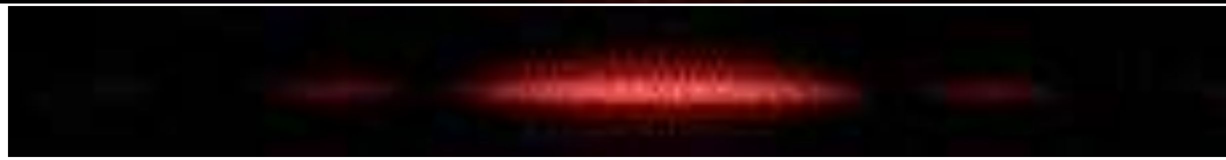
Interference Constructive - Destructive

2-slit

single slit



diff. grating

reflection

Note: λ difference = 0 difference

$$\frac{\lambda}{2} \rightarrow destructive \qquad \lambda \rightarrow constructive$$

+ n λ same condition

$$\Delta\ell = d\sin\theta \qquad \Delta\ell = \frac{d}{2}\sin\theta$$

reflection **tot. inter. = reflection + path length**

effect **effect** **effect**

n increase - λ/2

n decrease- no change

Newton-1666 Woolsthrope Eng.

Prism separates light into its component colors [different wavelength]

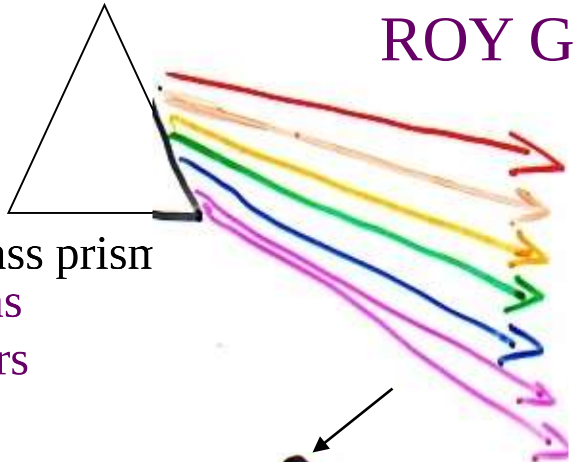
ROY G BIV To help remember

White

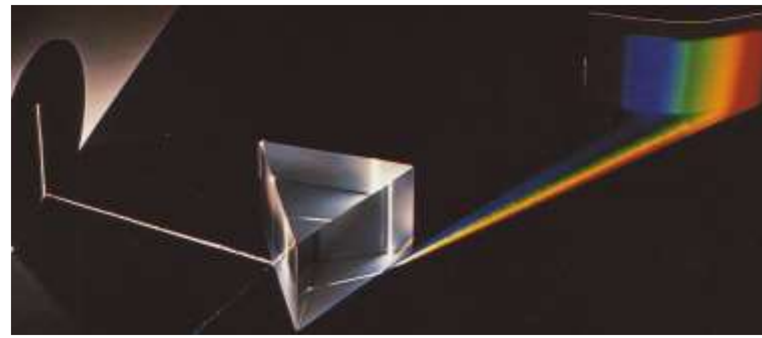
Light

glass prism

Contains all colors



RED
ORANGE
YELLOW
GREEN
BLUE
INDAGO
VIOLET



Circular

Elongated

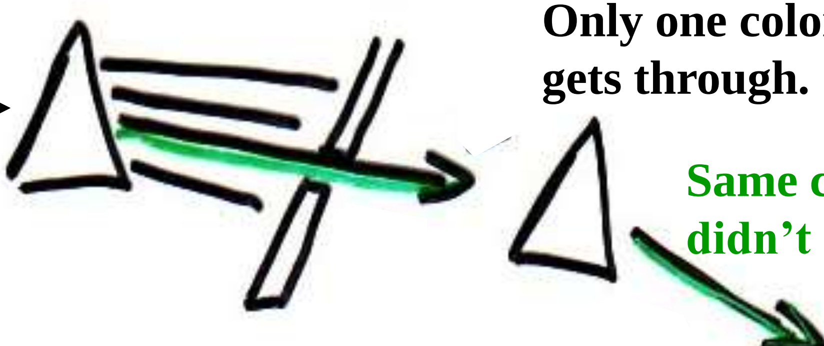
Before Newton it was thought that the glass 'darkened' the white light into colors.

Newton's proof of separating idea:

The Baroque Cycle
Neal Stephenson
Unusual characters !!
Newton…

White

Light

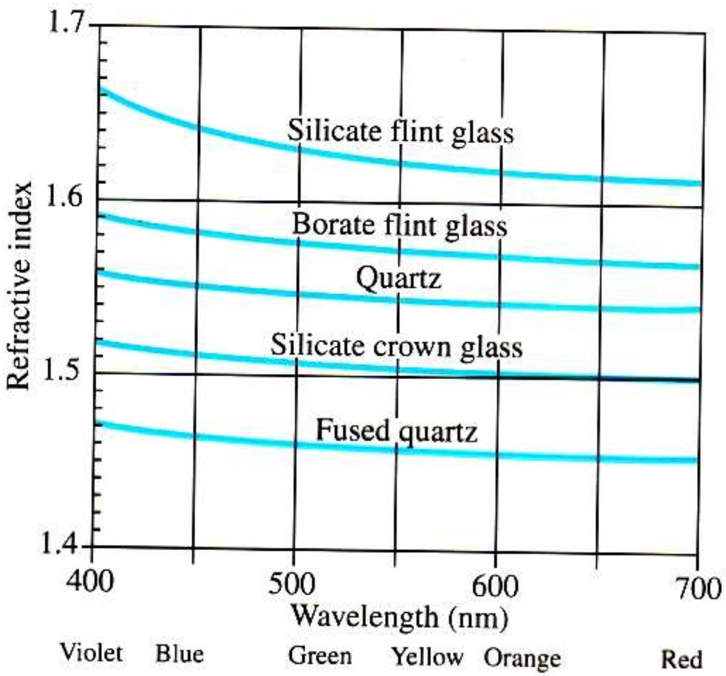


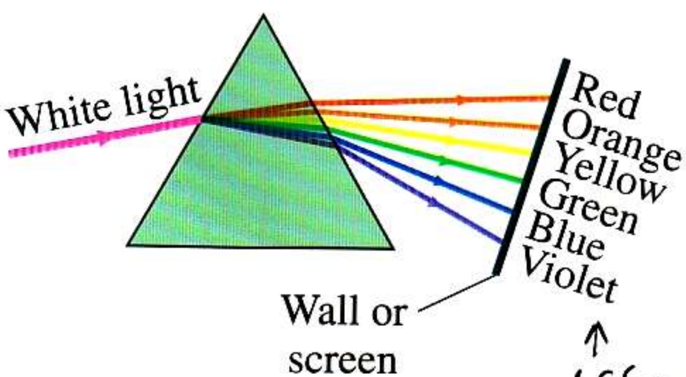
Only one color gets through.

Same color: 2nd prism didn't further darken light!

8-1

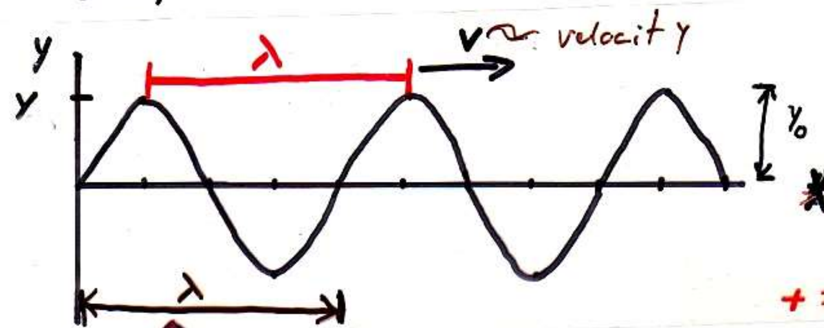
The index of refraction and dispersion





different wavelengths of waves !!!

Simple Harmonic Wave



wavelength

repeat distance

$$y = y_0 \sin\left[2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right)\right]$$

+ ⇒ moves in −x dir.

or $y = y_0 \sin\left[\frac{2\pi}{\lambda}(x \pm vt)\right]$ − ⇒ moves in +x dir

T = period = repeat time

sine wave in space and time

ie/ $t = 0 \Rightarrow y = y_0 \sin\left[2\pi\left(\frac{x}{\lambda}\right)\right]$

$x = 0 \Rightarrow y = y_0 \sin\left[2\pi\left(\pm\frac{t}{T}\right)\right]$

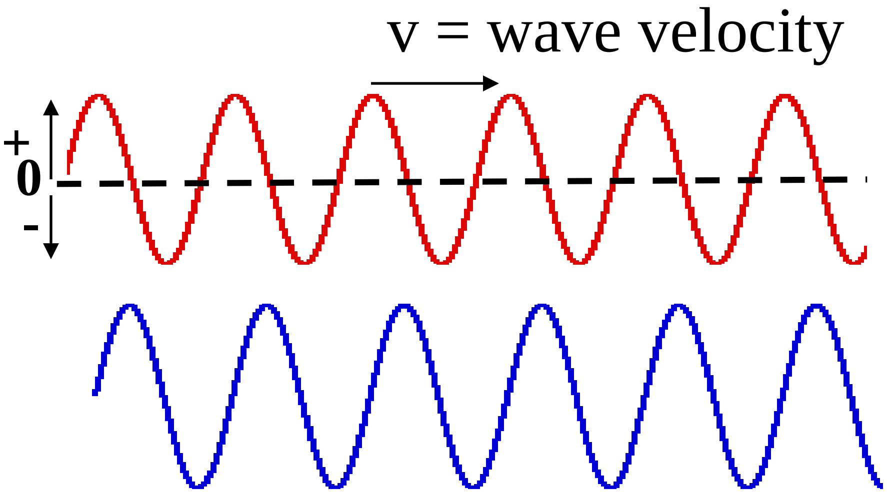
$v = \lambda \frac{1}{T} = \lambda f \qquad f = \frac{1}{T}$

f = frequency $\frac{1}{s} = Hz$

E & M Radiation

$$E = E_0 \sin 2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right)] \;!!$$

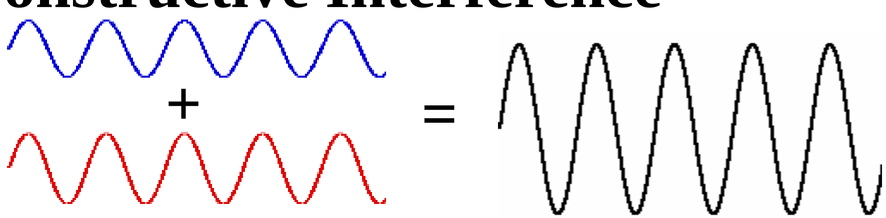
electric field
(and magnetic field)



Add up 2 waves point by point

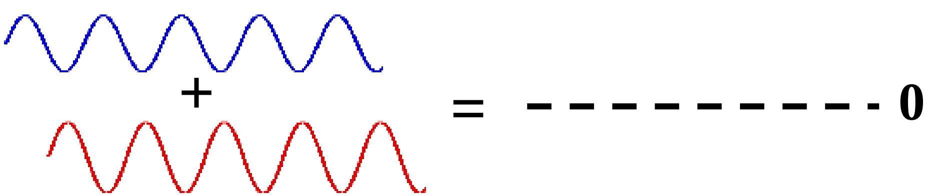
Interference – partial/total cancellation/increase

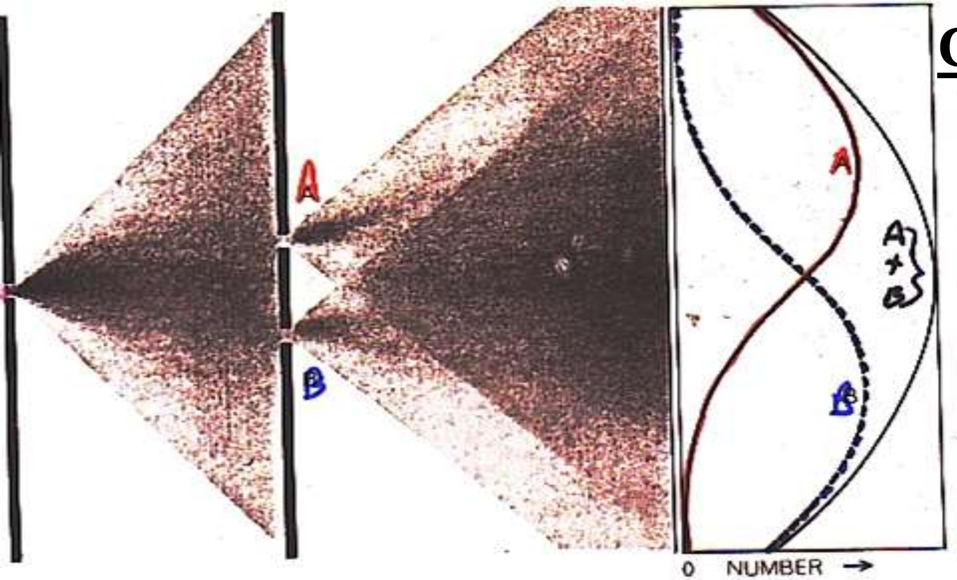
Constructive Interference



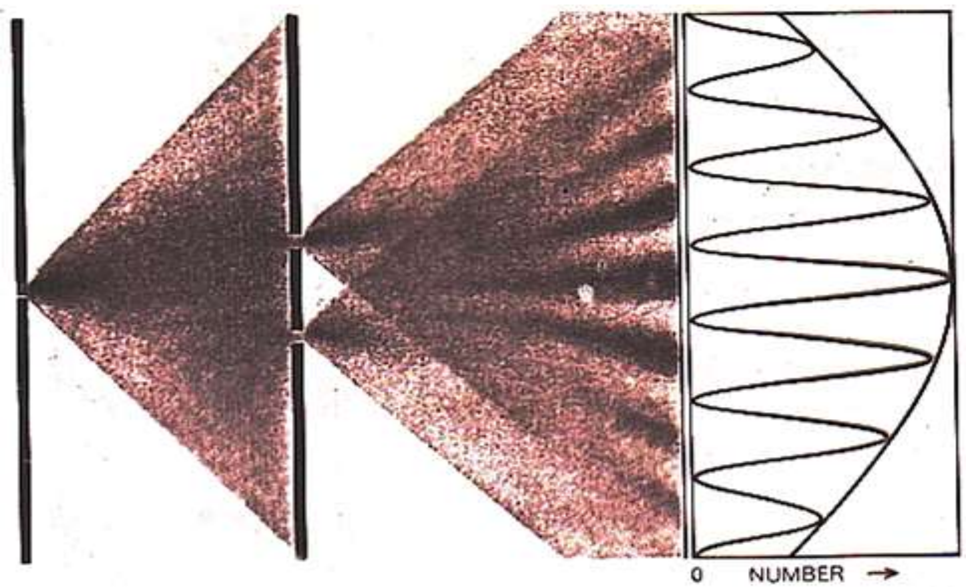
anything in-between

Destructive Interference





CLASSICAL PARTICLE DESCRIPTION of the two-slit experiment would say that the distribution of all particles that arrive at the screen is the sum (*black curve*) of the distribution curves for particles from the upper sli *solid color*) and lower slit (*broken color*).



QUANTUM PARTICLE DESCRIPTION says the distribution of particles will show a distribution pattern typical of wave phenomena, but only if *each* particle can go through *both* slits. Observation shows clearly that this description, and not the classical one, is correct.

Corpuscular or particle theory of light

2 particle sources add in simple way

(Newton believed this)

Christian Huygens proposed

Wave theory of light (correct)

2 wave sources

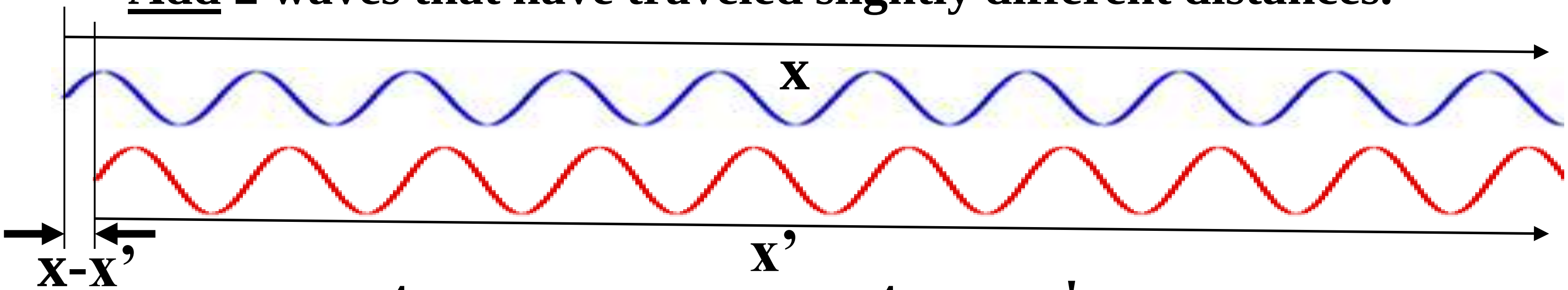
interference yields large intensity oscillations

Thomas Young

did the experiment 1801

observed light-wave interference

<u>Add</u> 2 waves that have traveled slightly different distances.



$$\mathrm{Sum} = E_0 \sin(\underbrace{2\pi\frac{t}{T} + 2\pi\frac{x}{\lambda}}_{\theta}) + E_0 \sin(\underbrace{2\pi\frac{t}{T} + 2\pi\frac{x'}{\lambda}}_{\theta'})$$

$\Downarrow$

trig. identity

$$\sin\theta + \sin\theta' = 2\sin\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)$$

$$\mathrm{Sum} = 2E_0 \underbrace{\cos(\pi\frac{x - x'}{\lambda})}\ \underbrace{\sin(2\pi\frac{t}{T} + \pi\frac{[x + x']}{\lambda})}_{\text{time dependence same with zero shift}}$$

Big difference with small x-x' !!

$$\frac{x - x'}{\lambda} = 0, 1, 2, 3, 4 \ldots m$$

$$\cos(m\pi) = \pm 1$$

constructive interference

$$\frac{x - x'}{\lambda} = 1/2, 3/2, 5/2, \ldots n/2 \qquad \cos(n\pi/2) = 0$$

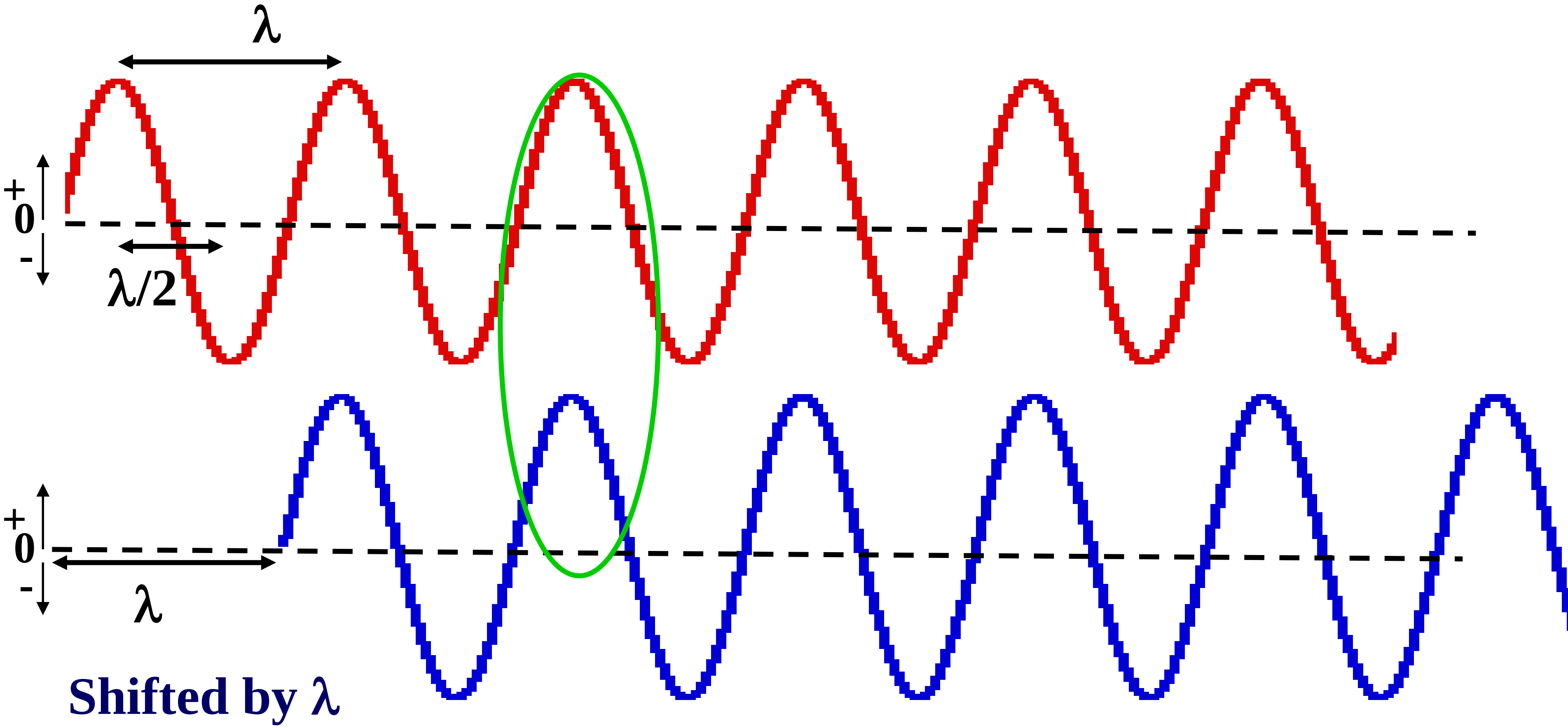
n odd !

destructive interference

forget time term

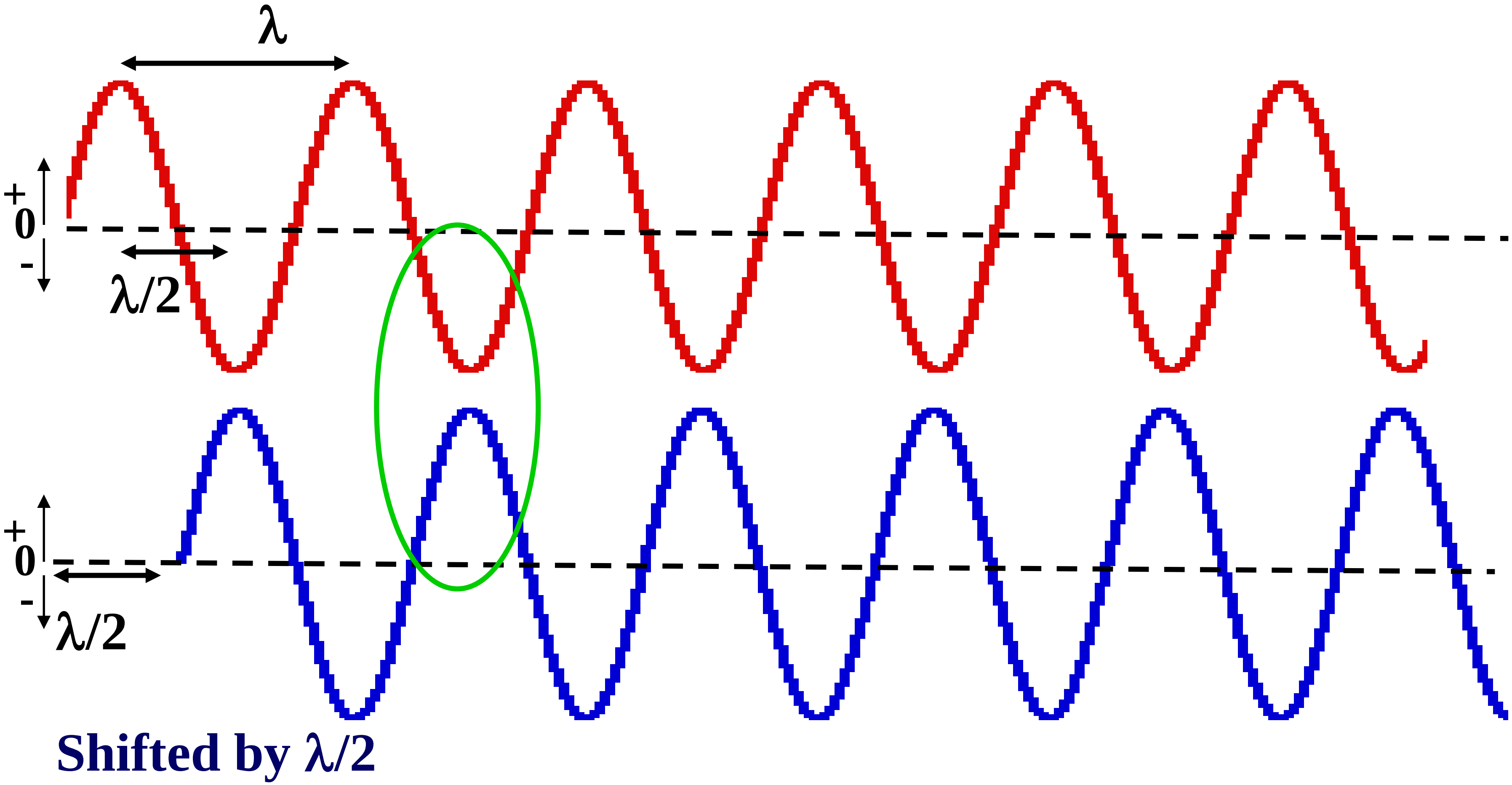
i.e. time average squared

$$I = [2E_0]^2 \cos^2(\pi\frac{x - x'}{\lambda})$$



(+A) + (+A) =2A Constructive interference

8-5a

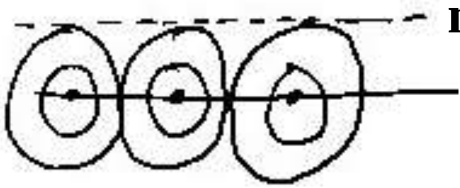


(+A) + (-A) =0 **Destructive interference**

8-5b

Huygens' Principle

Generate how wave propagates by summing up point sources



Huygens' point source

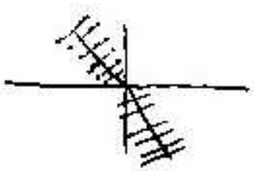
wave fronts (maxima) propagate in spherical shells

http://arana.cabrillo.edu/~jmccullough/Applets/Flash/Optics/Refraction.swf

Generate all of geometrical optics!!!

(1) Reflection $\theta_i = \theta_r$

(2) Refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$



$n_1 = 1$ c

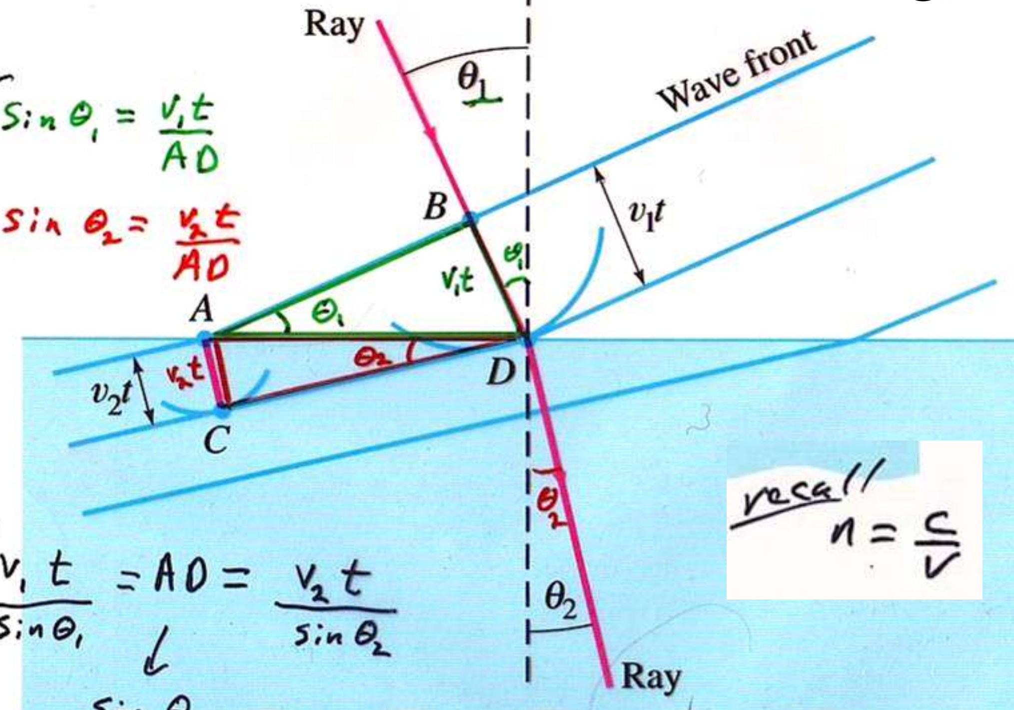
n_2 $v_2 = \frac{c}{n}$

$\lambda_2 = \frac{\lambda}{n}$

$c = \lambda f$

$v_1 = \lambda_1 f$

Snell's law follows from wave nature of light



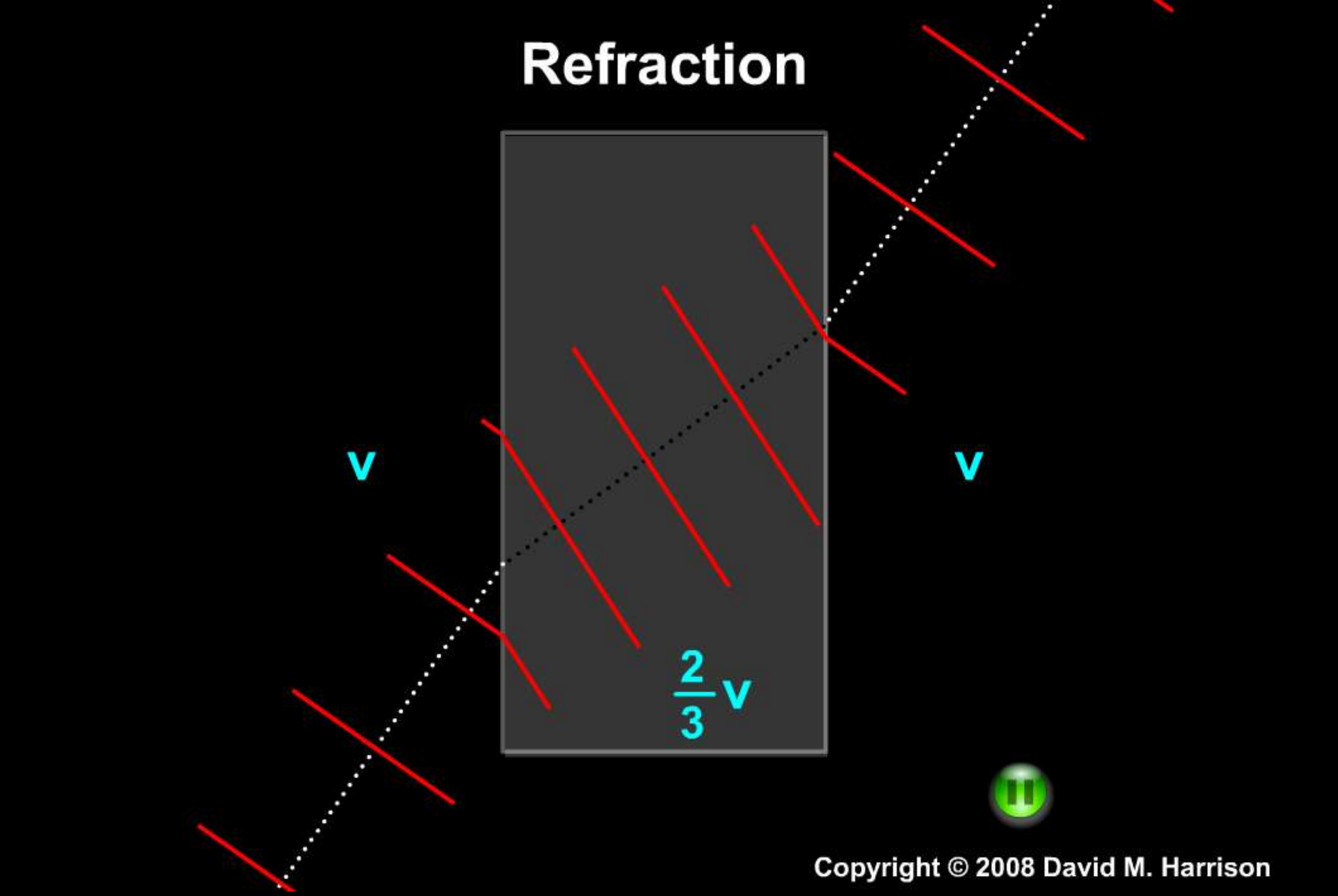
$\sin\theta_1 = \frac{v_1 t}{AD}$

$\sin\theta_2 = \frac{v_2 t}{AD}$

$$\frac{v_1 t}{\sin\theta_1} = AD = \frac{v_2 t}{\sin\theta_2}$$

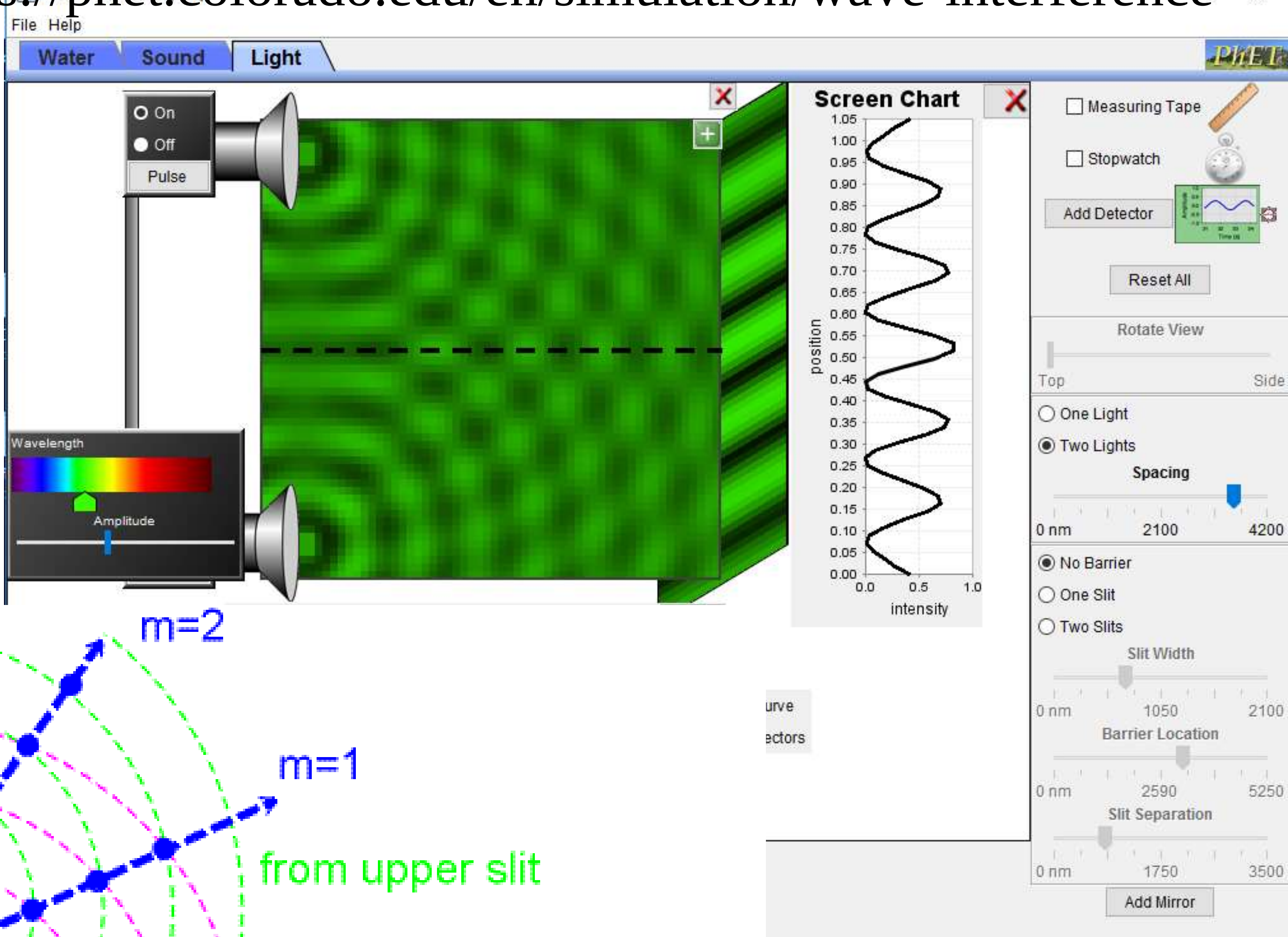
$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2} \longrightarrow \left[\frac{1}{v_2}\sin\theta_2 = \frac{1}{v_1}\sin\theta_1\right]\cdot c$$

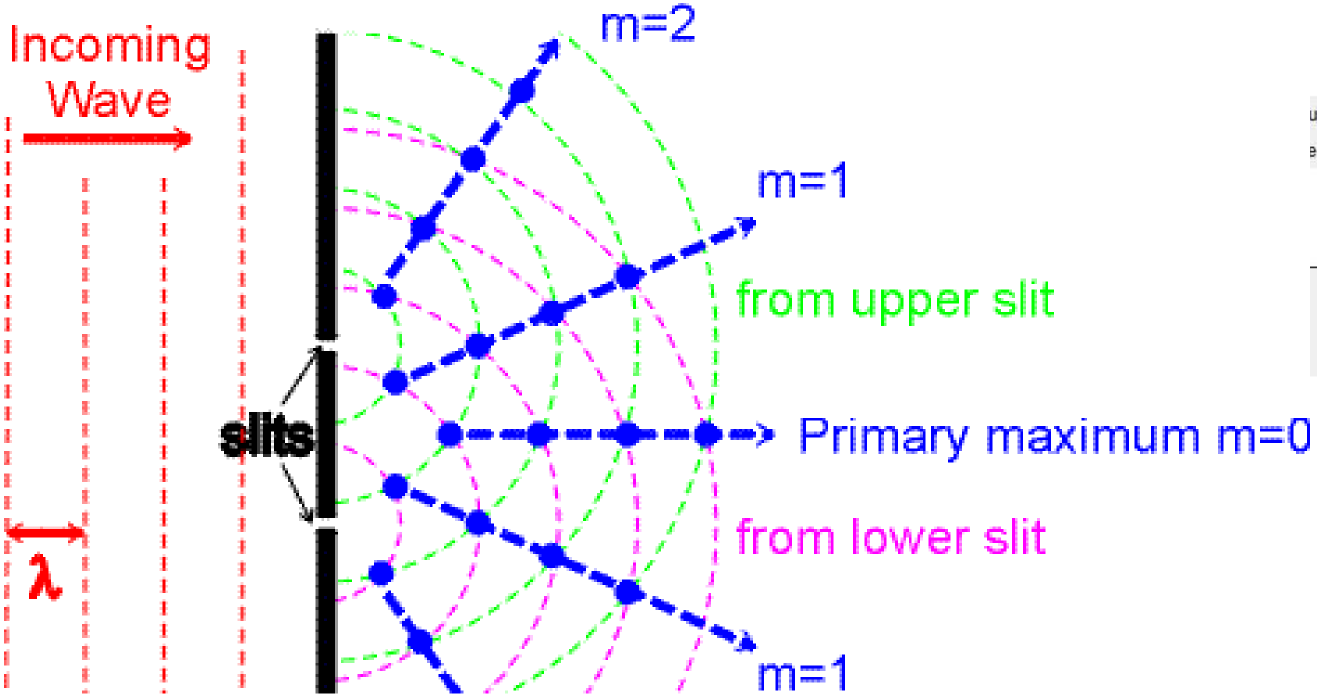
$$\boxed{n_2 \sin\theta_2 = n_1 \sin\theta_1}$$



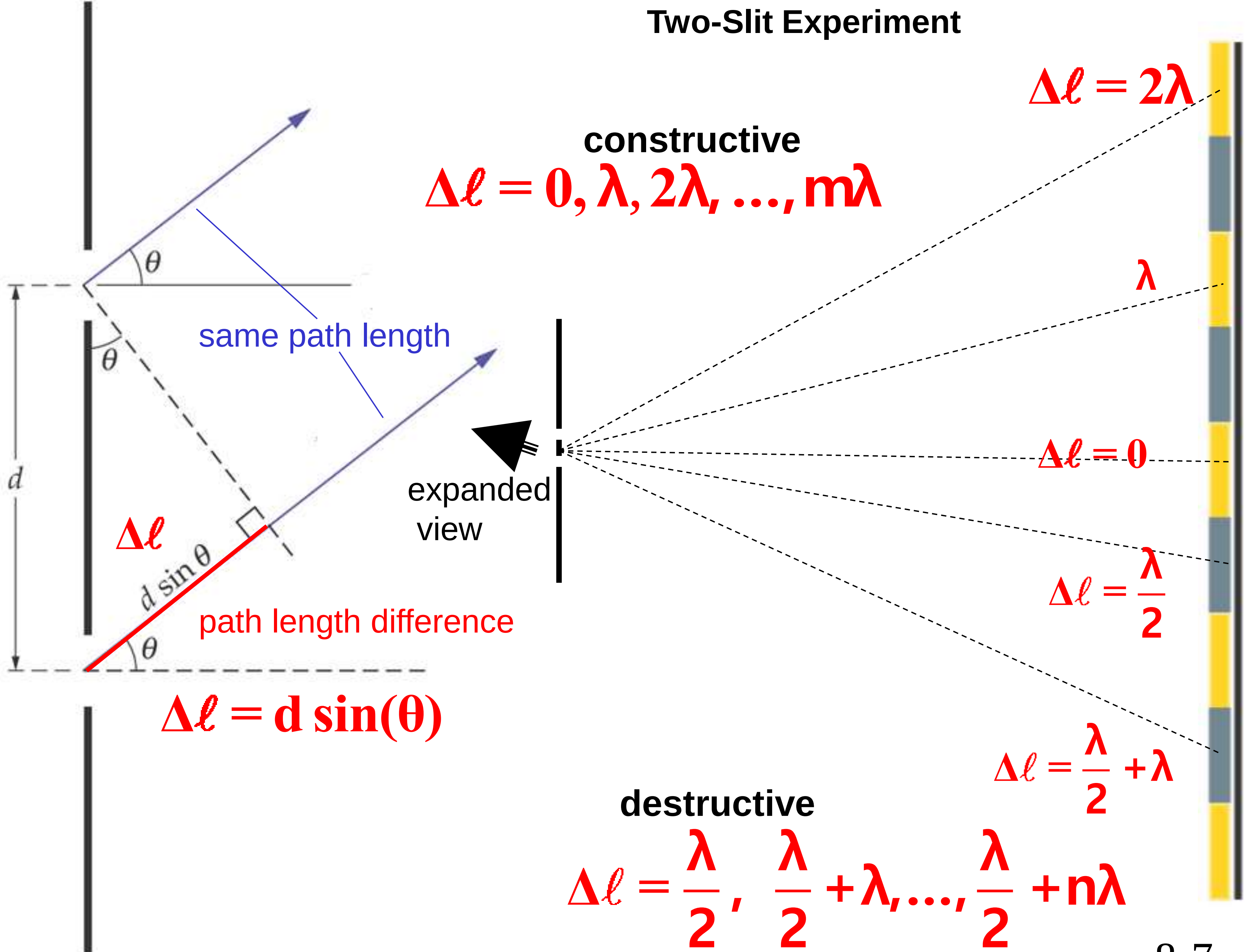
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Waves/Refraction/Refraction.swf

http://www.upscale.utoronto.ca/PVB/Harrison/Flash/Optics/Refraction/Refraction.swf https://phet.colorado.edu/en/simulation/wave-interference



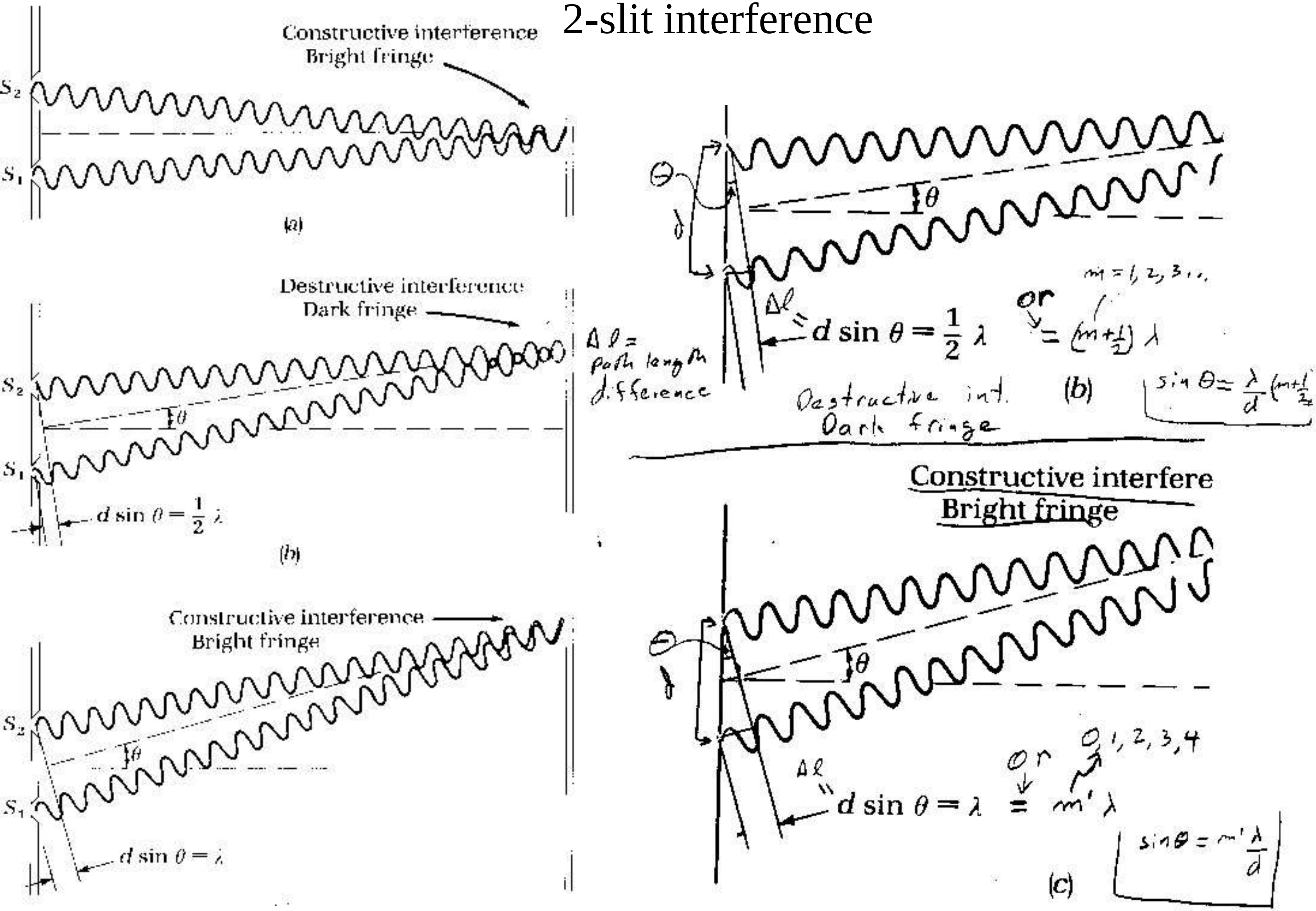


Two-Slit Experiment



http://www.walter-fendt.de/html5/phen/doubleslit_en.htm

2-slit interference



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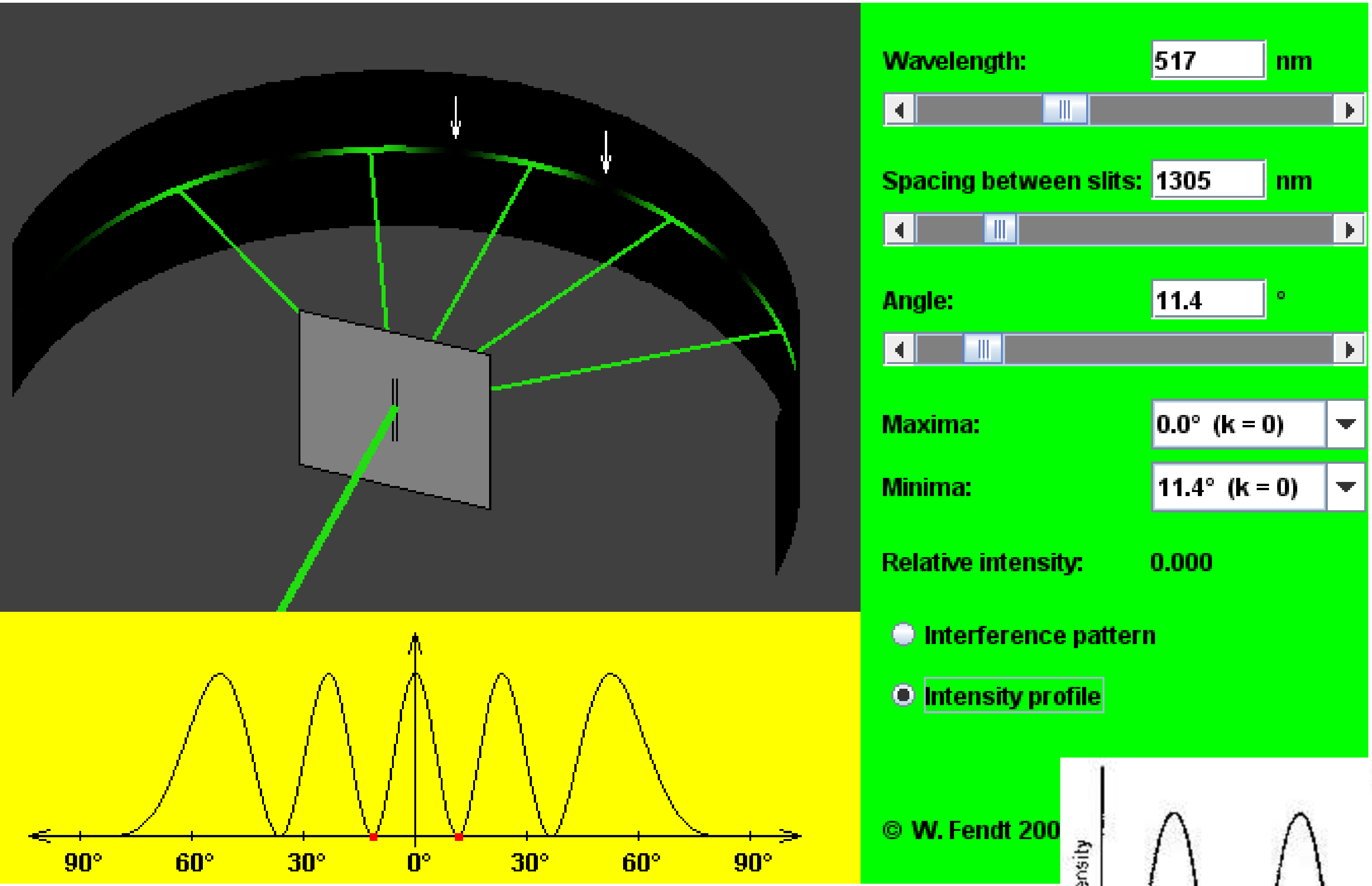
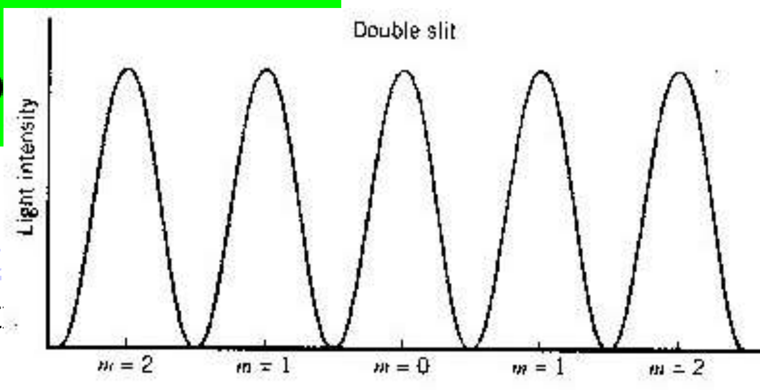
Interference of Light at a Double Slit





http://www.walter-fendt.de/html5/phen/doubleslit_en.ht

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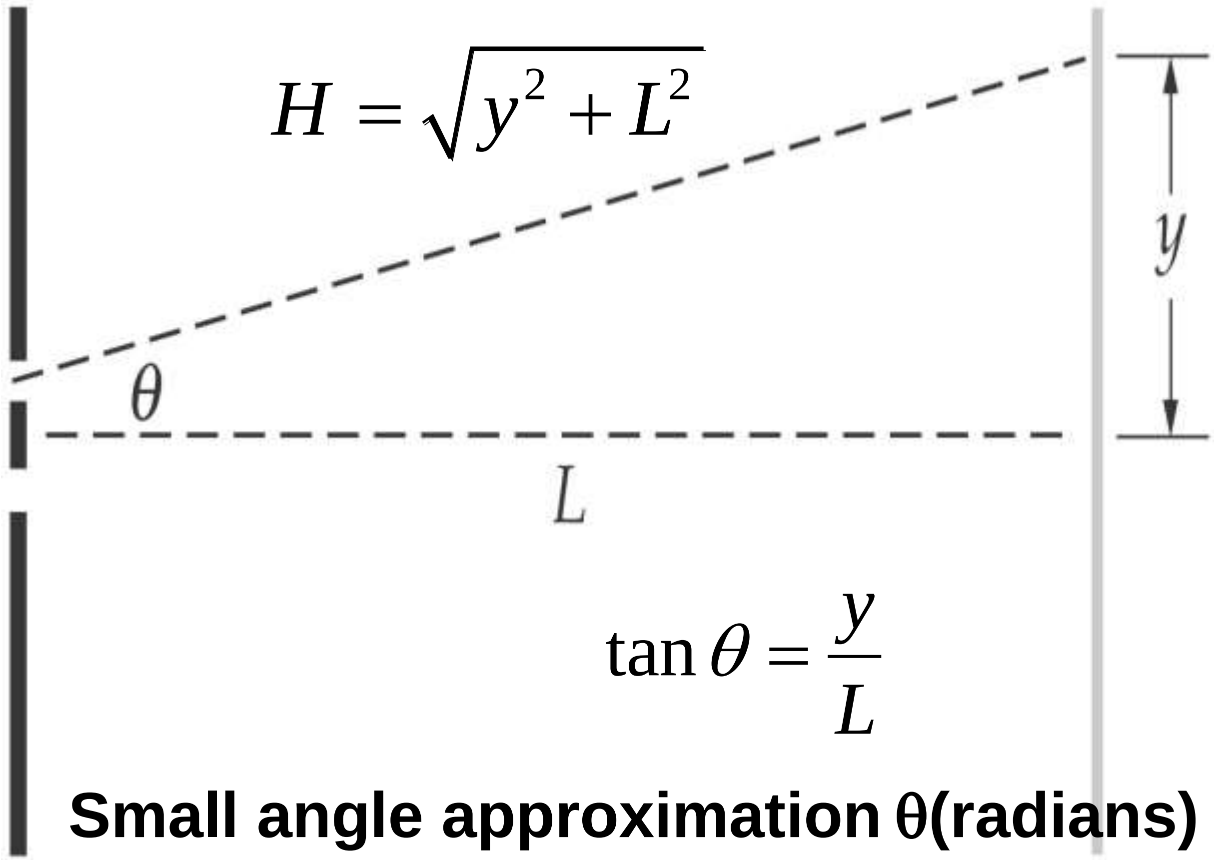
Linear Distance in an Interference Pattern



$$H = \sqrt{y^2 + L^2}$$

$$\tan\theta = \frac{y}{L}$$

Small angle approximation θ(radians) <~.1

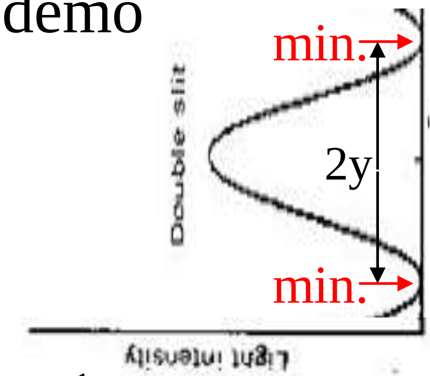
y<<H or L

$$\tan\theta \approx \sin\theta \approx \theta \approx \frac{y}{H} \approx \frac{y}{L}$$

2-slit interference demo

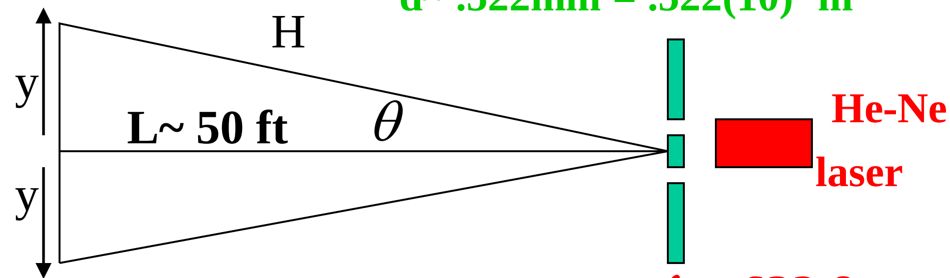
Measure 2y ~<u>2 cm</u>

$\Rightarrow$ **y~.1m**



Two-Slit Interference

d~ .522mm = .522(10)$^{-3}$m



$\lambda = 632.8\text{nm}$

$=6.33\,(10)^{-7}m$

$$\sin\theta = \frac{y}{H}$$

$$L = 50\,ft\,\frac{12in}{1\,ft}\,\frac{2.54cm}{1m}\,\frac{1m}{100cm} = 15.24m$$

$$d\sin\theta = \frac{\lambda}{2}$$

small angle approx.

$$H \approx L$$

$$\sin\theta = \frac{\lambda}{d}\frac{1}{2} = \frac{y}{H}$$

$$y = \frac{L\lambda}{2d} = \frac{(15.24)\;\;6.33(10)^{-7}}{2(.522)(10)^{-3}}m$$

$$= 92.4(10)^{-7+3}m$$

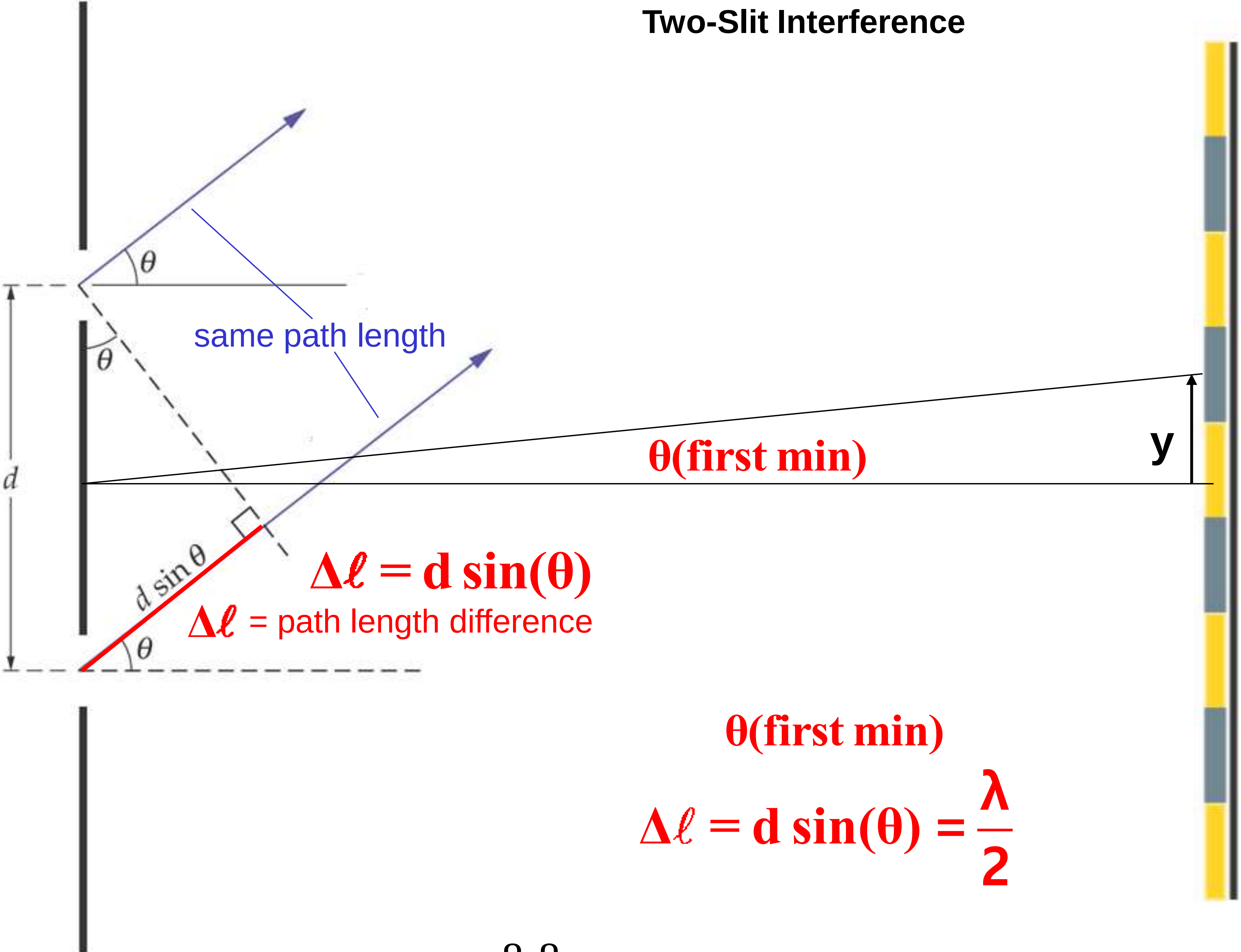
$$= .924(10)^{-7+3+2}m$$

$$y = .924(10)^{-2}m$$

$$\boxed{y = .924cm}$$ expect

$$\boxed{1\text{nm} \;\; =(10)^{-9}m}$$

Two-Slit Interference

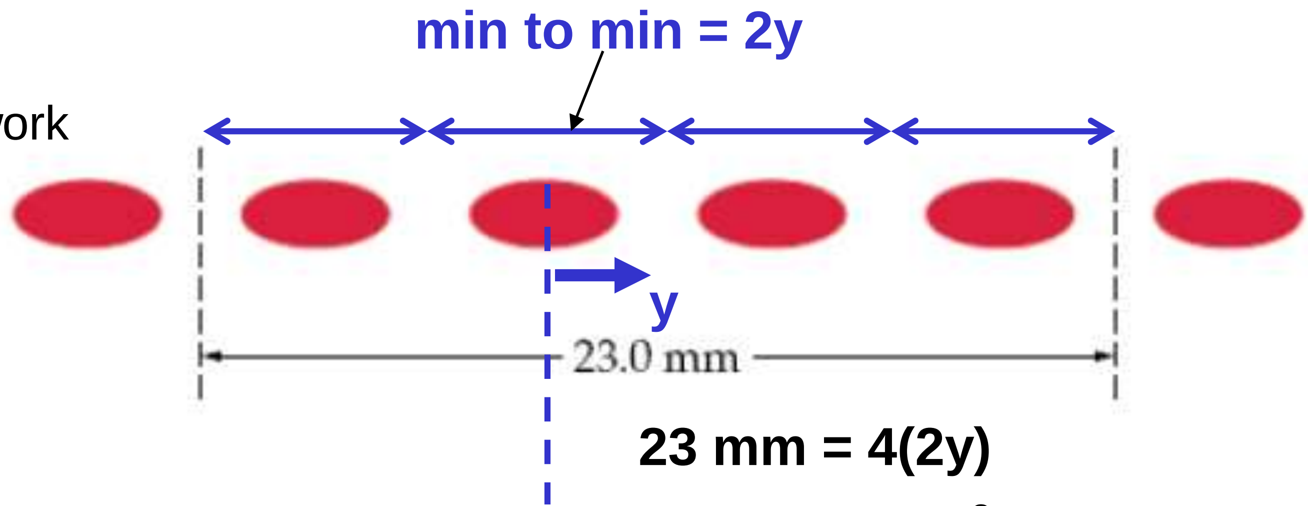


θ(first min)

$$\Delta\ell = d\sin(\theta) = \frac{\lambda}{2}$$

Two-Slit Interference

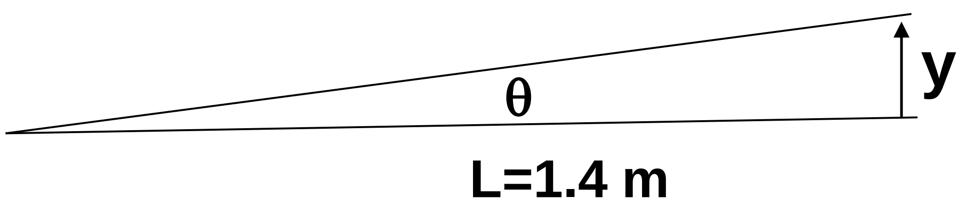
homework



23 mm = 4(2y)

$8y = 23\ (10)^{-3}$ m

$y = 2.875\ 10^{-3}$ m Use to find sin(θ)



$\tan(\theta) = y/L = 2.875\ 10^{-3}/1.4$

$\tan(\theta) = y/L = 2.05\ 10^{-3}$

$\tan(\theta) \sim \sin(\theta)$ here

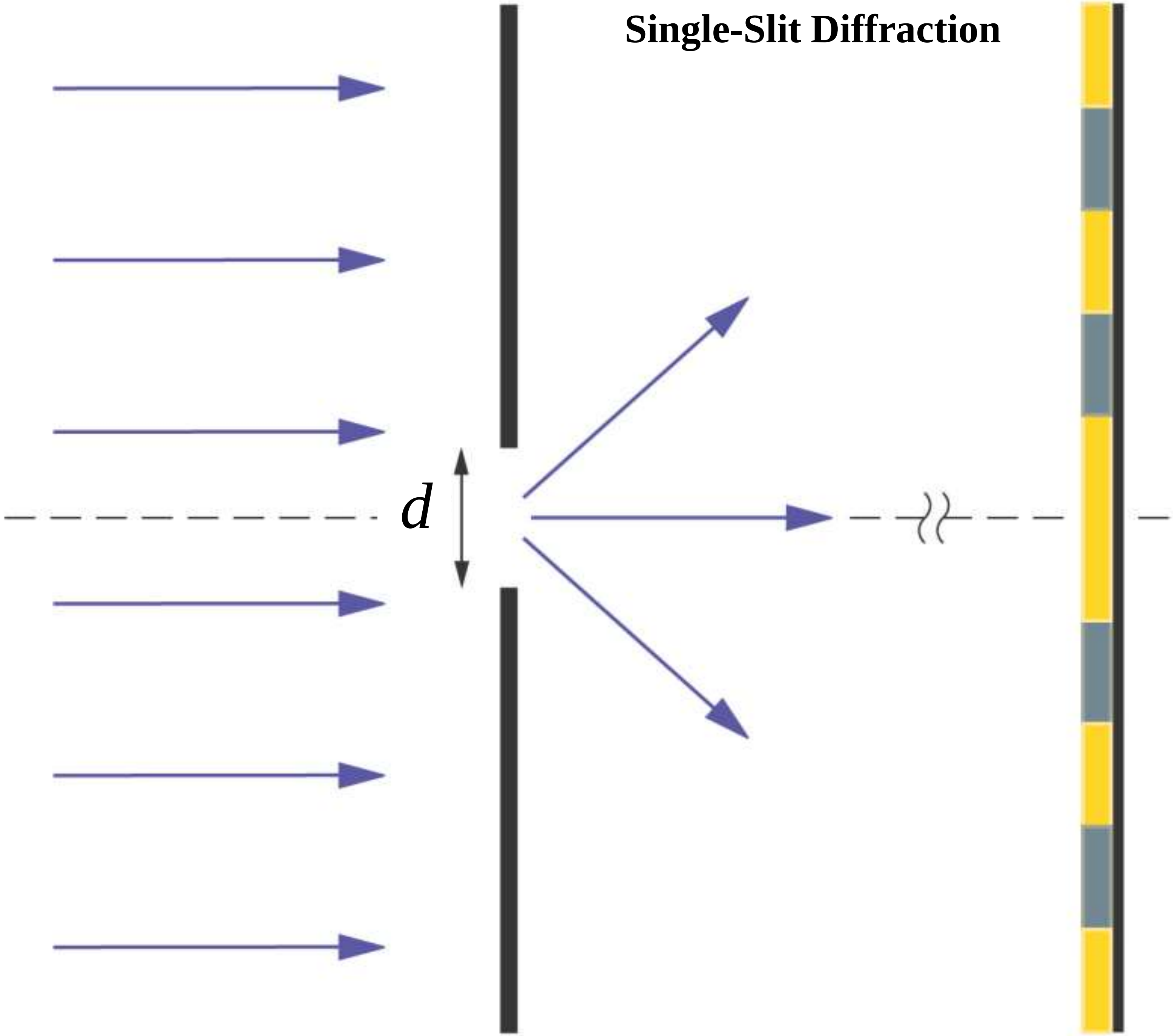
$d \sin(\theta) = \lambda/2$

$d = \lambda / [2 \sin(\theta)]$

$d = 632.9\ 10^{-9} / [2\ (2.05\ 10^{-3})\ ]$

$= 632.9\ 10^{-9} / 4.1\ (10^{3}) = 154\ 10^{-6}$ m

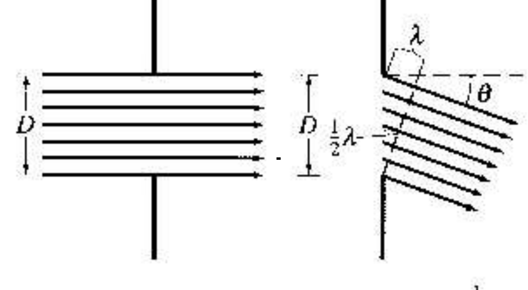
Single-Slit Diffraction



Single Slit Diffraction

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

First minimum



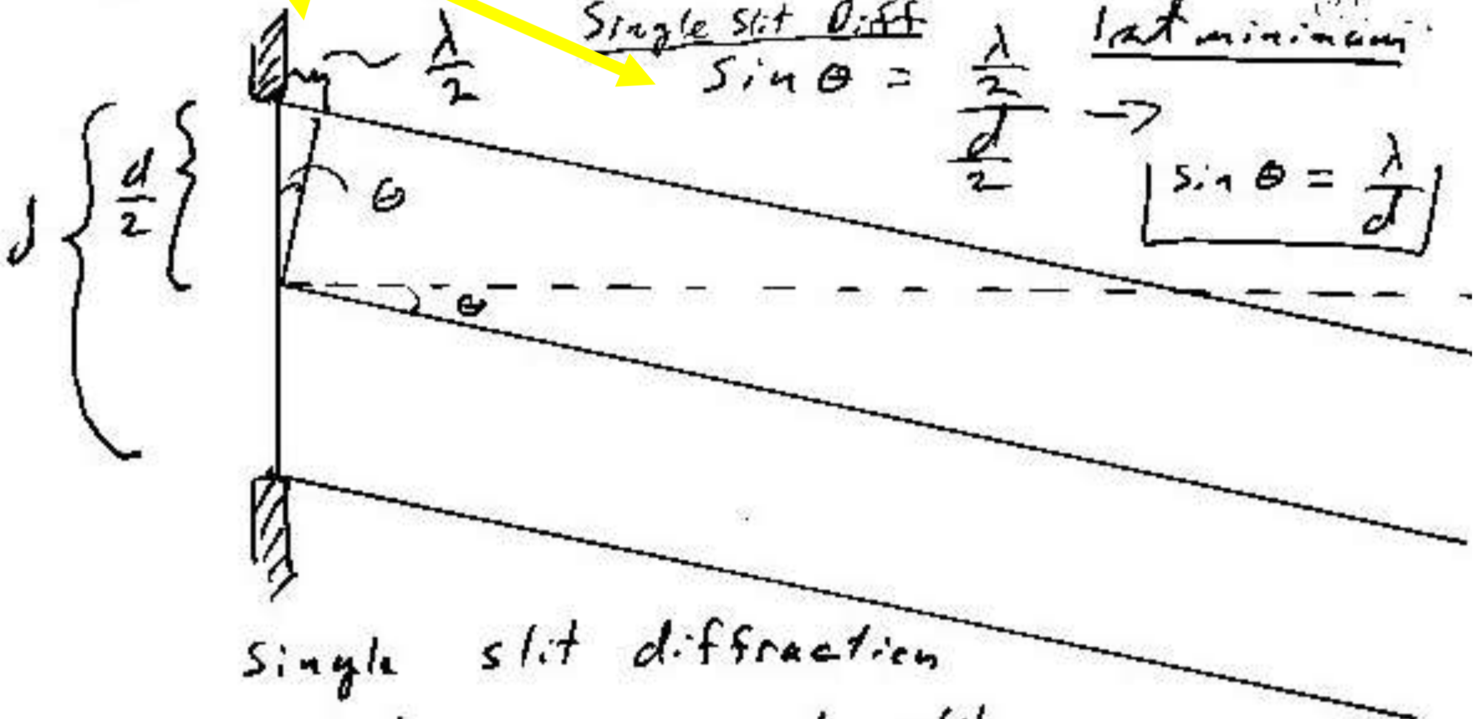
(a) $\theta = 0$
Bright

(b) $\sin\theta = \frac{\lambda}{D}$
Dark

Single slit Diff

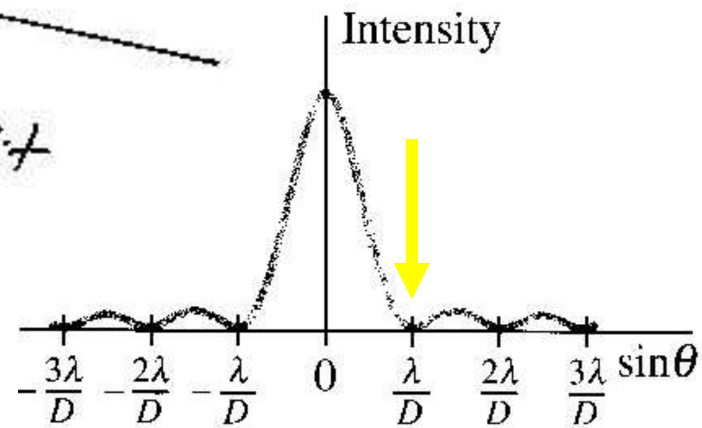
1st minimum

$$\sin\theta = \frac{\frac{\lambda}{2}}{\frac{d}{2}} \rightarrow \boxed{\sin\theta = \frac{\lambda}{d}}$$



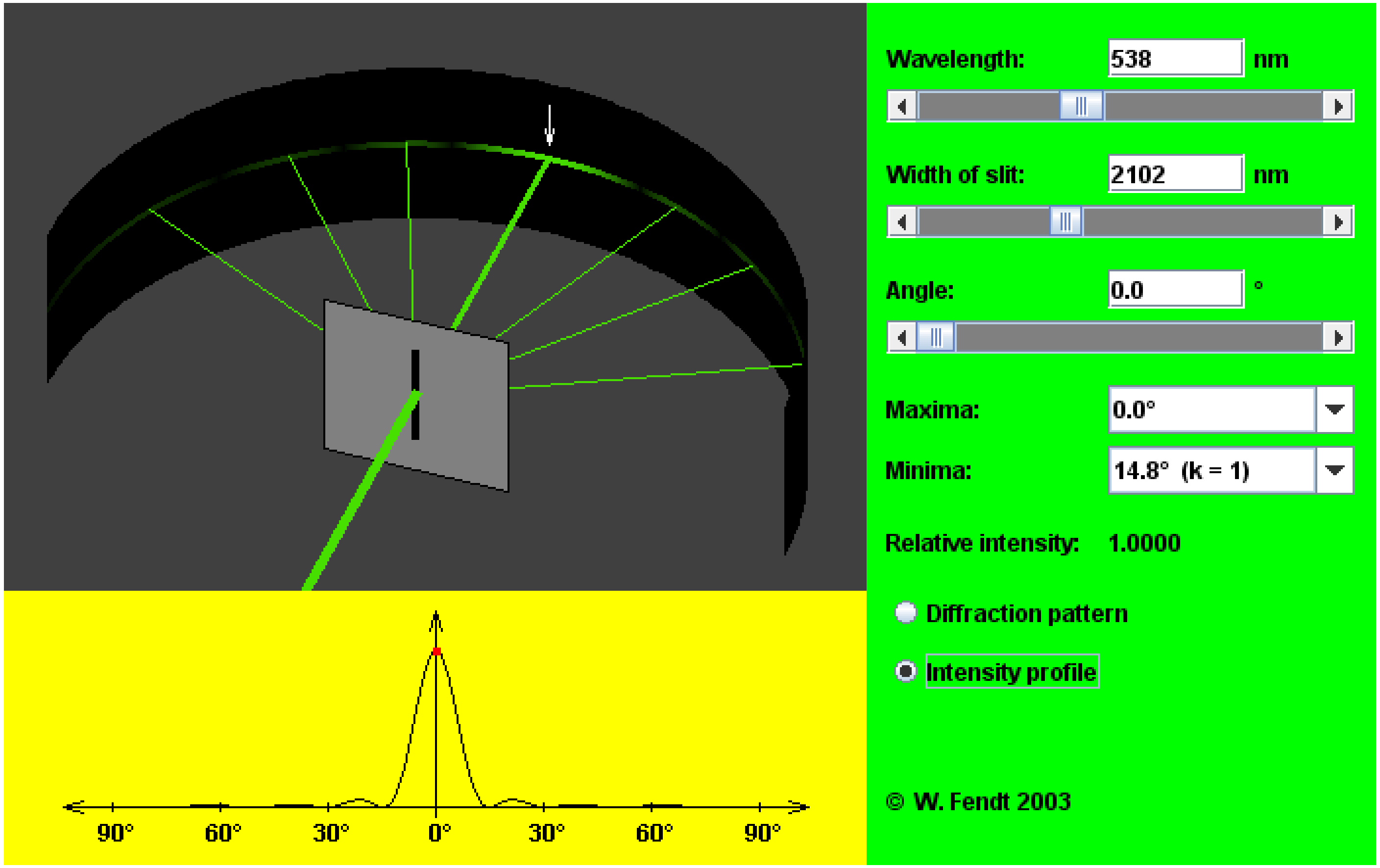
Single slit diffraction

1st min. top $\frac{1}{2}$ slit
cancels bottom $\frac{1}{2}$ slit



http://www.walter-fendt.de/html5/phen/singleslit_en.htm

Diffraction of Light by a Single Slit



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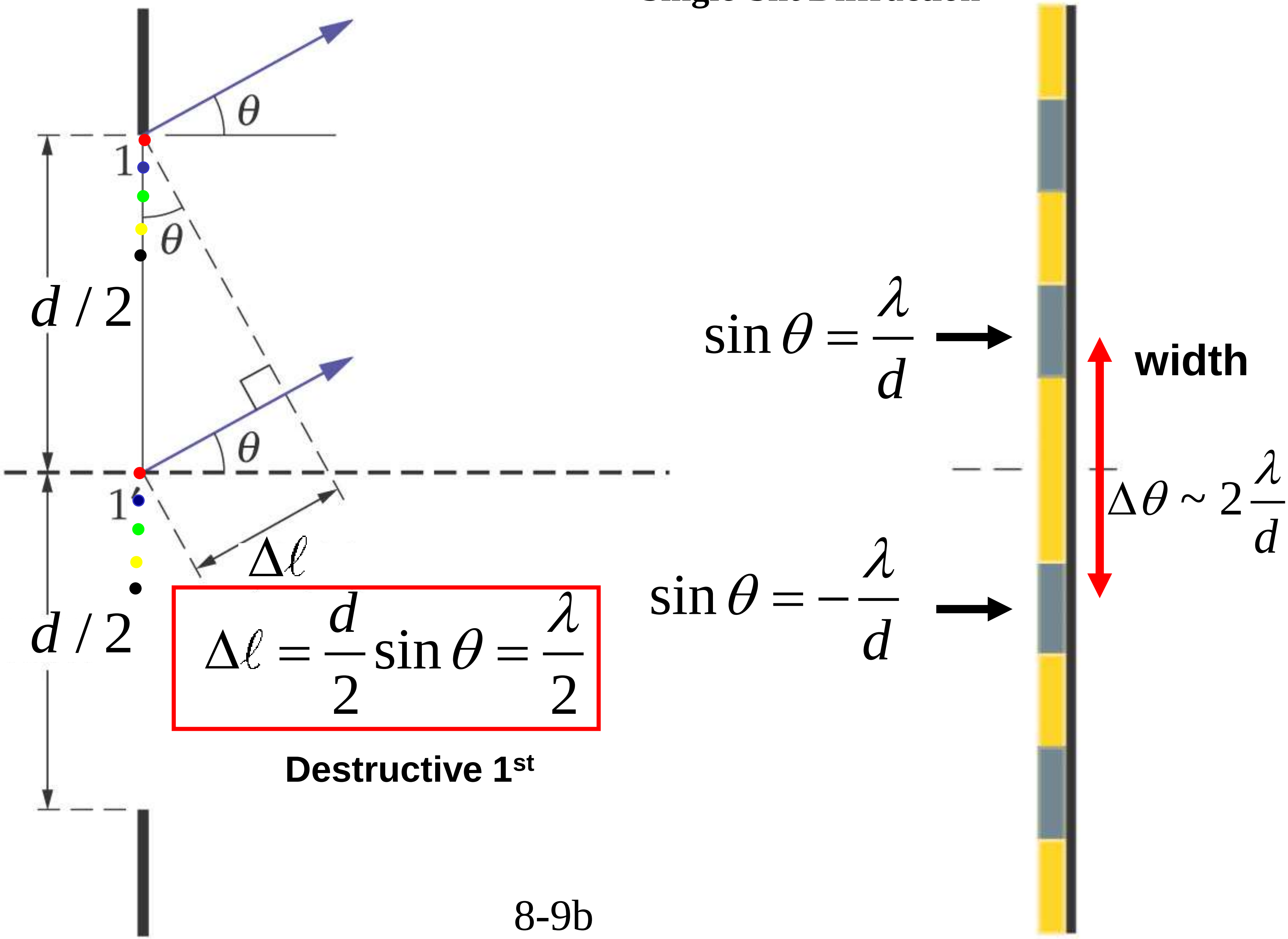
Single-Slit Diffraction



Single Slit Diffraction

$$\frac{d}{4}\sin\theta = \frac{\lambda}{2}$$

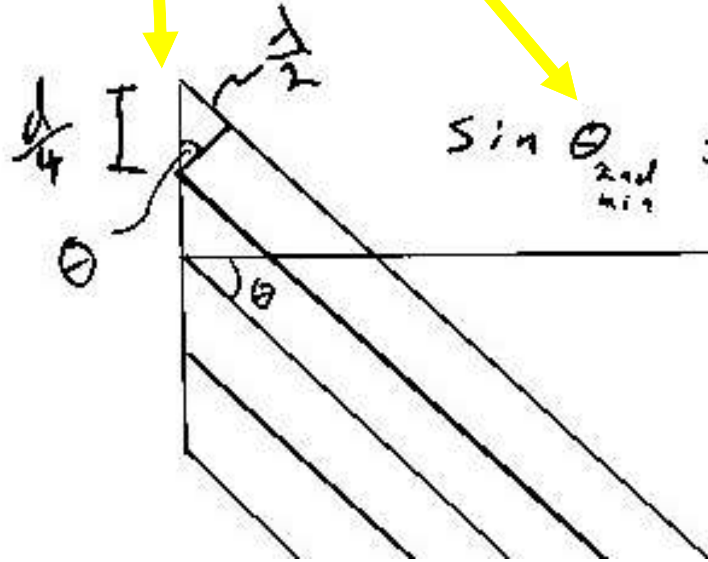
Single-Slit Diffraction

2nd minimum

Single slit diffraction 2nd min.

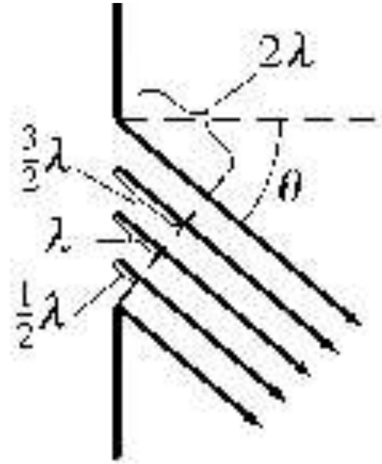
top $\frac{1}{4}$ cancels 2nd $1/4$

3rd $\frac{1}{4}$ cancels 4th $1/4$

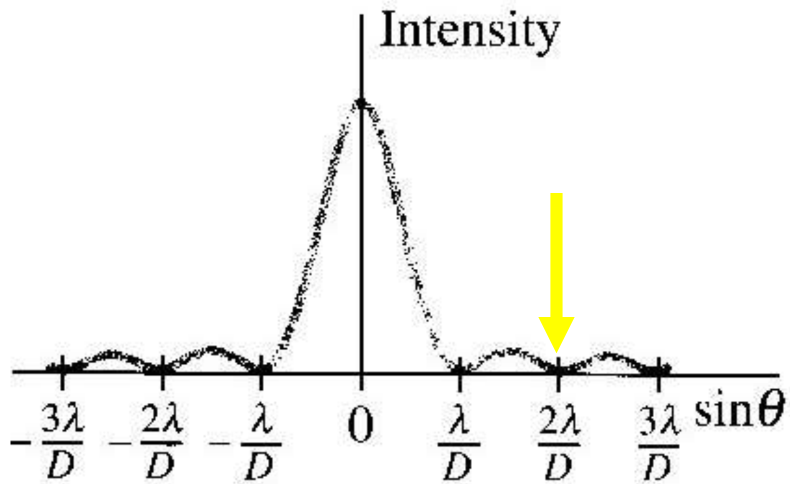


$$\sin\theta_{\substack{2nd\\min}} = \frac{\frac{\lambda}{2}}{\frac{d}{4}}$$

$$\left[\sin\theta_{\substack{2nd\\min}} = \frac{\lambda 2}{d}\right]$$

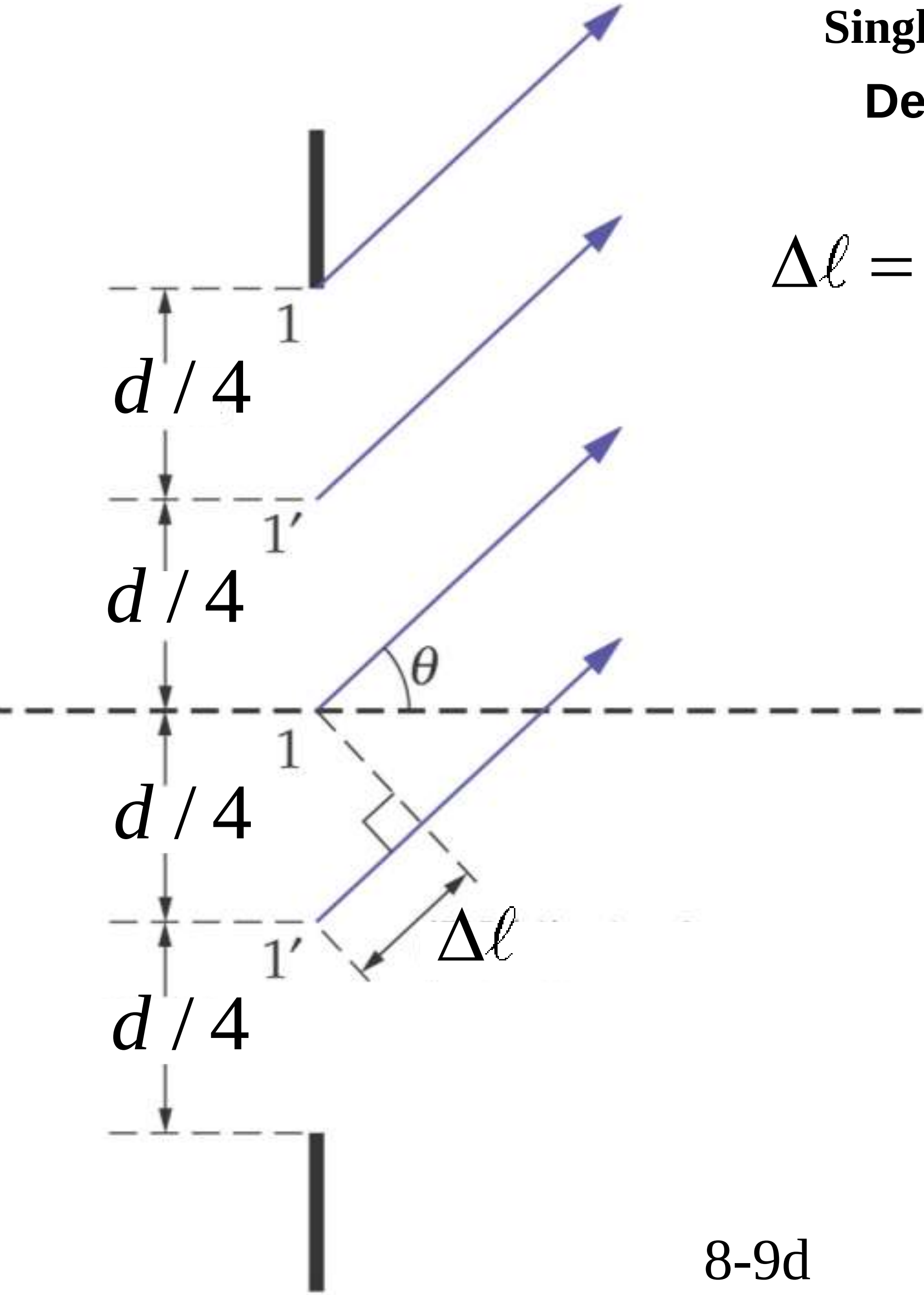


(d) $\sin\theta = \frac{2\lambda}{D}$
Dark



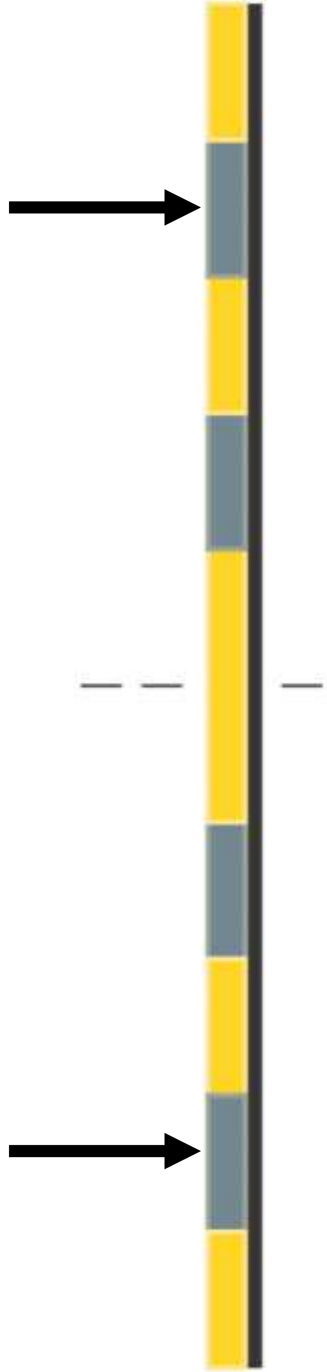
Single-Slit Diffraction

Destructive 2nd



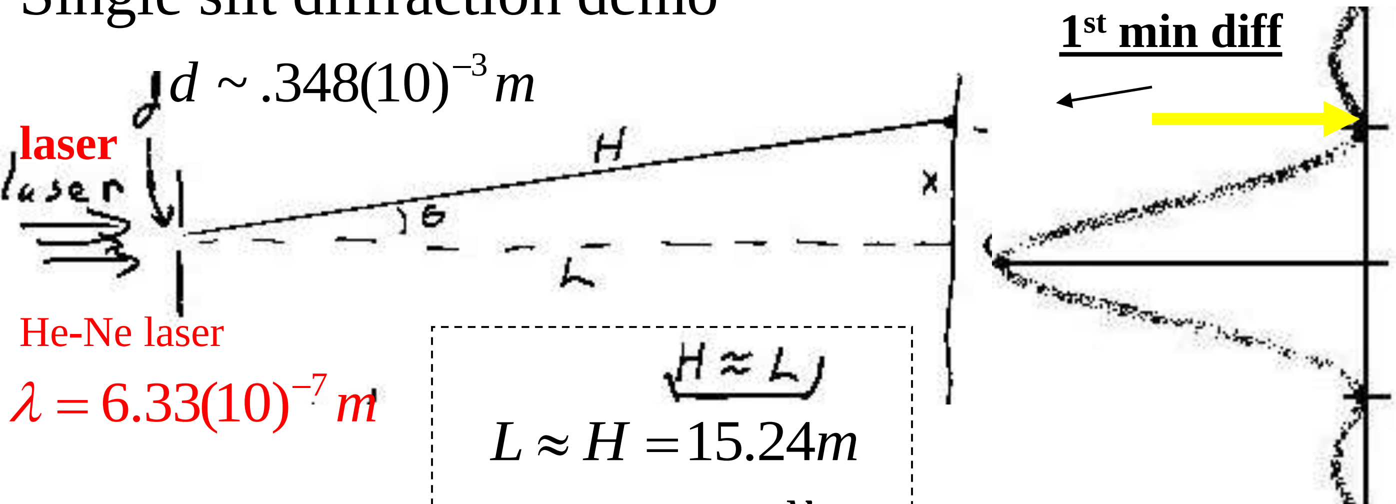
$$\Delta\ell = \frac{d}{4}\sin\theta = \frac{\lambda}{2}$$

$$\sin\theta = 2\frac{\lambda}{d}$$



Single slit diffraction demo

Single-Slit Diffraction



$d \sim .348(10)^{-3} m$

laser

1^{st} min diff

He-Ne laser

$\lambda = 6.33(10)^{-7} m$

$L \approx H = 15.24m$

$\sin\theta = \frac{y}{L}$ **small angle approx.**

First minimum

$$\boxed{\underline{\mathbf{1^{st}\ min}} \quad \Rightarrow \frac{d}{2}\sin\theta = \frac{\lambda}{2}}$$

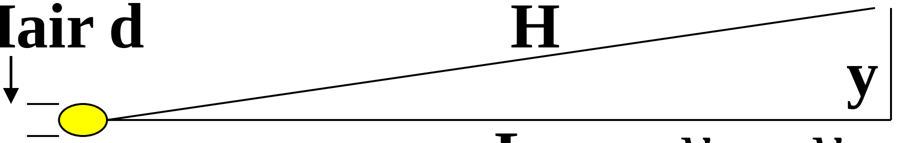
$$\frac{d}{2}\frac{y}{L} = \frac{\lambda}{2} \rightarrow y = \lambda\frac{L}{d}$$

$$y = \frac{(6.33)(10)^{-7} m(15.24m)}{.348(10)^{-3} m}$$

$$= 277(10)^{-4} m = 2.8cm = 3\text{cm}$$

Width of Prof. Croft's Hair

Hair d

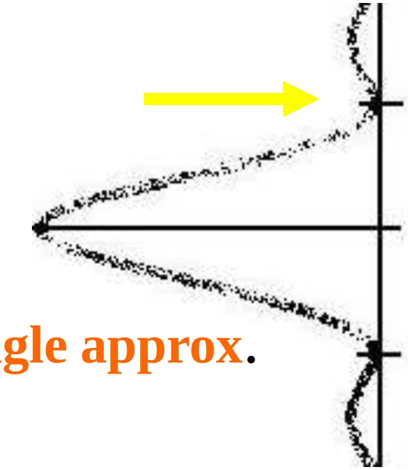


Laser

635 mn

6.35 $(10)^{-7}$ m

$$\sin\theta \sim \frac{y}{L} \sim \frac{y}{H}$$ small angle approx.



1st min

1st min at y=7mm= $7(10)^{-3}$m

With L~H~1m

<u>1st min diff</u>

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

$$\sin\theta = \frac{\lambda}{d}$$

$$\frac{y}{L} = \frac{\lambda}{d} \rightarrow d = \frac{\lambda}{y}L$$

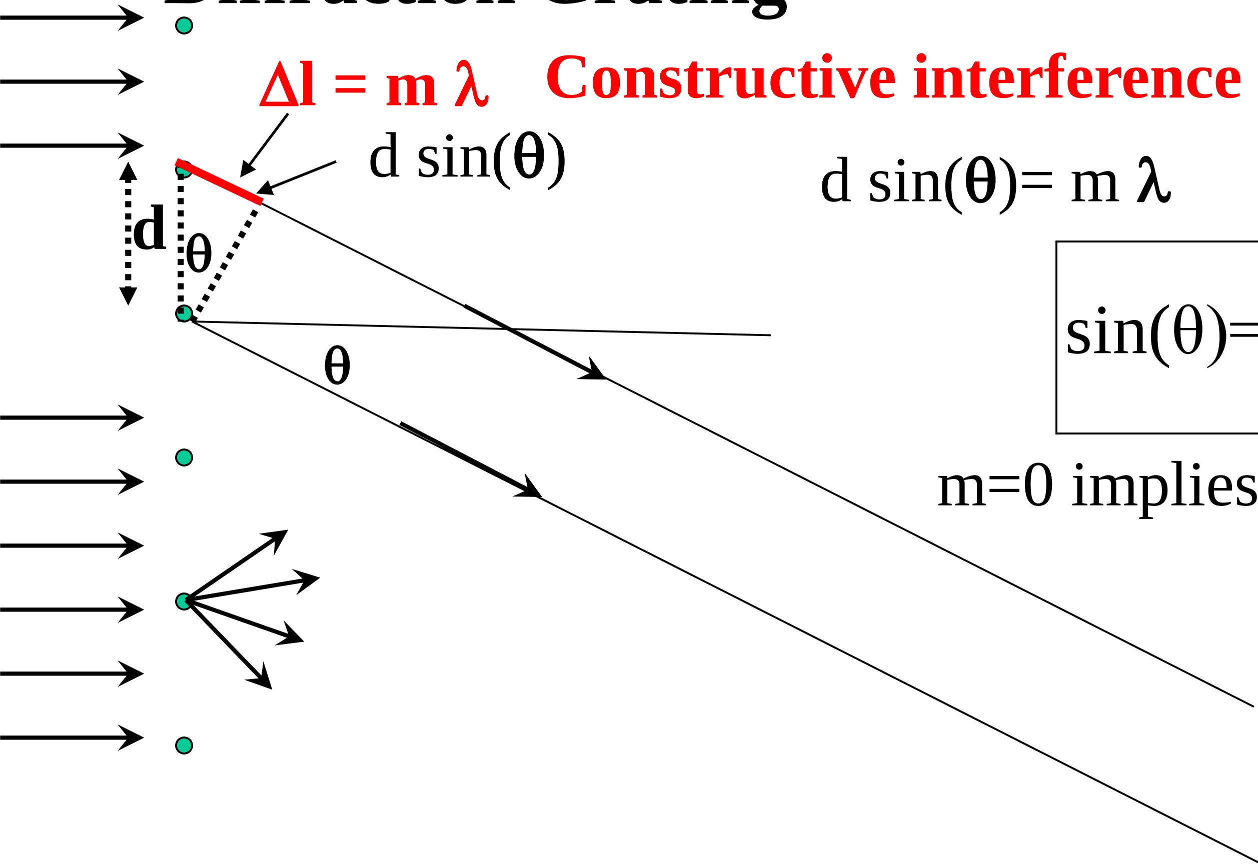
Blocking screen shape gives same pattern as hole in screen

$$d_{hair} = \frac{6.35(10)^{-7}\,m\;1m}{7(10)^{-3}m}$$

$$d_{hair} = .9(10)^{-4}\,m$$

$$= .09(10)^{-3}\,m = .09mm \quad \text{or } =.1\text{mm}$$

Diffraction Grating



Constructive interference m = 0, 1, 2….

$d \sin(\theta) = m\,\lambda$

$$\boxed{\sin(\theta) = m\frac{\lambda}{d} \quad m = 0, 1, 2\ldots.}$$

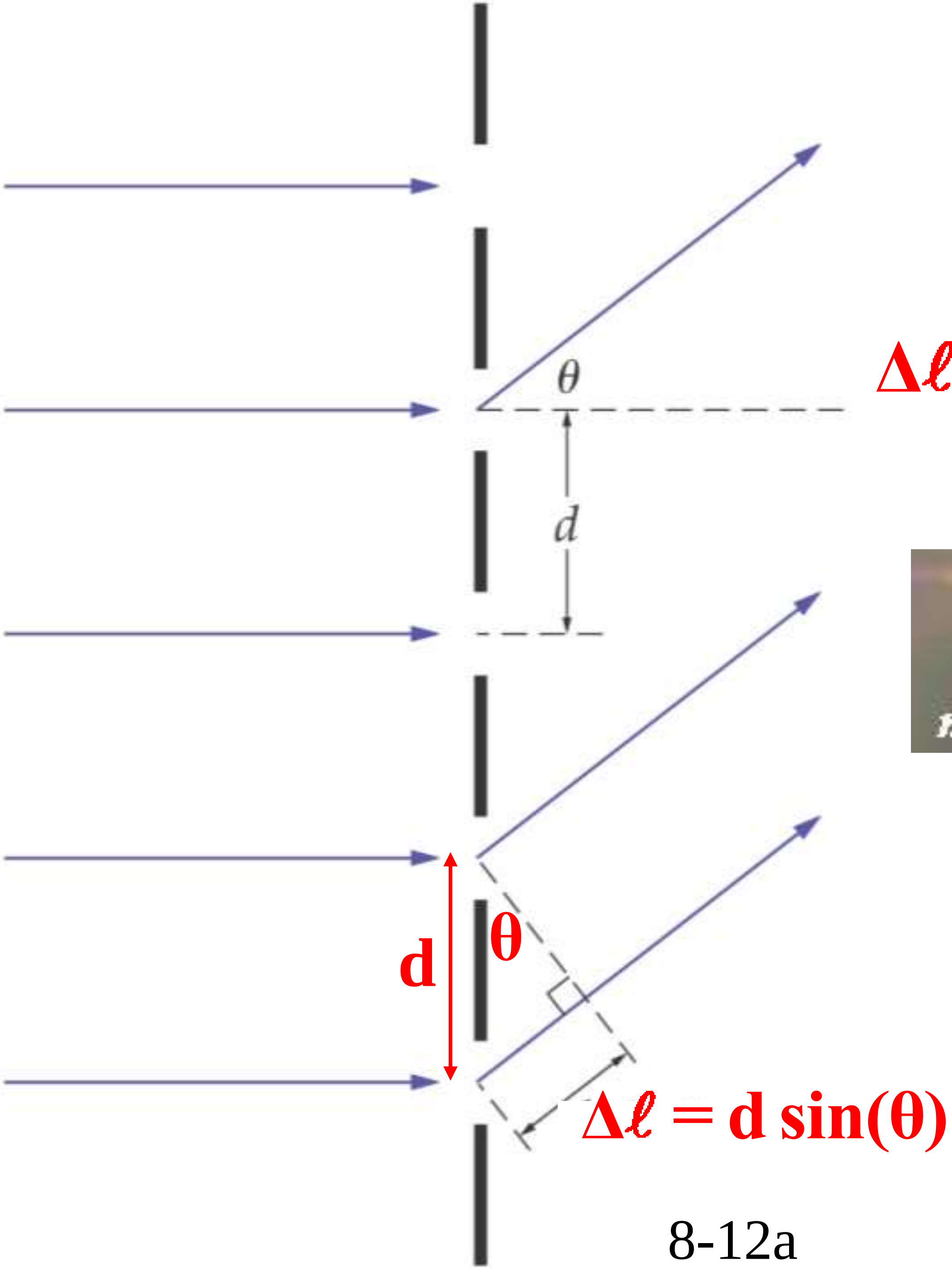
m=0 implies **θ=0 (straight through)**

m>0 depends on λ

Can use to diffract visible light from

$4000 - 6000\ \overset{0}{\mathrm{A}}\ [\overset{0}{\mathrm{A}} = 10^{-8} cm = 10^{-10} m]$

Diffraction Grating



$$\Delta\ell = d\sin(\theta)$$

constructive

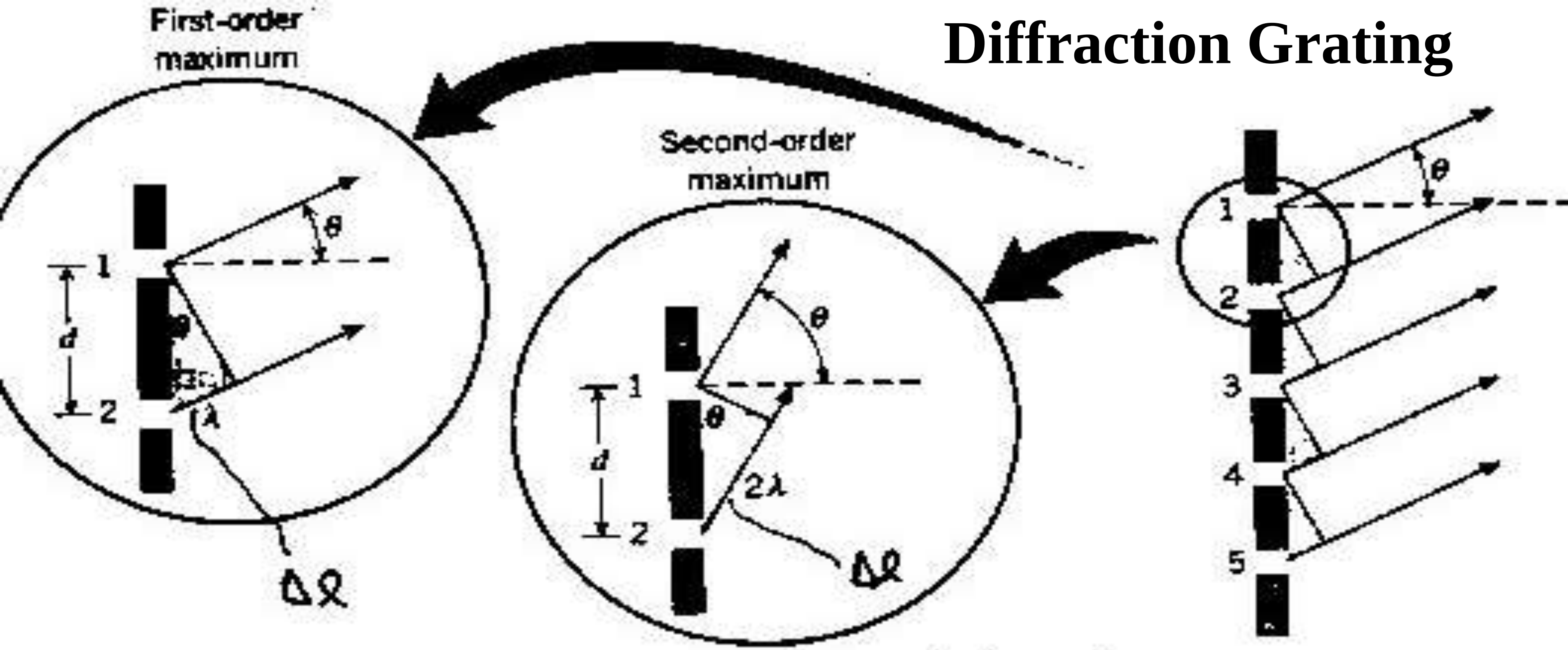
$$\Delta\ell = 0, \lambda, 2\lambda, \ldots, m\lambda$$

“white light” many λ’s

different λ ⇔ different θ

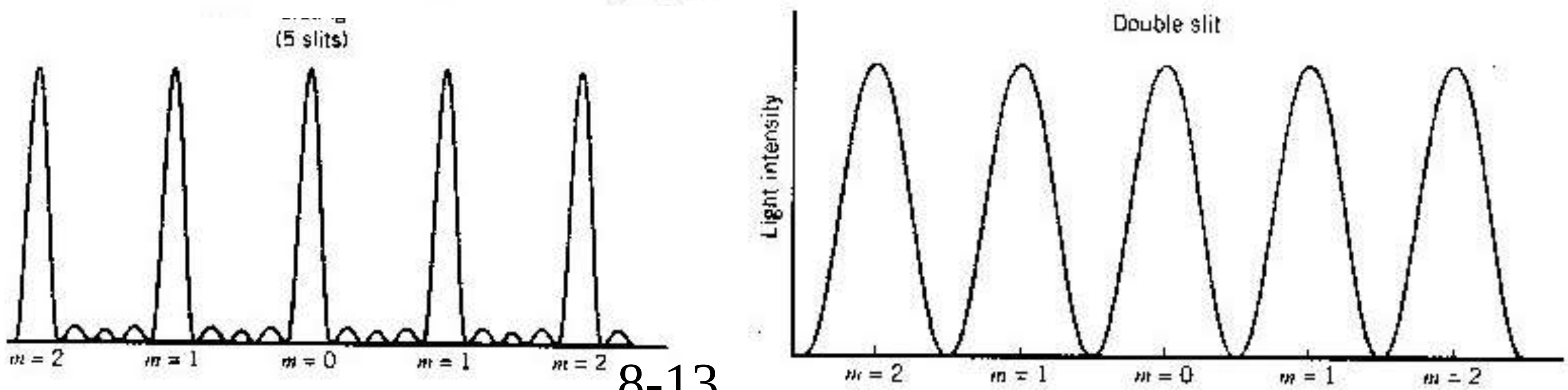
Diffraction Grating separates λ’s

Diffraction Grating



$$\Delta \ell = d \sin\theta = m\lambda$$

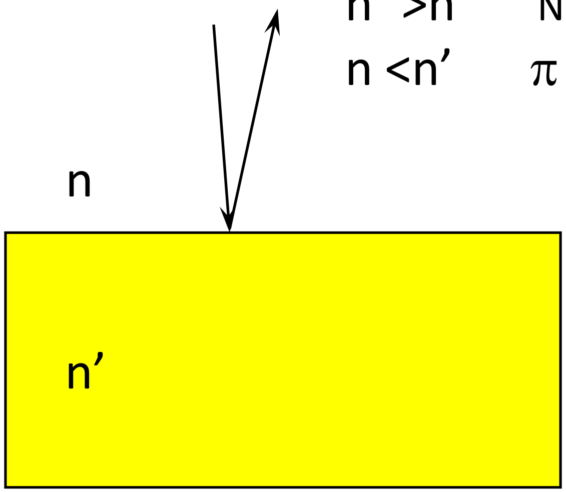
Grating
(5 slits)



Rules for interface reflection

n >n' NO PHASE CHANGE

n <n' π PHASE CHANGE**= EXTRA PATH OF** λ /2



Transmission slows down

n <n'

Reflection

π PHASE CHANGE**= EXTRA PATH OF** λ /2

Transmission speeds up

n >n'

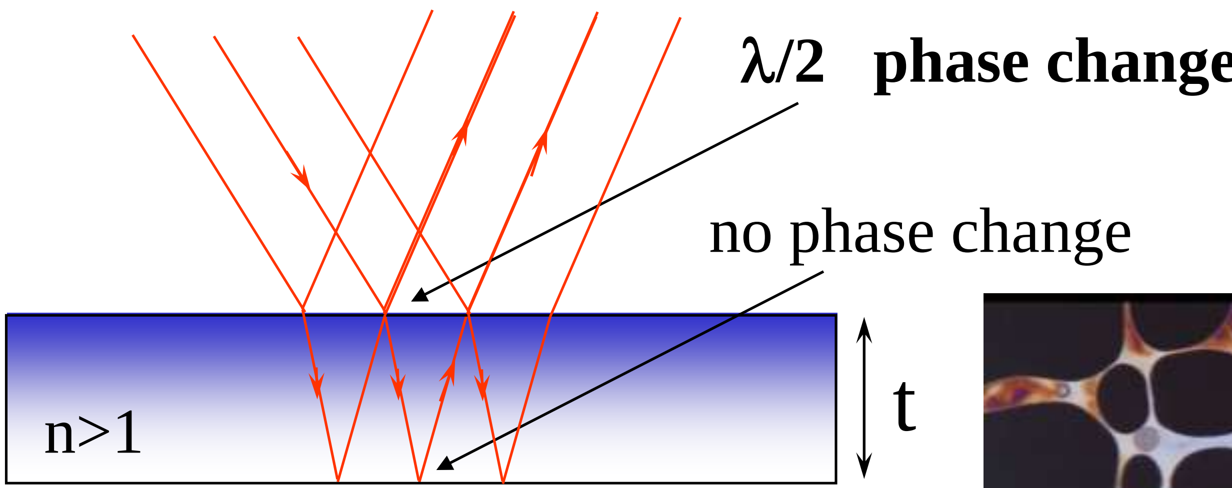
Reflection

NO PHASE CHANGE

http://www.kettering.edu/physics/drussell/Demos/reflect/reflect.html

Simple effect – interface reflection <u>only</u> interference

Just before the soap film pops, it goes dark



Black regions t << λ

If thickness t << λ, then

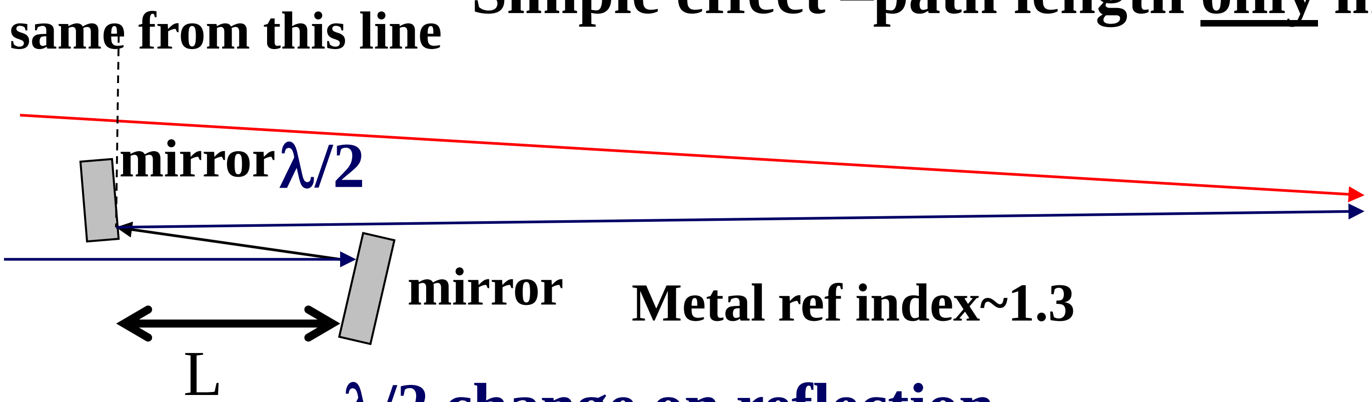
$2nt/\lambda << 2$

Destructive interference for all wavelengths

No reflected light

https://www.youtube.com/watch?v=6qzQIsg5eTE&feature=youtu.be

Simple effect –path length only interference



λ/2 change on reflection

$$\Delta\ell = 2\,L$$ **Path length difference**

λ/2+ λ/2= λ : 2 reflections
No reflection effect

$$\Delta\ell = 2\,L = \lambda/2$$ **destructive interference (1st)**

$$\Delta\ell = 2\,L = \lambda$$ **constructive interference (1st)**

$$\Delta\ell = 2\,L = \lambda\ \mathrm{m} \qquad \mathrm{m}=1,2,3,4,5\ldots$$ **constructive interference**

$$\Delta\ell = 2\,L = (\,\mathrm{m}\lambda + 1/\,2) \qquad \mathrm{m}=0,1,2,3,4,5\ldots$$ **destructive interference**

Note: λ difference = 0 difference

interference between rays reflected

THIN FILM INTERFERENCE

- reflected from top
- & from bottom interface.

depends on

$n_1 > n_2 > n_3$

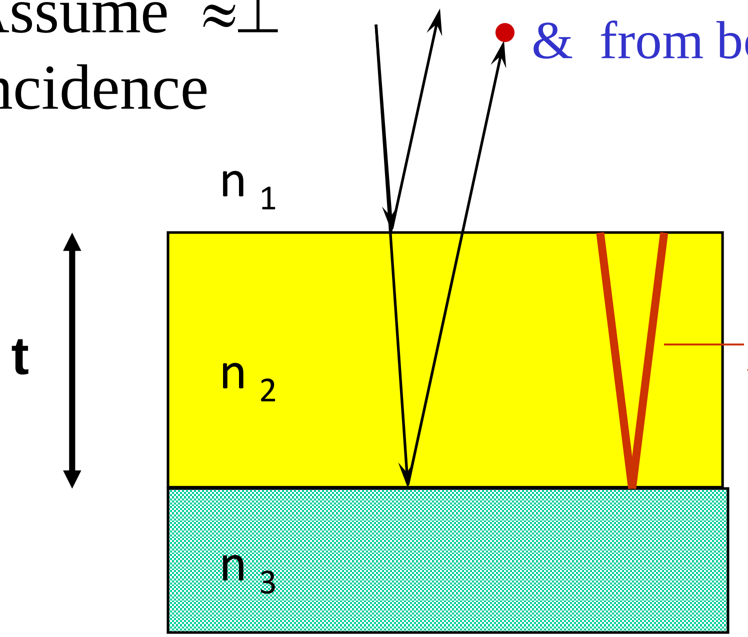
$n_1 > n_2 < n_3$

$n_1 < n_2 < n_3$

$n_1 < n_2 > n_3$

...

Assume $\approx\perp$ incidence



PATH DIFF. IN FILM

$\Delta \ell = 2\,t$

tot. inter. effect	=	reflection effect	+	path length effect
specify		**figure out**		**choose to fit**

Case $\mathbf{n_1 < n_2 < n_3}$

$\pi : \lambda/2$ change $\qquad \pi : \lambda/2$ change

reflection changes – no effect

Case 1a

constructive

"thinnest"

$\Delta \ell = 2\,t = \lambda_{n2} = \lambda / n_2$

$t = \lambda / 2n_2$

Case 1b

destructive

"thinnest"

$\Delta \ell = 2\,t = \lambda_{n2}/2 = \lambda / 2n_2$

$t = \lambda / 4n_2$

 Just path length gives effect

Example anti-reflection coating in green 532 nm

Want λ/2 for destructive interference

Want to interfere **destructive** (cancel)

Air	$n_1 = 1$	
MnF2	$n_2 = 1.38$	t $\Delta\ell = 2t$
glass	$n_3 = 1.50$	

$n_1 < n_2$ $\pi : \lambda/2$ change

same

$n_2 < n_3$ $\pi : \lambda/2$ change

have no interface reflection effect

tot. inter. = reflection + path length
effect effect effect

0

Want λ/2 for destructive interference must come from path length

min. thickness

$$\frac{\lambda_{n2}}{2} = \Delta\ell = 2t$$

$$t = \left(\frac{1}{2}\right)\frac{\lambda_{n2}}{2} \longleftarrow \lambda_{n2} = \frac{\lambda}{n_2}$$

$$t_{min} = \frac{1}{4}\frac{\lambda}{n_2}$$

$$\mathbf{t_{min} = 96.4\ nm} \longleftarrow \mathbf{t_{min} = \frac{1}{4}\frac{532}{(1.38)}\ nm}$$

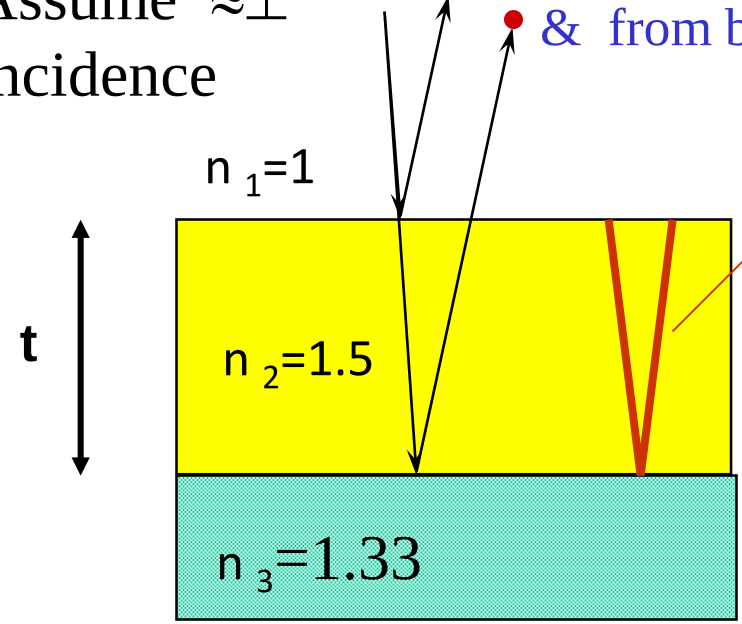
Note: λ difference = 0 difference

THIN FILM INTERFERENCE

interference between rays reflected

- reflected from top
- & from bottom interface.

Assume $\approx\perp$ incidence



PATH DIFF. IN FILM

$\Delta \ell = 2\,t$

example

Constructive " enhanced" reflection

tot. inter. effect	=	reflection effect	+	path length effect
specify		figure out		choose to fit

λ

constructive "thinnest"

$\lambda/2$

must undo

$\Delta \ell = 2\,t = \lambda_{n2}/2 = \lambda/2\,n_2$

$t = \lambda / 4n_2$

Case $n_1 < n_2 > n_3$

$n_1=1$ $\quad$ $n_2=1.5$ $\quad$ $n_3=1.33$

$\pi : \lambda/2$ change $\quad$ no change

Reflection change

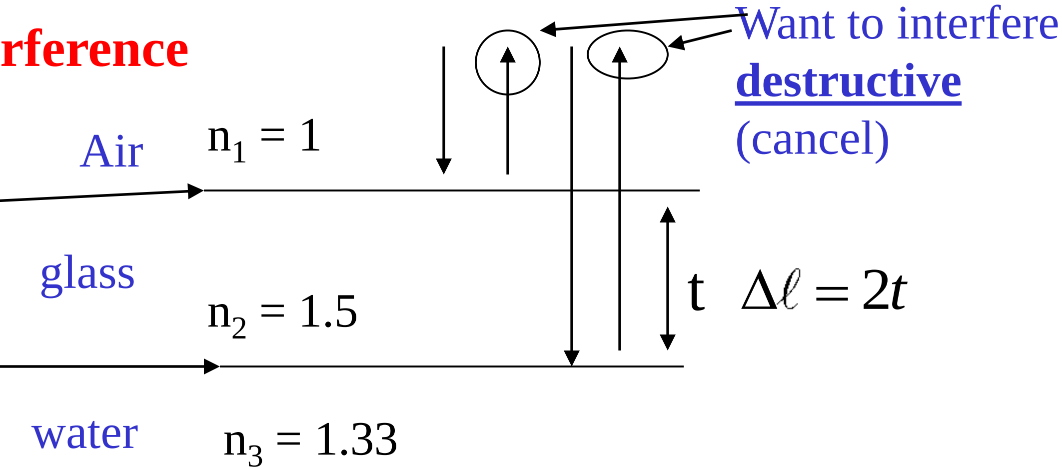
$\pi : \lambda/2$

Note: λ difference = 0 difference

Example anti-reflection coating in green 532 nm

Want $\lambda/2$ for destructive interference

$n_1 < n_2$ $\pi : \lambda/2$	change
$n_2 > n_3$	no change



$\lambda/2$ for destructive interference from reflection only

Want λ from path length

min. thickness

tot. inter. effect = reflection effect + path length effect

Want $\lambda/2$ | **$\lambda/2$** | **need λ**

$$\lambda_{n2} = \Delta\ell = 2t$$

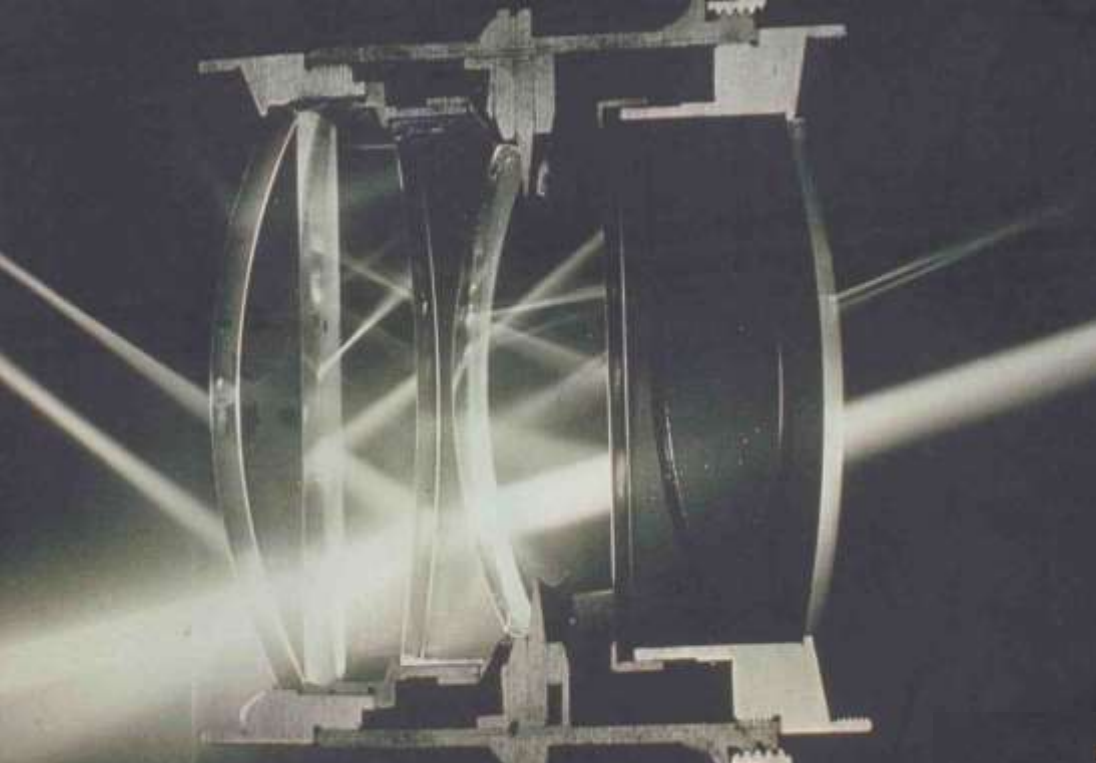
$$t = \frac{\lambda_{n2}}{2} \longleftarrow \lambda_{n2} = \frac{\lambda}{n_2}$$

$$t_{\min} = \frac{1}{2}\frac{\lambda}{n_2}$$

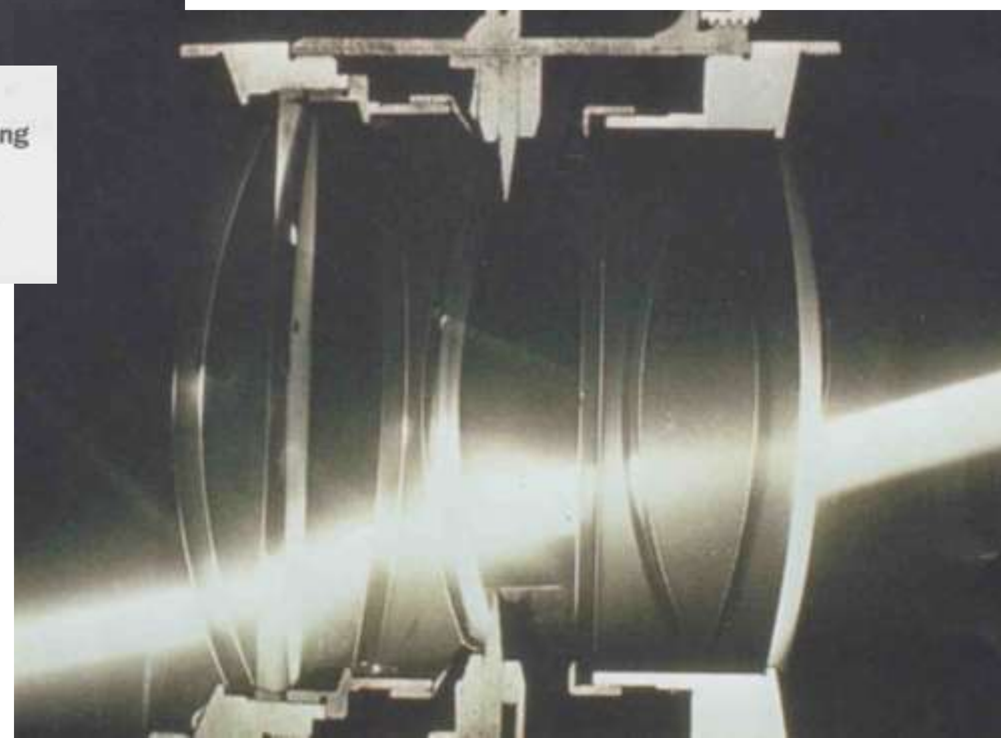
$$\mathbf{t_{min}} = \frac{1}{2}\frac{532}{(1.5)}\,\mathbf{nm}$$

$$\boxed{\mathbf{t_{min}} = 177\ \mathbf{nm}}$$

Note: λ difference = 0 difference

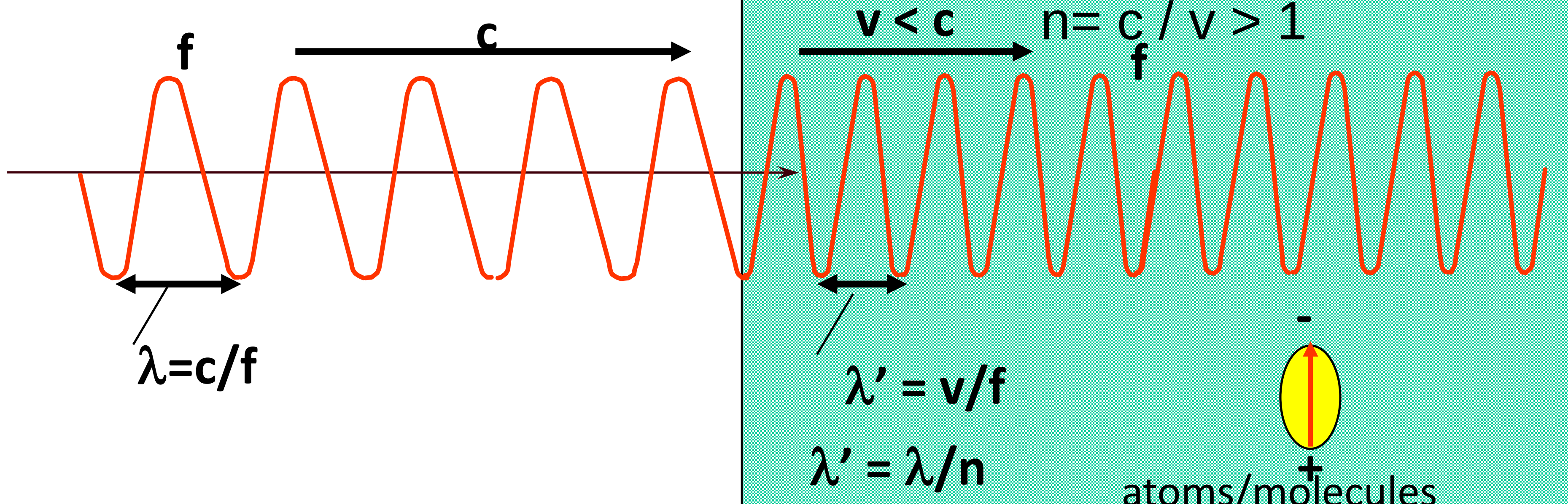


Uncoated Lenses and Coated Lenses.
Top: A cross section of compound lenses in a camera shows light scattering from uncoated glass surfaces, *Bottom*: and their suppression by a thin antireflective surface coating on the lenses, greatly improving the image. Goro Studio, New York, 1944.



EM wave in vacuum

EM wave in material
light slower in medium.
Index of refraction:



atoms/molecules
must distort
- slows propagation

IM MATERIAL
FREQUENCY UNCHANGED.
WAVELENGTH IS SHORTER

	n
AIR	1.000293
ICE	1.309
WATER	1.333
GLASS	1.523
DIAMOND	2.419

http://stwww.weizmann.ac.il/Lasers/laserweb/Ch-1/F1s1t2p2.htm

Brewster's Angle