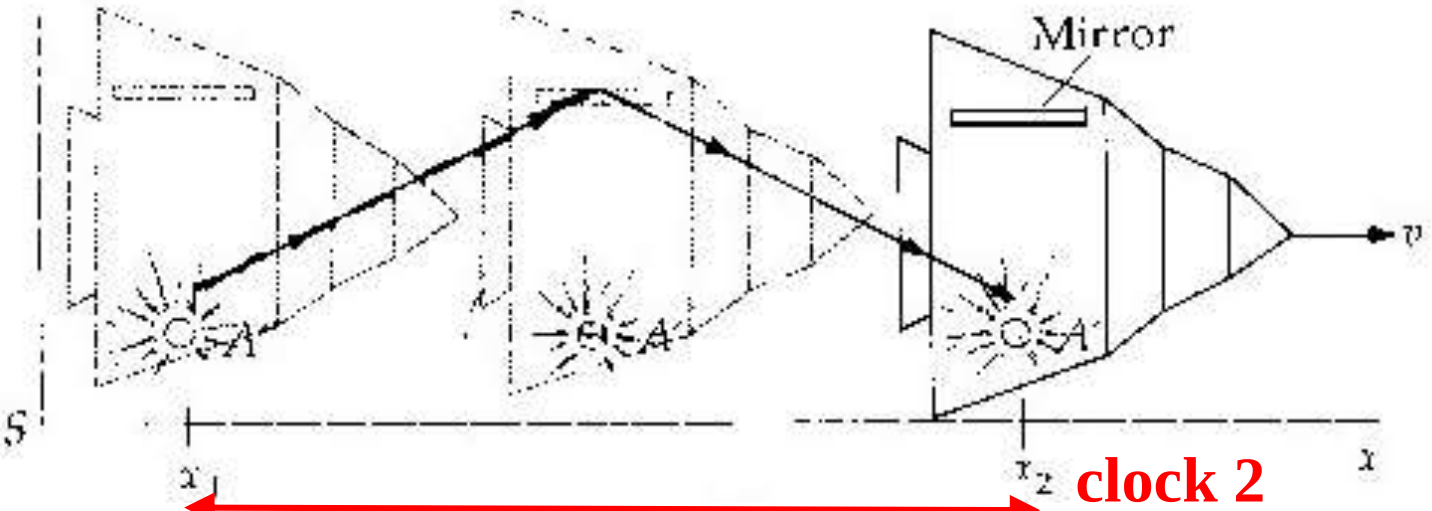
clock 2
record time

clock 1 & 2

a) had to be synchronized before experiment (at same place)

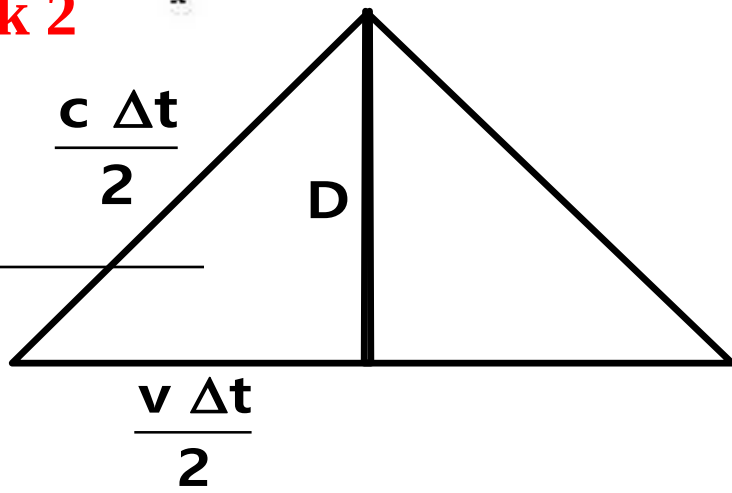
b) have to be brought together and times compared after

# Time Dilation




9-6a

clock 1 — moving clock frame (2 clocks) — clock 2




$$\left(\frac{c\Delta t}{2}\right)^2 = D^2 + \left(\frac{v\Delta t}{2}\right)^2$$

recall $D = \frac{c\Delta t_0}{2}$

$$D^2 = \frac{\Delta t^2\, c^2}{2^2}\left(1 - \frac{v^2}{c^2}\right)$$

$$\therefore \quad \frac{\Delta t_0{}^2\, c^2}{2^2} = \frac{c^2 \Delta t^2}{2^2}\left(1 - \frac{v^2}{c^2}\right)$$

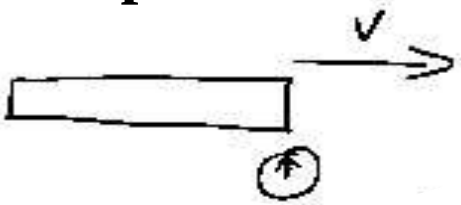
$$\Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

**time dilated in moving ref. frame !!!**

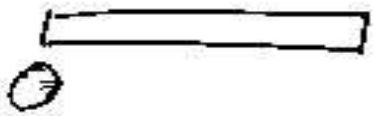
# Experiment: measure the length of a ruler with a moving clock



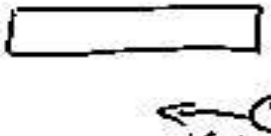
## Length contraction




Clock start



Clock stop




Clock start

Clock stop
(when it reaches other end

rest frame for clock:
moving frame for ruler

$$L = \{v\ \Delta t_0\}$$

rest frame for ruler:
moving frame for clock (actually 2 clocks)

$$L_0 = v\,\Delta t$$

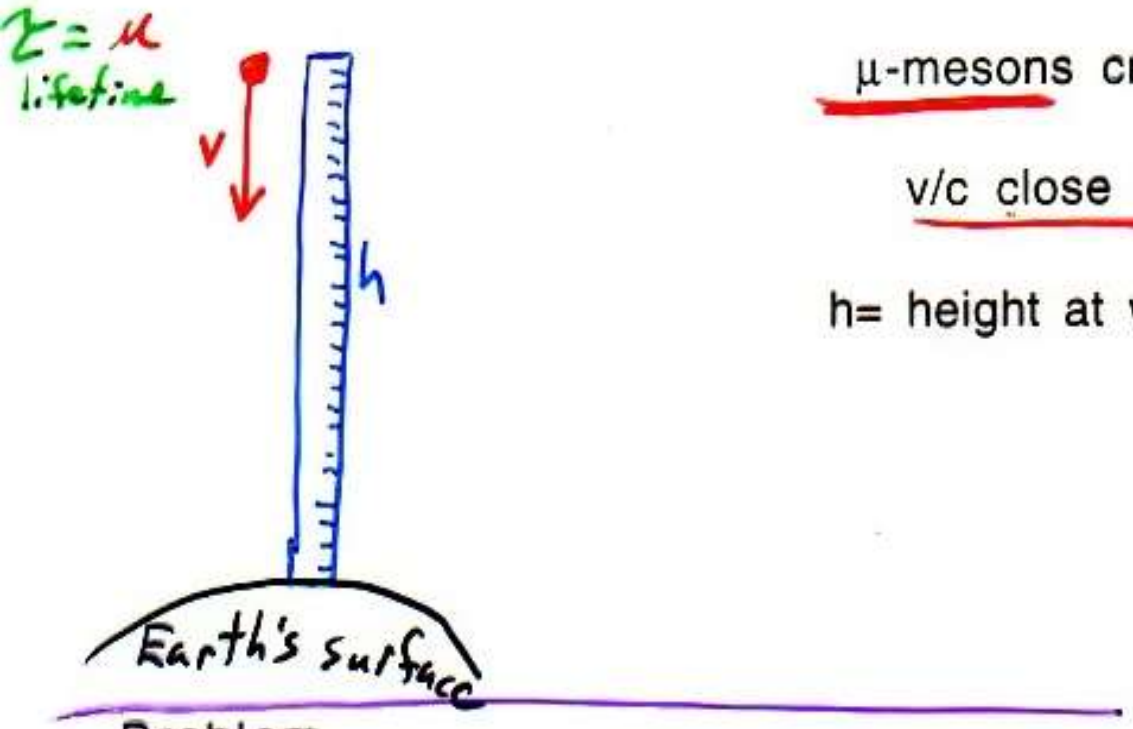
$$\therefore \quad L_0 = \frac{\{\ v\ \Delta t_0\}}{\sqrt{1 - v^2/c^2}}$$

but $$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

so $$L_0 = \frac{L}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \quad L = L_0\sqrt{1 - v^2/c^2}$$ less than 1 ( length contracts)

Classic example of problem
The case of the μ-mesons that live too long




μ-mesons created in upper atmosphere

v/c close to 1 so relativity important

h= height at which μ-meson created

Problem
- know h at which μ-meson created
- know speed v
- know how long μ-mesons live (half life) ~ τ

--> ≈ **None of the μ-mesons should reach the earth's surface**
(they should die first)

**But lots of μ-mesons reach the surface !!!!!**

# μ-meson problem (time dilation viewpoint)



clock at rest on the μ-meson - lives life at one spot (from its view point)

"proper or rest frame of reference" for the clock

- all reference frames moving with respect to the proper frame will measure a dilated or lengthened time

clock rest frame

**μ-meson "thinks" he lives just the right lifetime**


clock moving frame

time measured is dilated longer time

**observer "thinks" μ-meson lives too long**

## μ-meson problem (length contraction viewpoint)

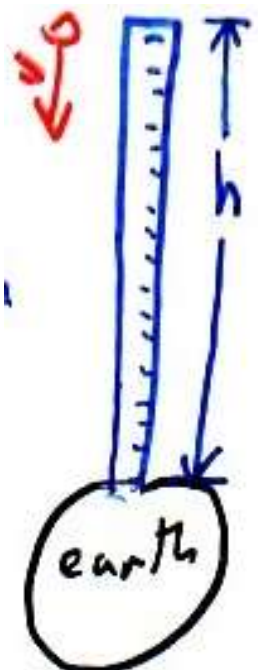
ruler at rest with respect to the earth

earth is rest frame for ruler

all moving frames will measure a contracted (shorter) length

ruler rest frame

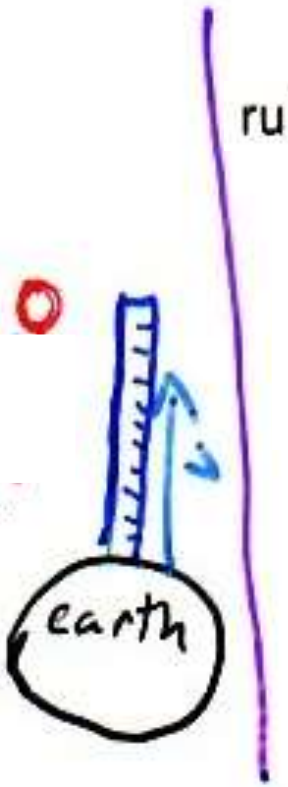
ruler - full length = h




ruler moving frame

**μ-meson sees contracted ruler running past**

μ-meson "says" it is easy to cover this shortened distance in his lifetime!!!



# μ meson: time dilation view

$\mu$ - meson

$\tau_0 \approx 2.2(10)^{-6}$ s

**v = .998 c = (.998) 3 (10)$^8$ m/s**

**Guess ( wrong) how far it should travel**

$\tau_0 \cdot v = (.998)\ 3(10)^8\ 2.2(10)^{-6} = 660$ m

**Actually we see μ born at 1 place and die at another**

**∴ we are in the moving reference frame for the clock.**

**Time dilation**

$$\tau = \frac{\tau_0}{\sqrt{1 - v^2/c^2}} = \frac{\tau_0}{\sqrt{1-(.998)^2}} = \tau_0\, 16$$

**∴ we see it travel**

$$\tau v = \tau_0 \cdot 16 \cdot v \approx 11 \text{ km} \quad !!$$

# μ meson: length contraction view



$\tau_0 = 2.2(10)^{-6}$ s is lifetime

$L_0$

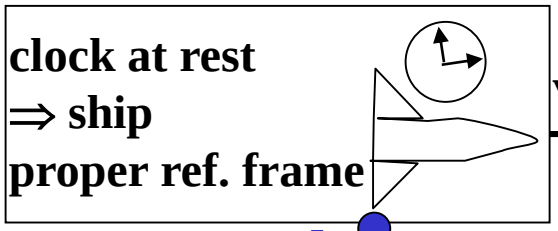
**How long does the 11 km distance to Earth look to the μ-meson who views it in a moving reference frame?**

$$L = L_0\sqrt{1 - v^2/c^2} \quad \left.\begin{array}{l} \\ v = .998c \end{array}\right\} \Rightarrow L = L_0 \frac{1}{16}$$

$$\therefore \quad L = \frac{11\ \text{km}}{16} = \underline{660\ \text{m}}$$

$\tau_0\ (.998c) = 660$ m  everything is ok for μ

# Space ship travel to 100 light year (ly) distant star




v=0.999 c

earth

star

100 ly [= {3(10)$^8$ m/s} {365 d} {24 hr/d} {3600 s/hr}= 9.5 10$^{15}$ m]

clock at 2 places ⇒ earth-star in moving ref. frame

$$\Delta t_{\text{earth-star}} = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

time dilation view

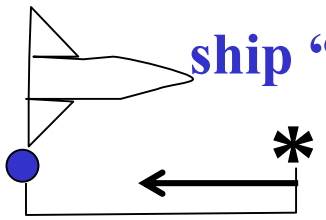
$$1-\frac{v^2}{c^2} = 1-0.999^2 = 1-0.998001 = 0.001999$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 22.4 \qquad \sqrt{1-\frac{v^2}{c^2}} = .0447$$

100 years = 22.4 $\Delta t_0$ $\quad \Delta t_0$ = 100/22.4 yr= 4.47 yr

## Length contraction view

earth — ruler at rest ⇒ 100 ly proper length — star




moving ruler ⇒ contracted

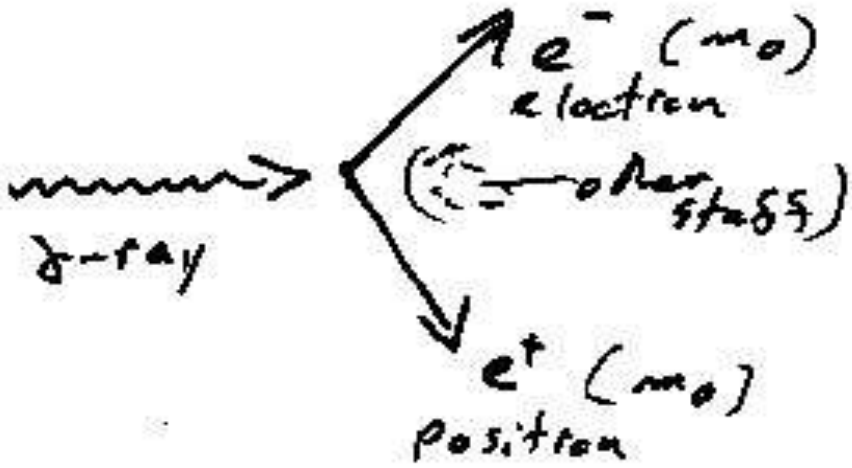
$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

L=100 (.0447)=4.47 ly

# Simple Example of $E = mc^2$



## Pair creation

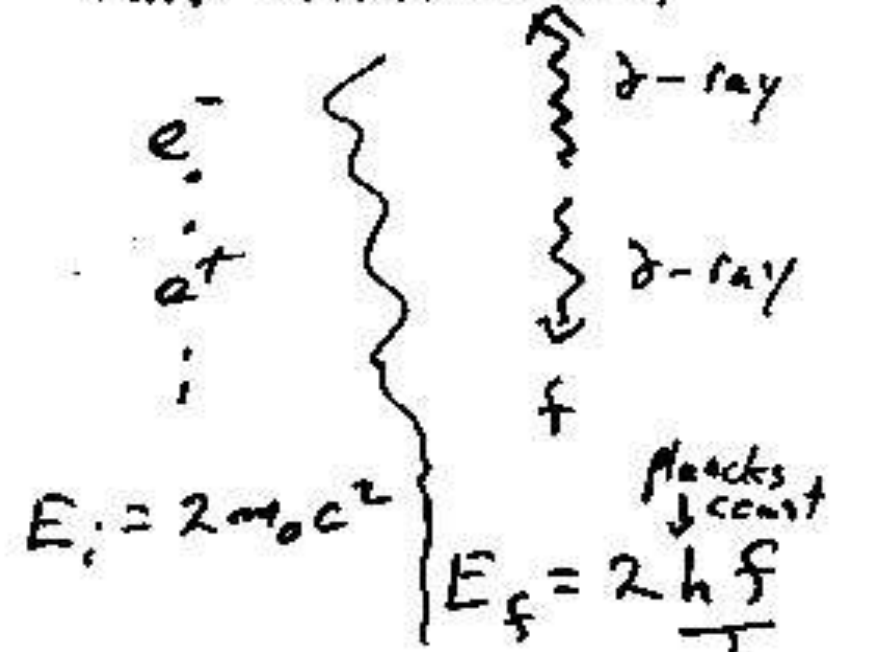



Min. energy of $\gamma$-ray needed

$E = m_0c^2 + m_0c^2$

$E_{min} = 2m_0c^2$

## Pair annihilation




$E_i = E_f$

$2m_0c^2 = 2hf$

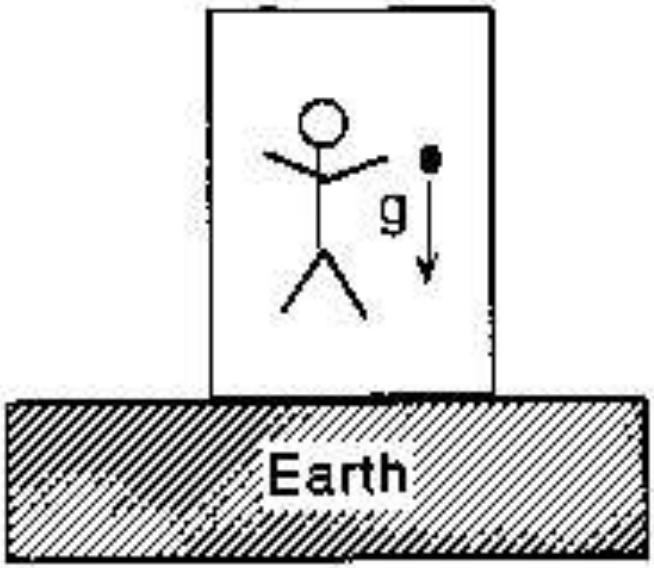
@ what is the freq. $f = \frac{m_0c^2}{h} = \frac{9(10)^{-31}\text{kg}[3(10)^8\text{m/s}]^2}{6.6(10)^{-34}\text{Js}} = \frac{81(10)^{-31}(10)^{16}\text{J}}{6.6(10)^{-34}\text{Js}} = 1.2(10)^{20}\text{Hz}$

# Einstein's General Theory of Relativity (1915)

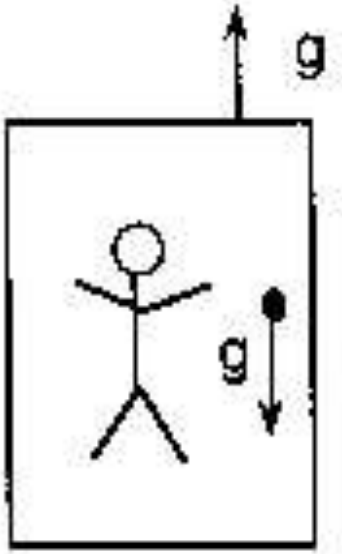


- considers gravity and accelerating reference frames on same footing

**in General Theory of Relativity**
**Gravitational forces ⇒ curvature of space**




**On Earth: person sees object accelerate down with g**




**in deep space**

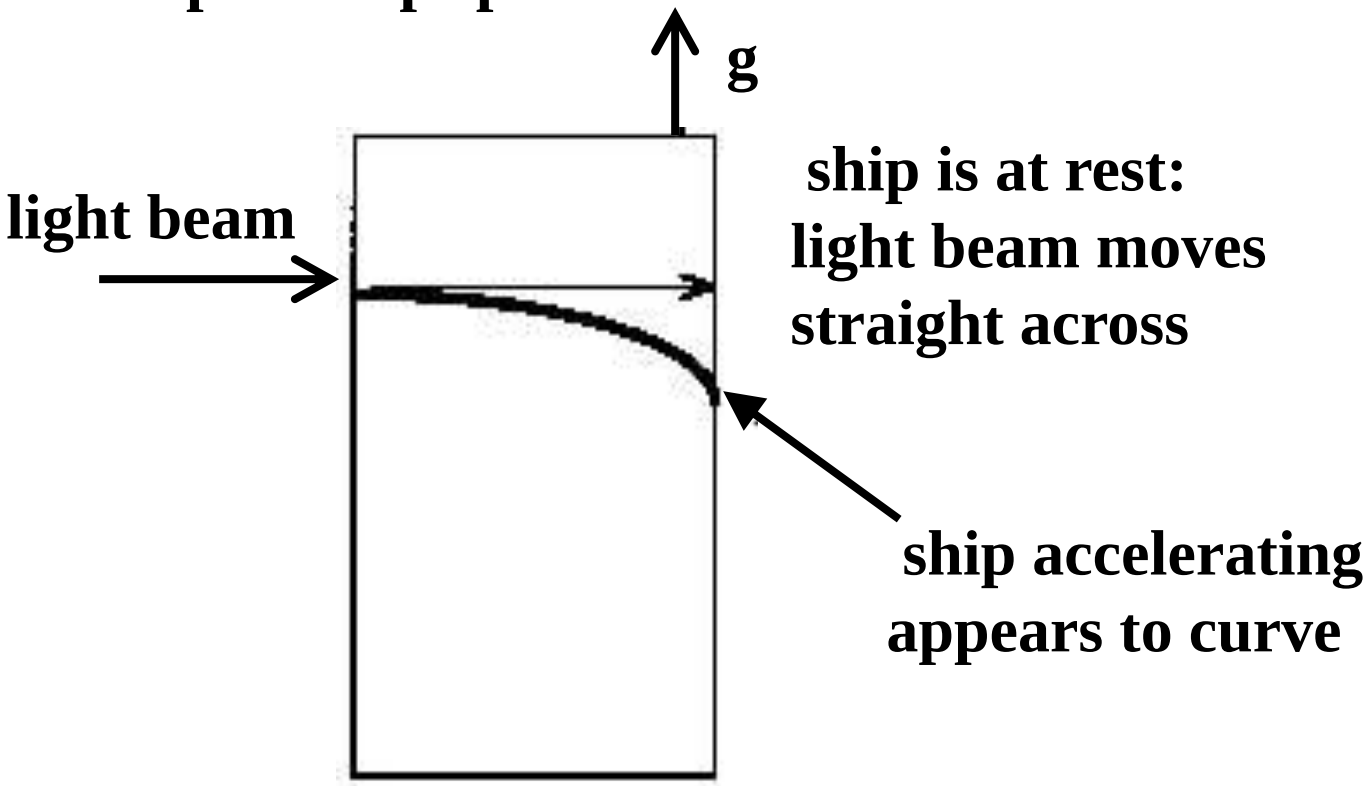
**In space ship accelerating at a person sees object accelerate down with g**

- no way to tell whether gravity or acceleration of box/ship (reference frame) is causing the object's acceleration

**Principal of Equivalence !!!**

Now what about light.

**space ship in deep space**

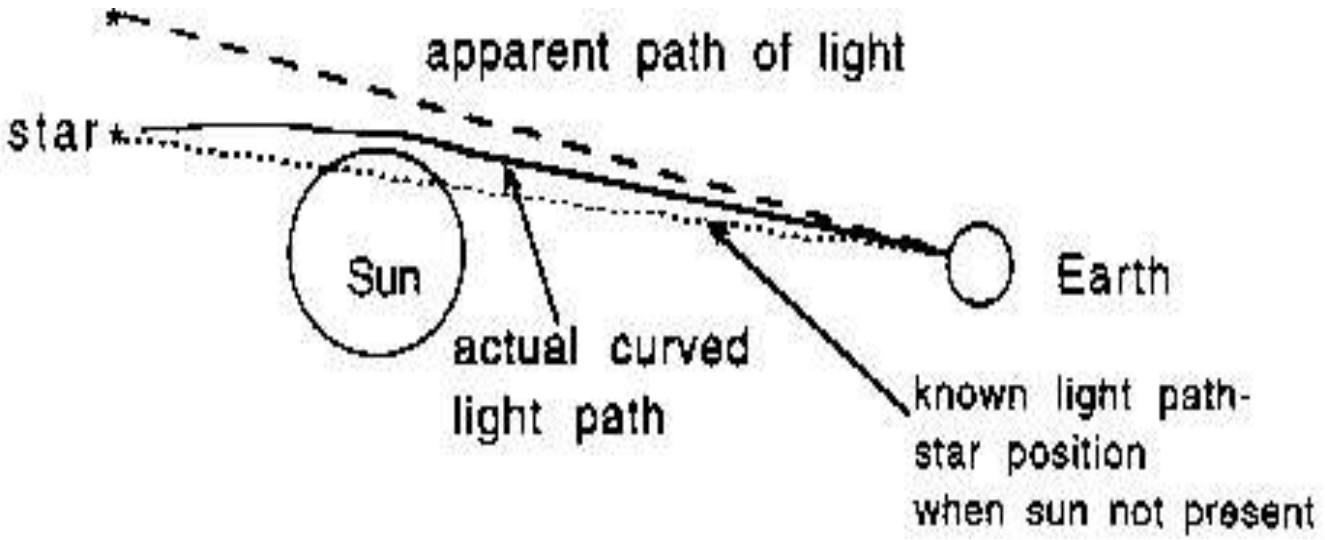



- **light follows curved path in accelerating reference frame !!!**

**"Principle of Equivalence" therefore implies**

- **Light follows curved path in a gravitational field !!!**

**"Principle of Equivalence" therefore implies**




LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When

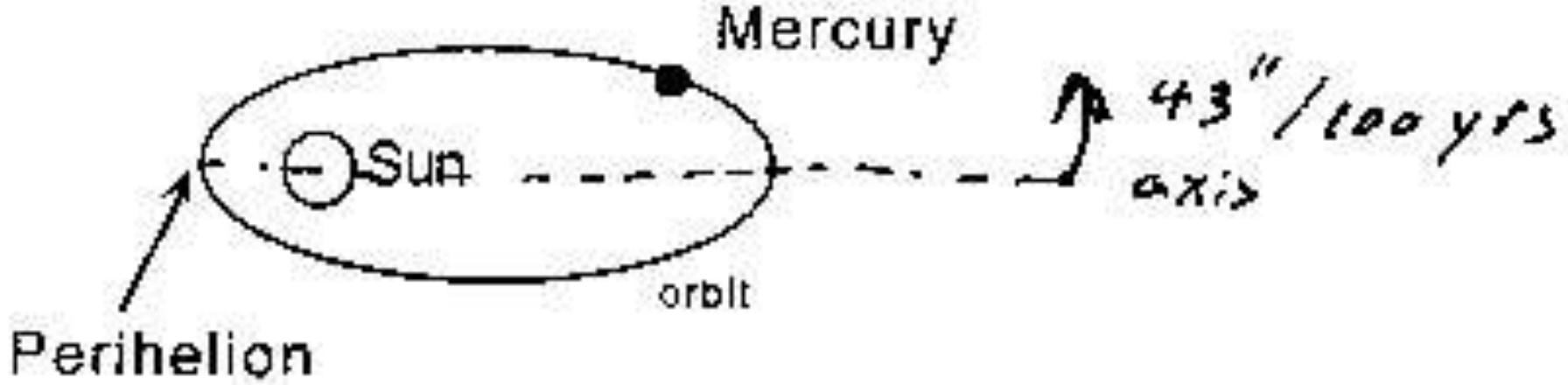
The New York Times

Published: November 10, 1919

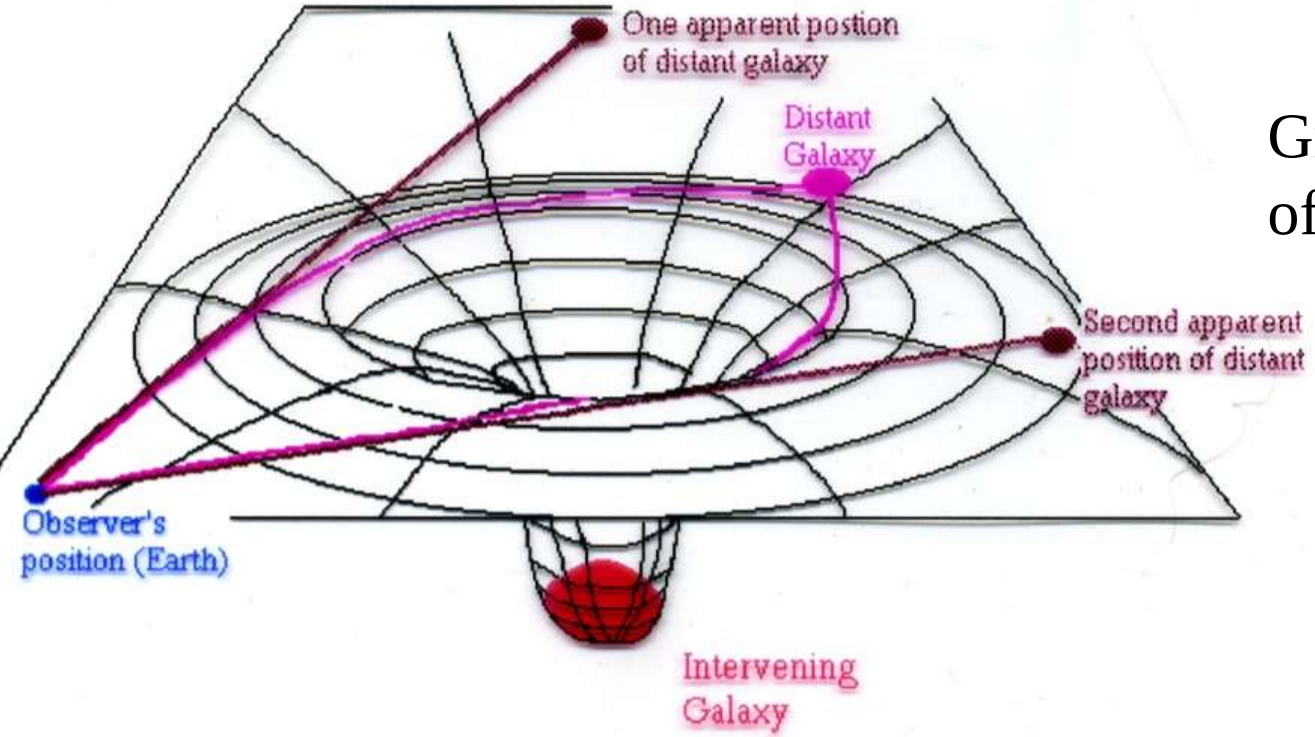


- **observation of deflection of star light path at solar eclipse confirmed Einstein's ideas in the General Theory of Relativity**
- **Einstein's General Theory of Relativity**
  **- gravitational forces replaced by the curvature of space**
  **- light just follows the curvature of space**

**Advance of Mercury's orbit also proves quantitative confirmation of Einstein's GR Theory**

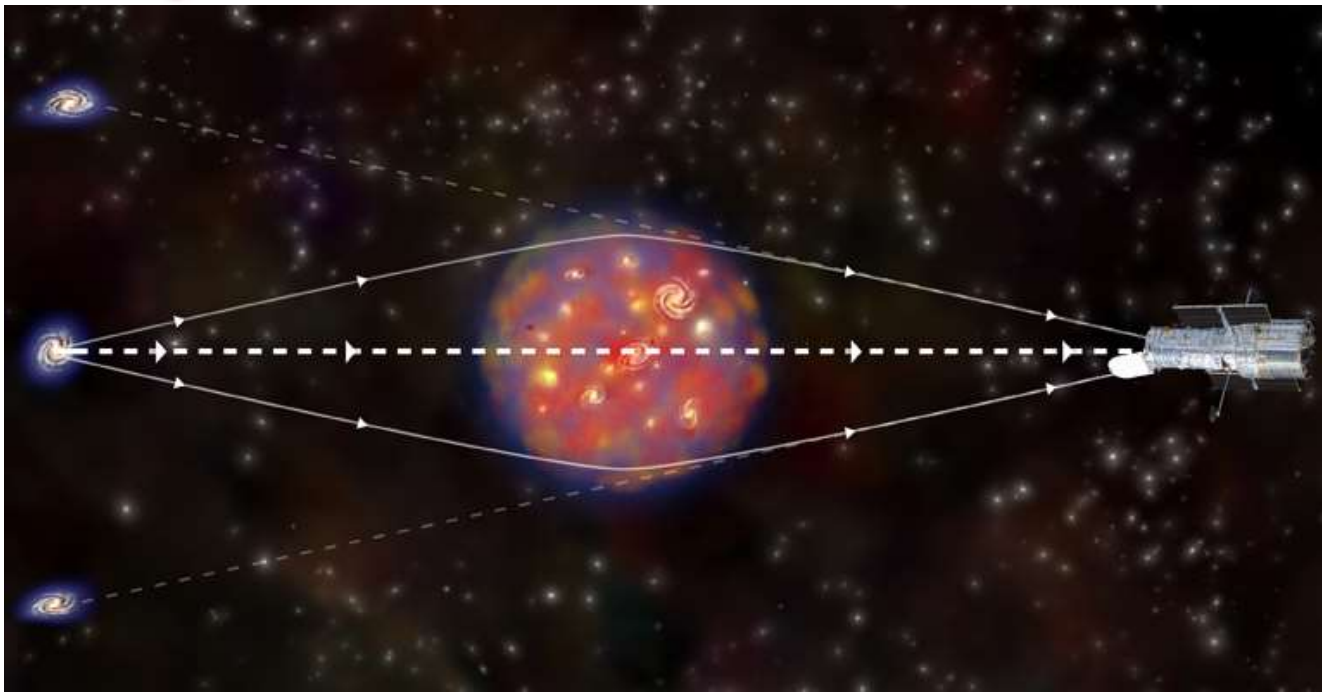



**Advance of the perihelion of Mercury had been long known deviation from Newton's Laws.**



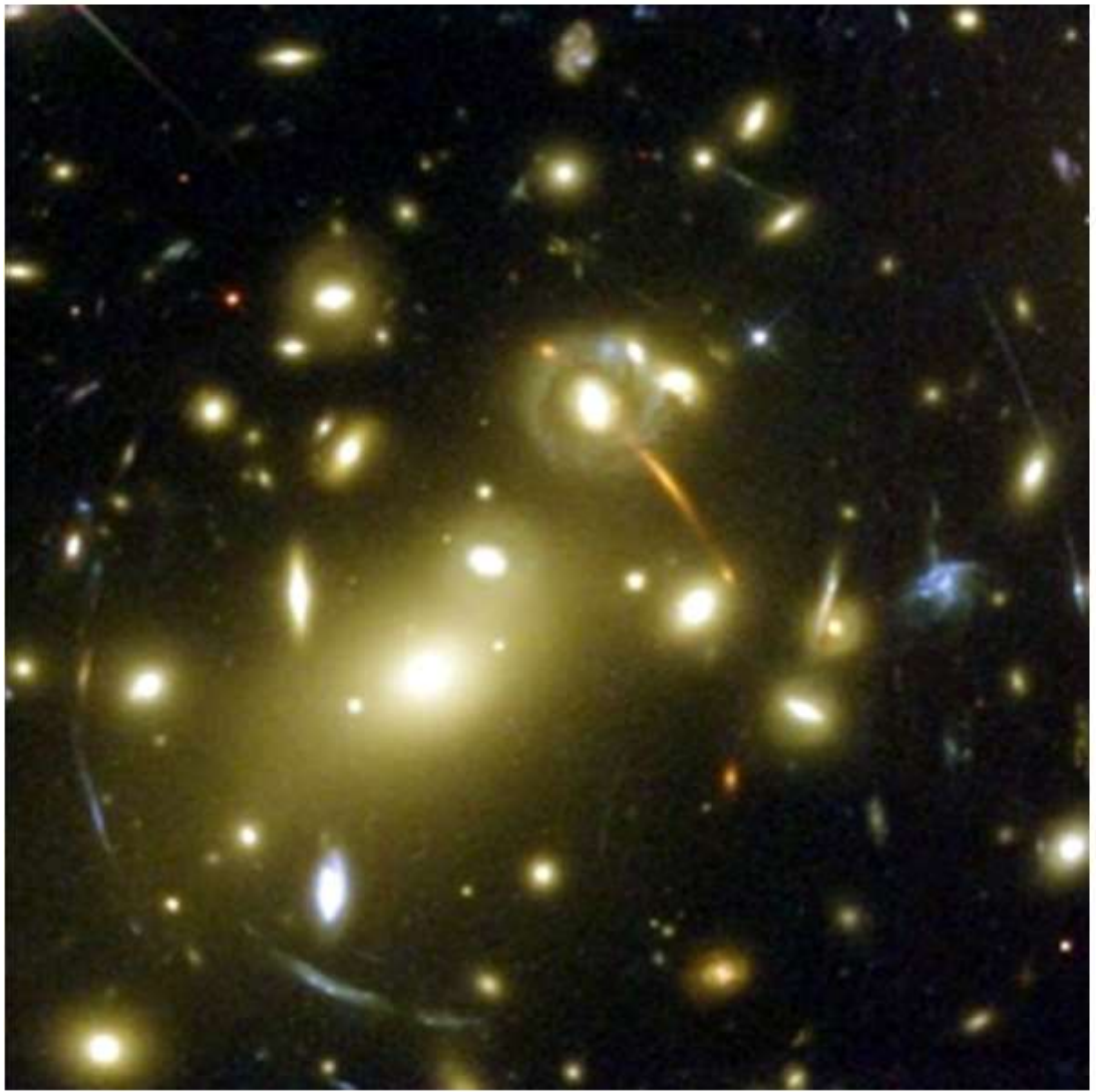

# Gravitational Lensing of distant objects





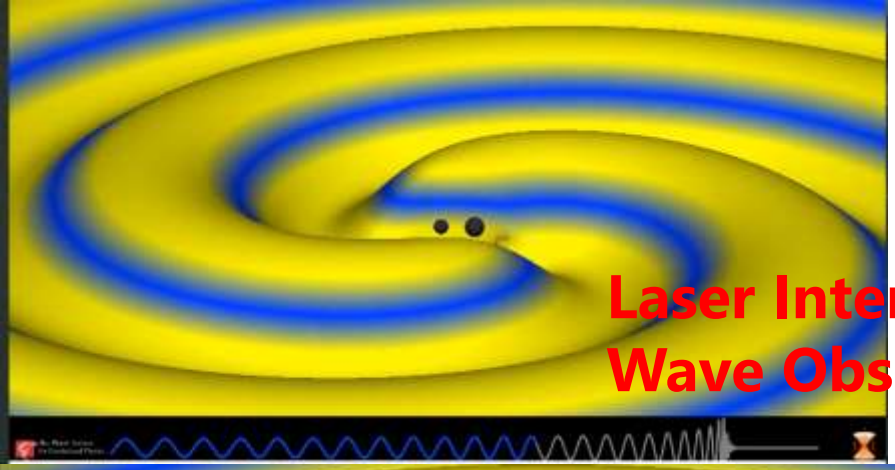
C. Keaton Rutgers

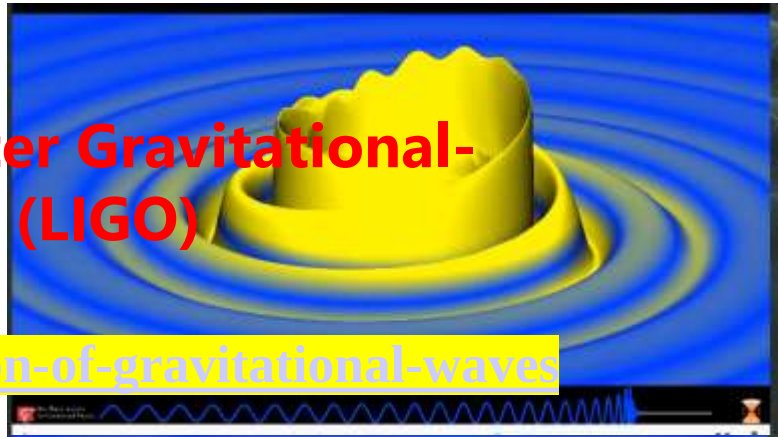

**4x splitting of background galaxy image by foreground galaxy**



**Arc distortions of background galaxies images by foreground galaxy (at center)**

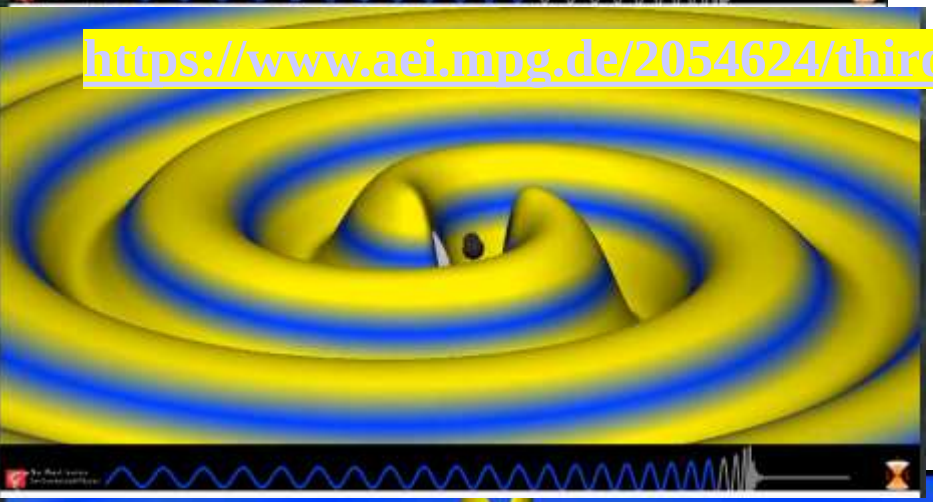
# Gravitational-Waves from Black Hole merger

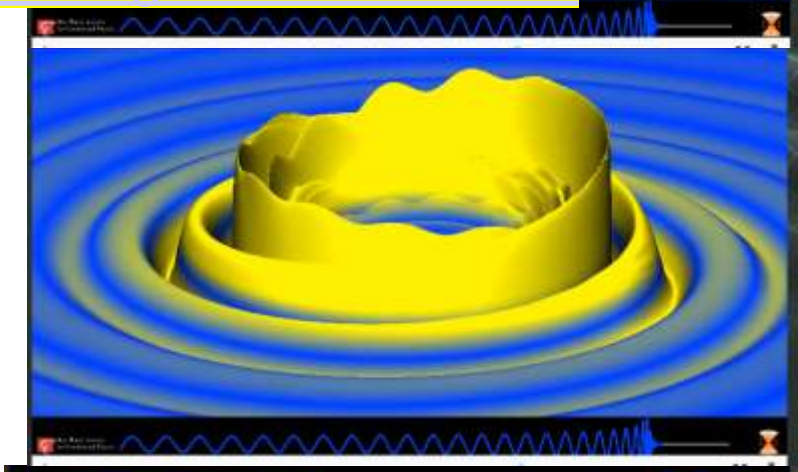


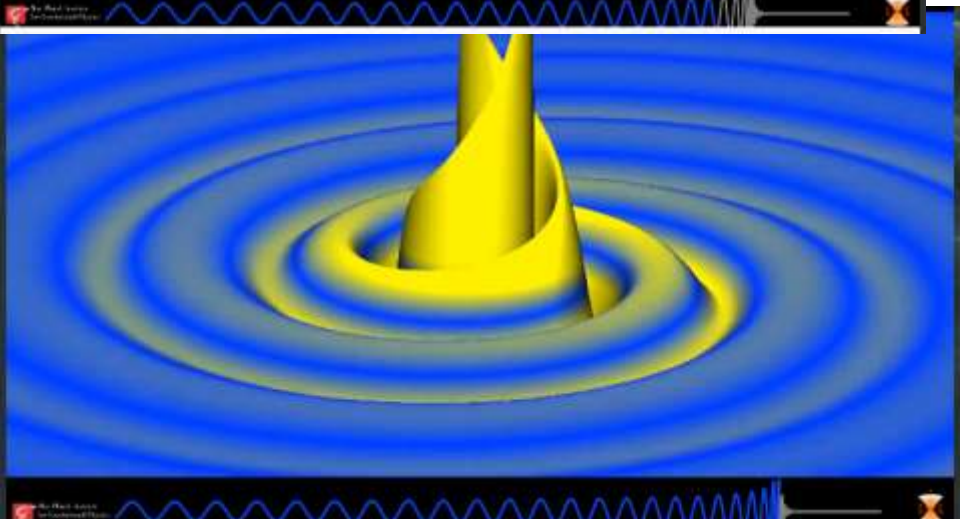


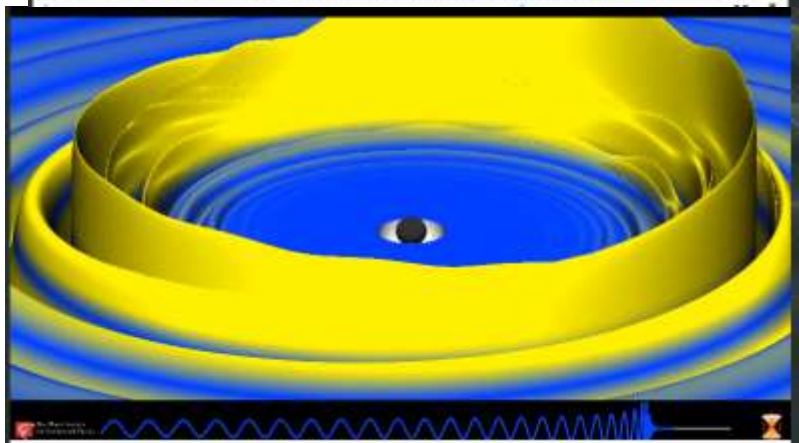
**Laser Interferometer Gravitational-Wave Observatory (LIGO)**

https://www.aei.mpg.de/2054624/third-detection-of-gravitational-waves



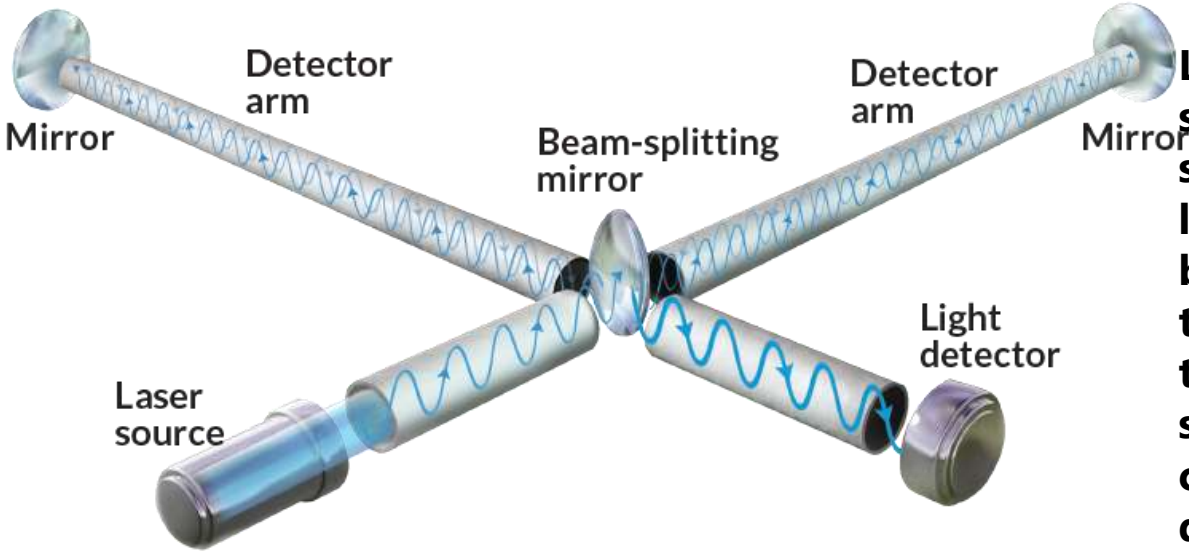




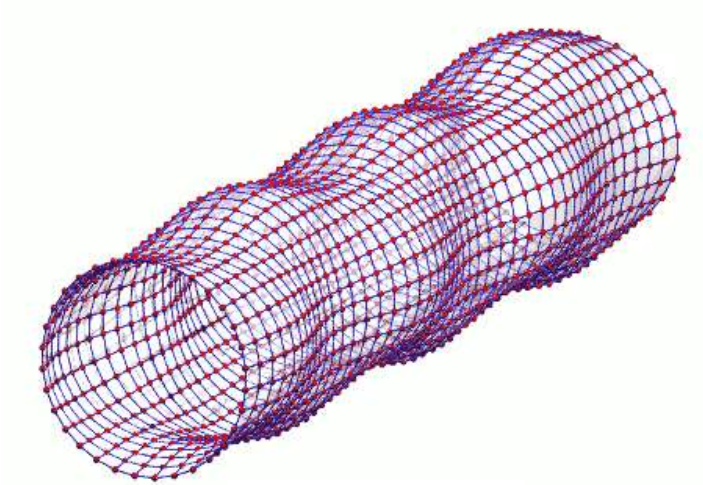


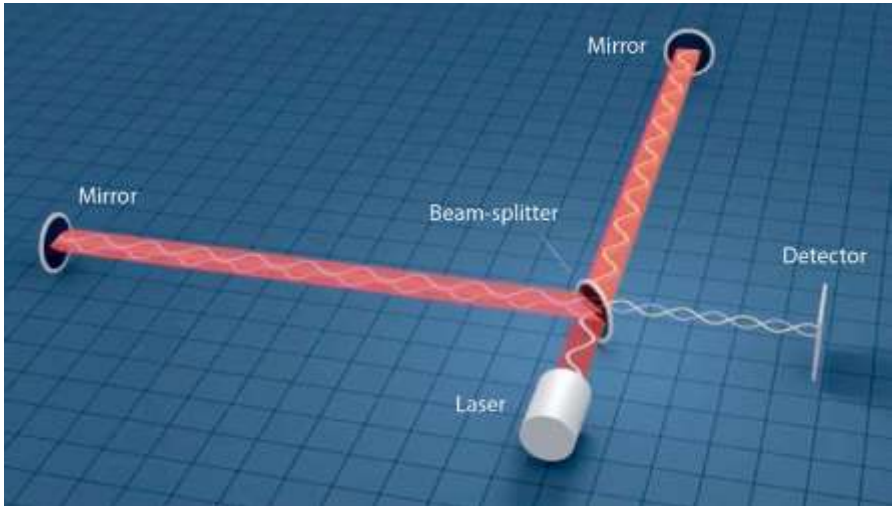
# Laser Interferometer Gravitational-Wave Observatory (LIGO)

https://www.sciencenews.org/article/trio-wins-physics-nobel-prize-gravitational-wave-detection



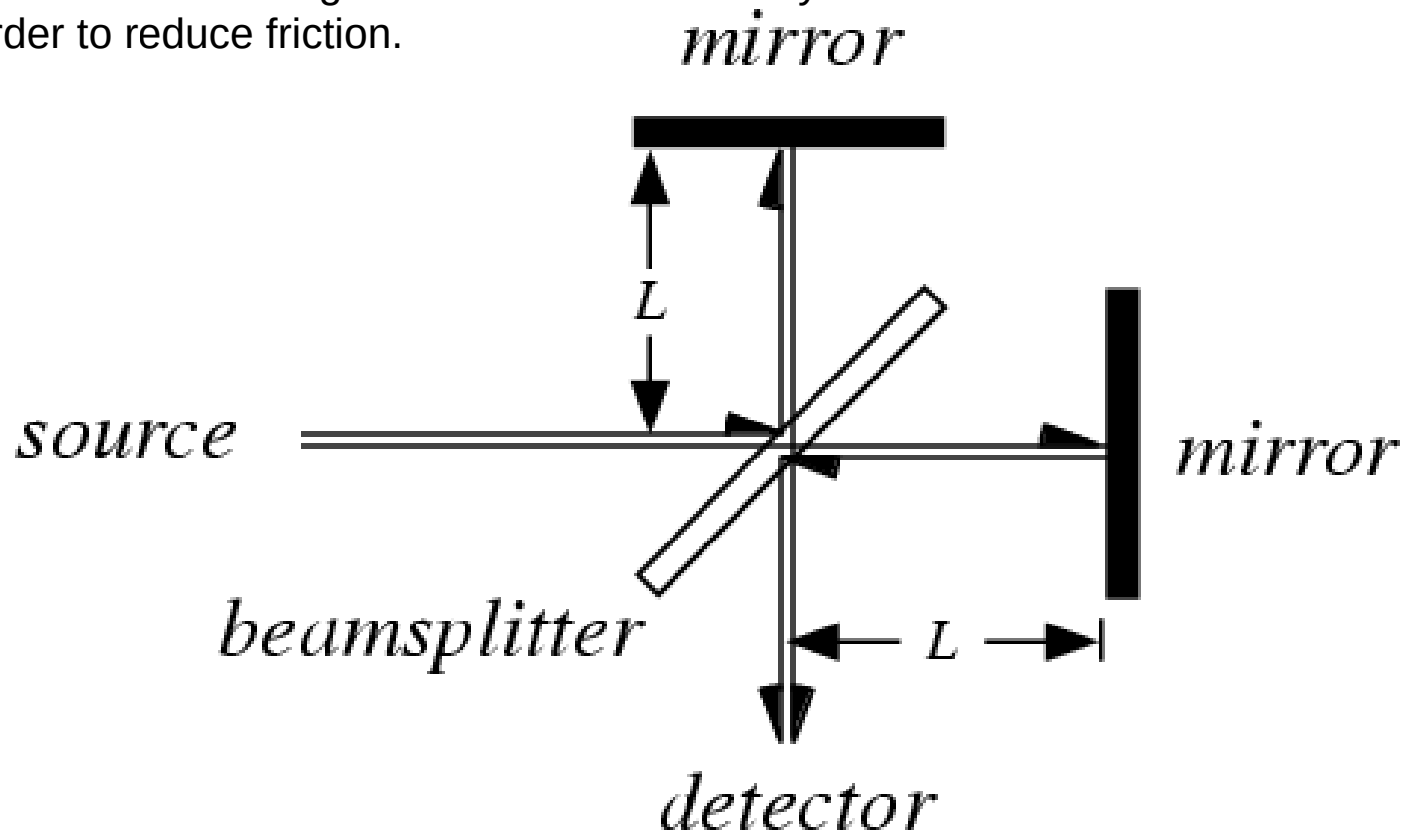

**LIGO detects gravitational waves by splitting a laser beam in two, sending light down each arm. The light reflects back and forth between mirrors in the arms, before the beams recombine and are sent to a detector. If the arms are the same length, the light beams will cancel each other out. Any length difference — such as that caused by gravitational waves stretching one arm while squeezing the other — will allow some light through to the detector.**



After the development of Maxwell's theory of electromagnetism, several experiments were performed to prove the existence of ether and its motion relative to the Earth. The most famous and successful was the one now known as the Michelson-Morley experiment, performed by Albert Michelson (1852-1931) and Edward Morley (1838-1923) in 1887.

Michelson and Morley built a Michelson interferometer, which essentially consists of a light source, a half-silvered glass plate, two mirrors, and a telescope. The mirrors are placed at right angles to each other and at equal distance from the glass plate, which is obliquely oriented at an angle of 45° relative to the two mirrors. In the original device, the mirrors were mounted on a rigid base that rotates freely on a basin filled with liquid mercury in order to reduce friction.




http://scienceworld.wolfram.com/physics/Michelson-MorleyExperiment.html