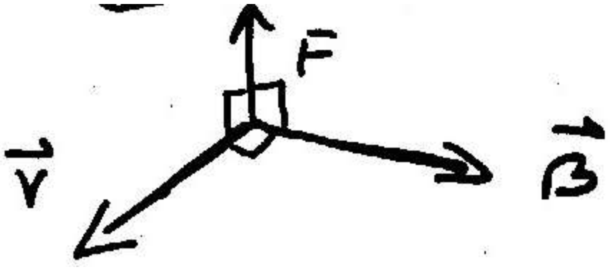
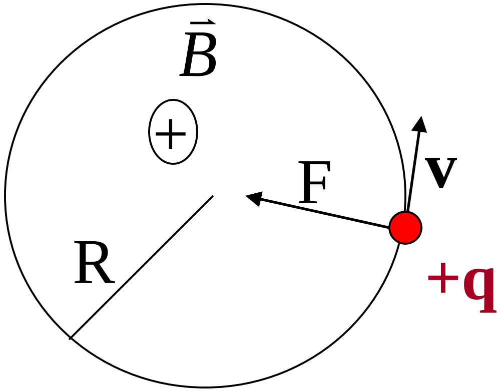


$F = qv_\perp \; B = qv\, B_\perp$



$$\mathbf{F = I\,\ell\,B_\perp = I\,\ell\,B\sin(\theta)}$$

$$\mu_0 \; I = \sum_{\text{edge}} B_{\|} \; \Delta l$$ **Ampere's Law**

$$\mu_o I = B\, 2\pi R \qquad \Rightarrow B = \frac{\mu_o\, I}{2\pi R}$$

$$B = I\, n\, \mu_0$$

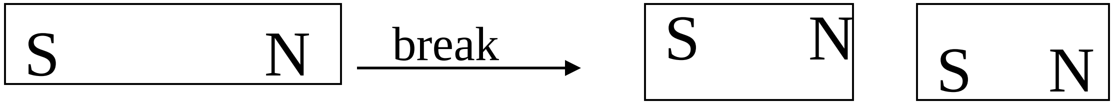
Magnetic Interactions

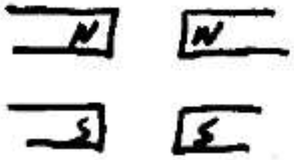
1.) Force at a distance

2.) Repulsive and attractive

Like electrostatics!

Permanent magnets





repulsive

attractive!

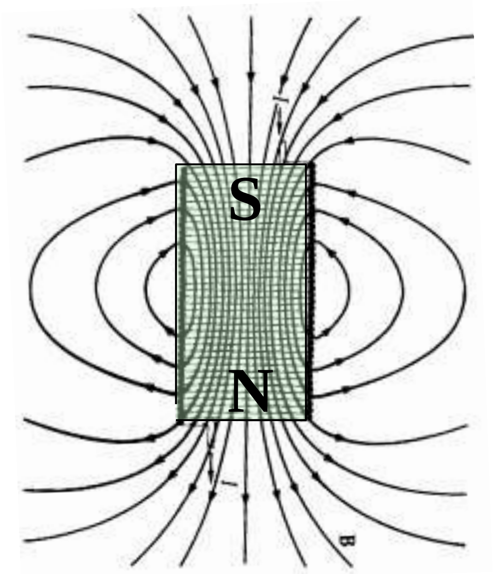
NO MAGNETIC MONOPOLES

Magnets and Magnetic fields : Dipole Fields

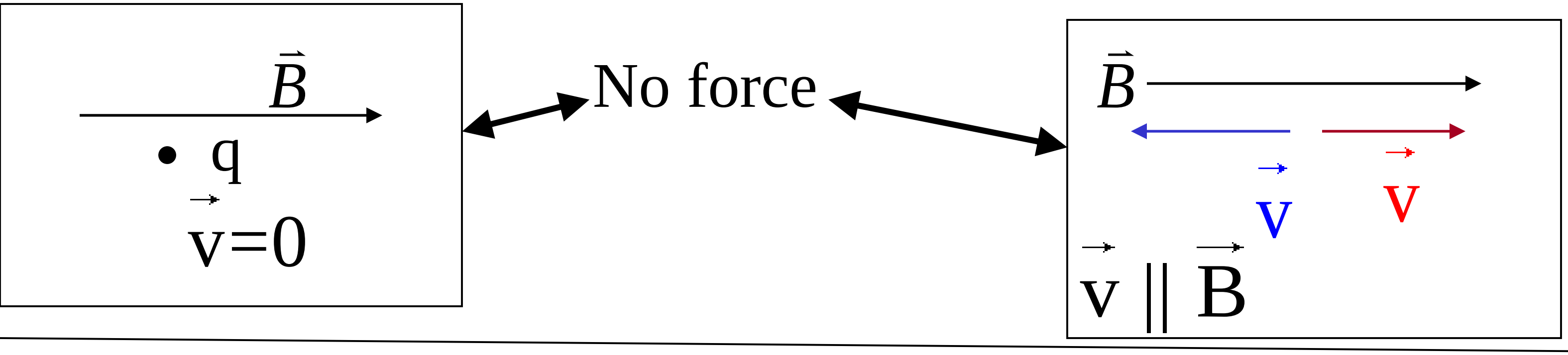
Magnetic field $\vec{B}$ {Like E}

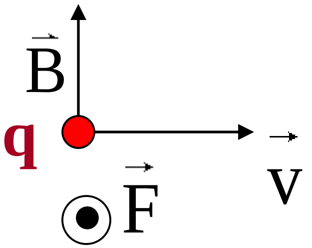
Magnetic field lines

$$B - \text{ units } \ \text{Tesla} = \frac{\text{kg}}{\text{CS}}$$



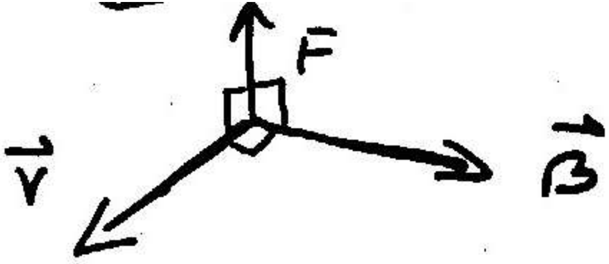
Forces on charge due to $\vec{B}$ (magnetic field)





$\vec{v} \perp \vec{B}$ ∃ F force

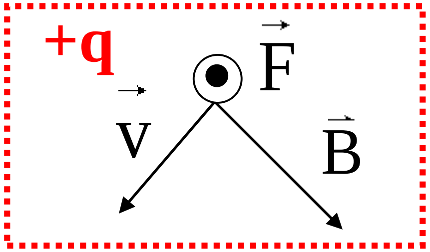
direction by **right hand rule**!

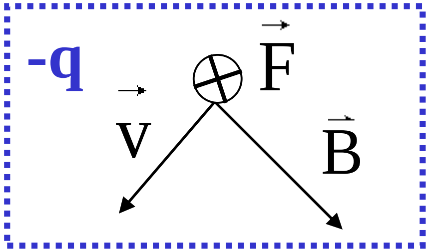


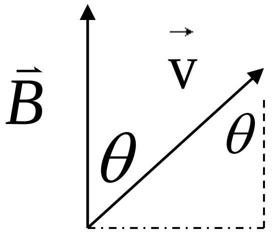
note: ⊙ =vector at you

⊗ = vector into page

+q & -q opposite







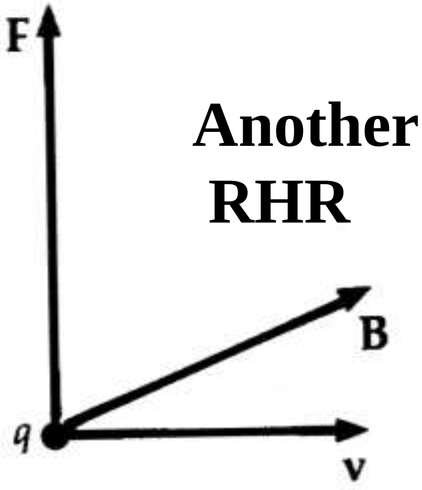
$v_\perp = v \sin\theta$

$$F = q v_\perp B = q v B_\perp$$

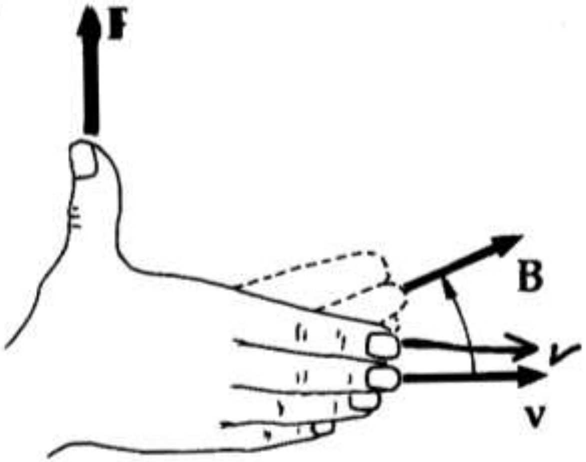
$$= q\, v\, B \sin\theta$$

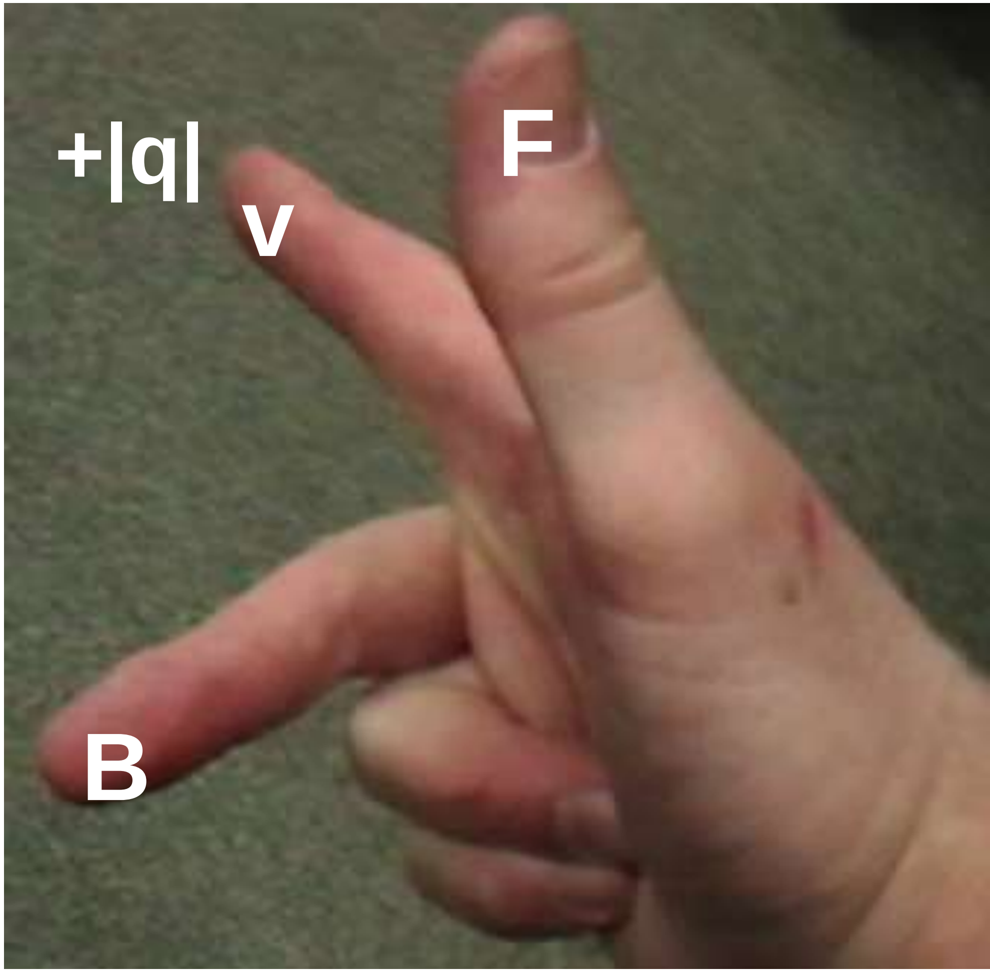
Croft preferred RHR

direction by **right hand rule**!



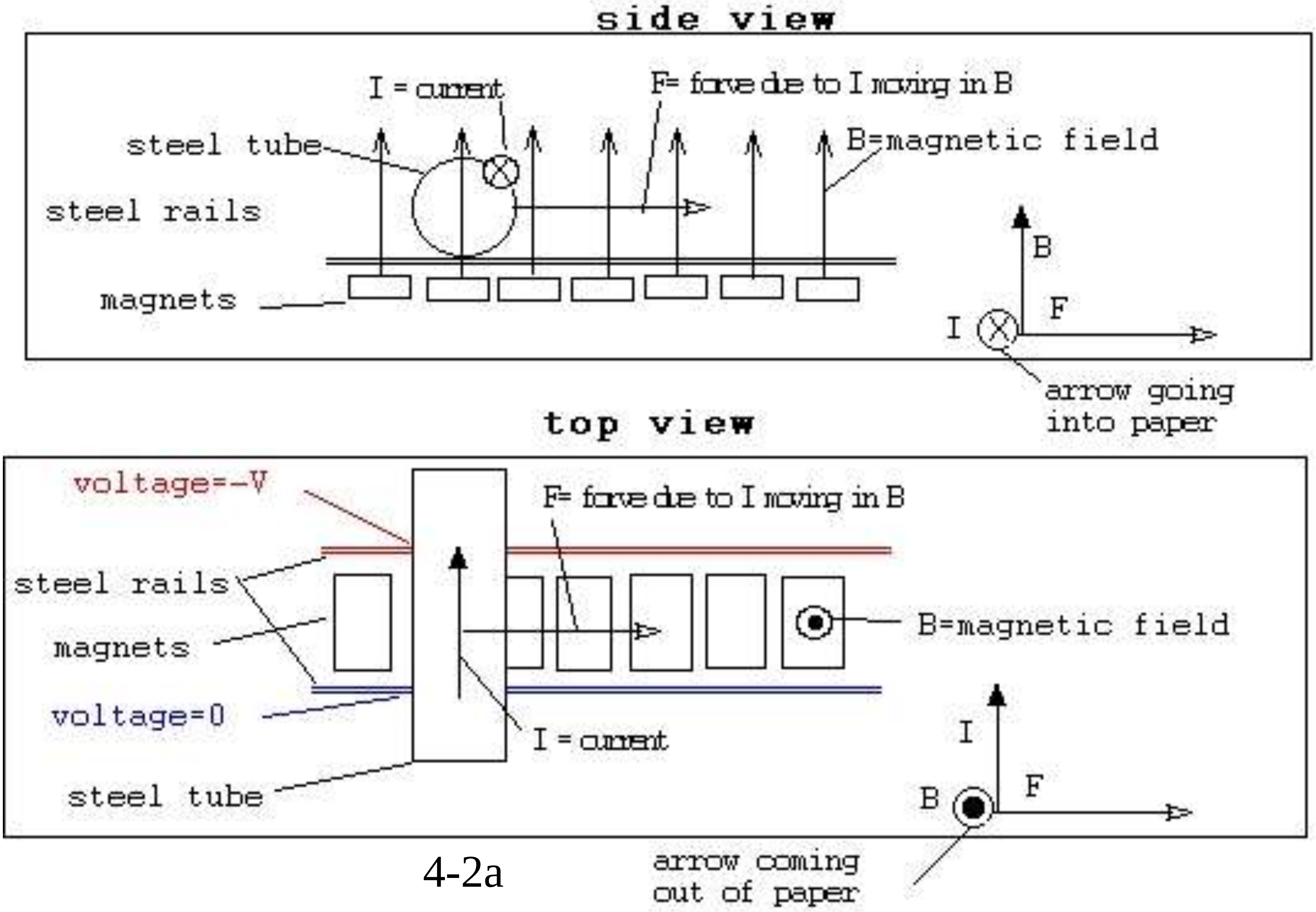
Another equivalent RHR



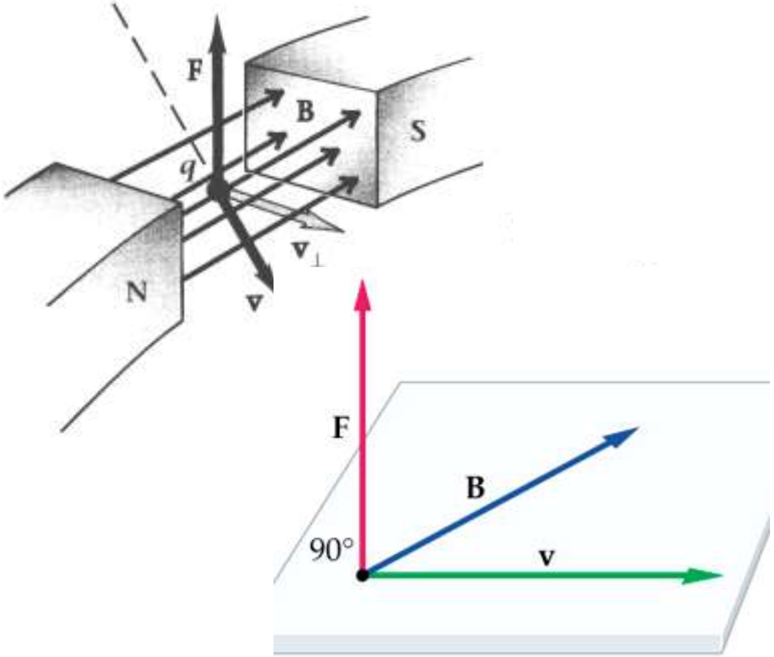


-|q| flip thumb

Faraday Motor



4-2a

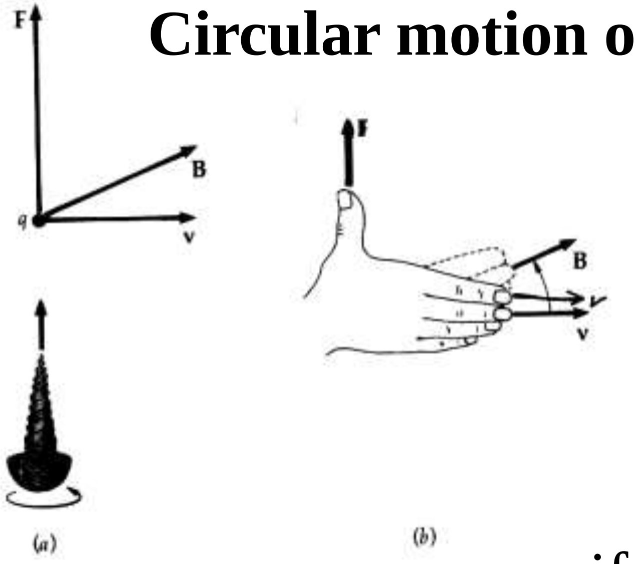


$F = q\, v_\perp B$ Units B field **tesla**

$$\therefore\ N = C\,\frac{m}{s}\,T$$

$$\Rightarrow T = \frac{NS}{mC} = kg\,\frac{m}{s^2}\,\frac{s}{mC} = \boxed{\frac{kg}{Cs} = T}$$

Circular motion of charge moving ⊥ to magnetic field

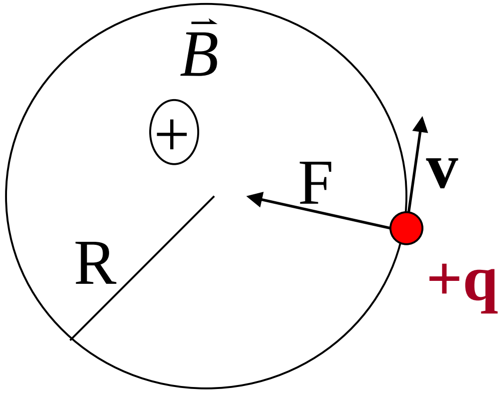


(a)

(b)

$$F = m\,a = m\,\frac{v^2}{R}$$

$$qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB}$$



uniform B field → $\vec{v} \perp$ to $\vec{B} \Rightarrow$ circular motion.

17 Circular motion of a charge in a uniform magnetic field

Recall

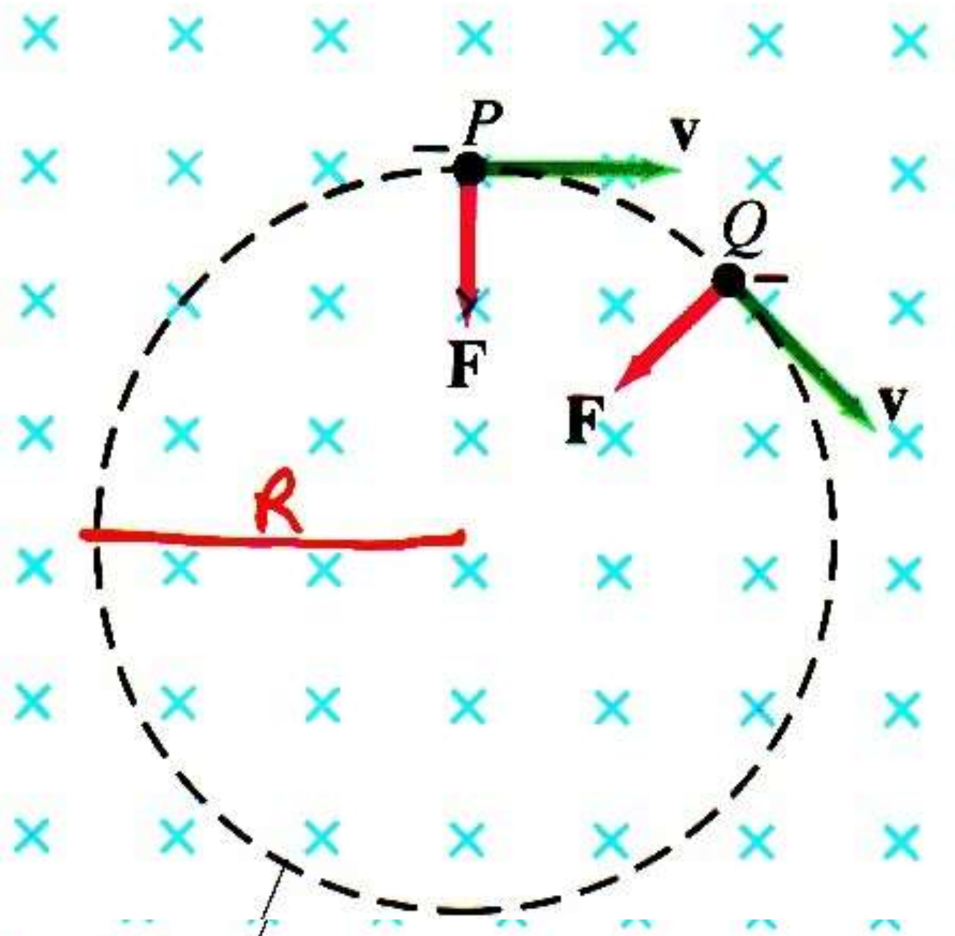
$$F = ma = \underbrace{\frac{mv^2}{R} = \overset{\text{electron}}{e}vB}$$

$$\downarrow$$

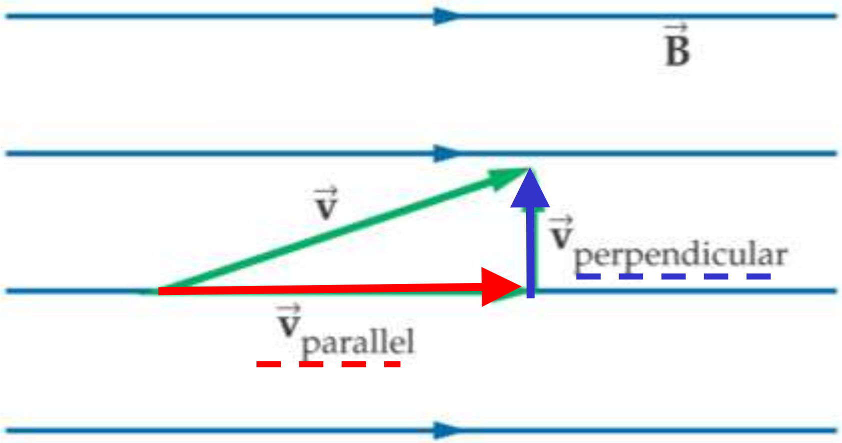
$$\frac{e}{m} = \frac{v}{RB}$$

or

$$\underbrace{v = \left(\frac{e}{m}\right)RB}_{\text{save for later}}$$



General motion of charged particle in a magnetic field



(a)

|| direction

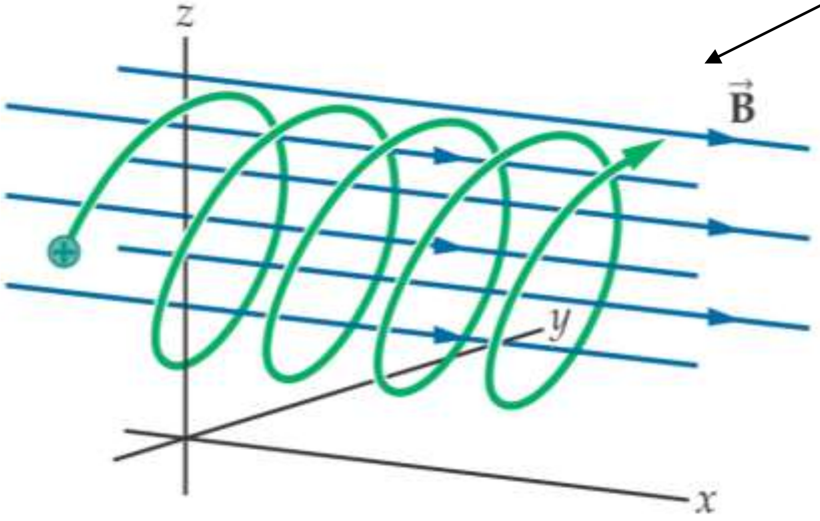
$v_{\parallel}$ = constant

magnitude & direction

⊥ direction

circular motion

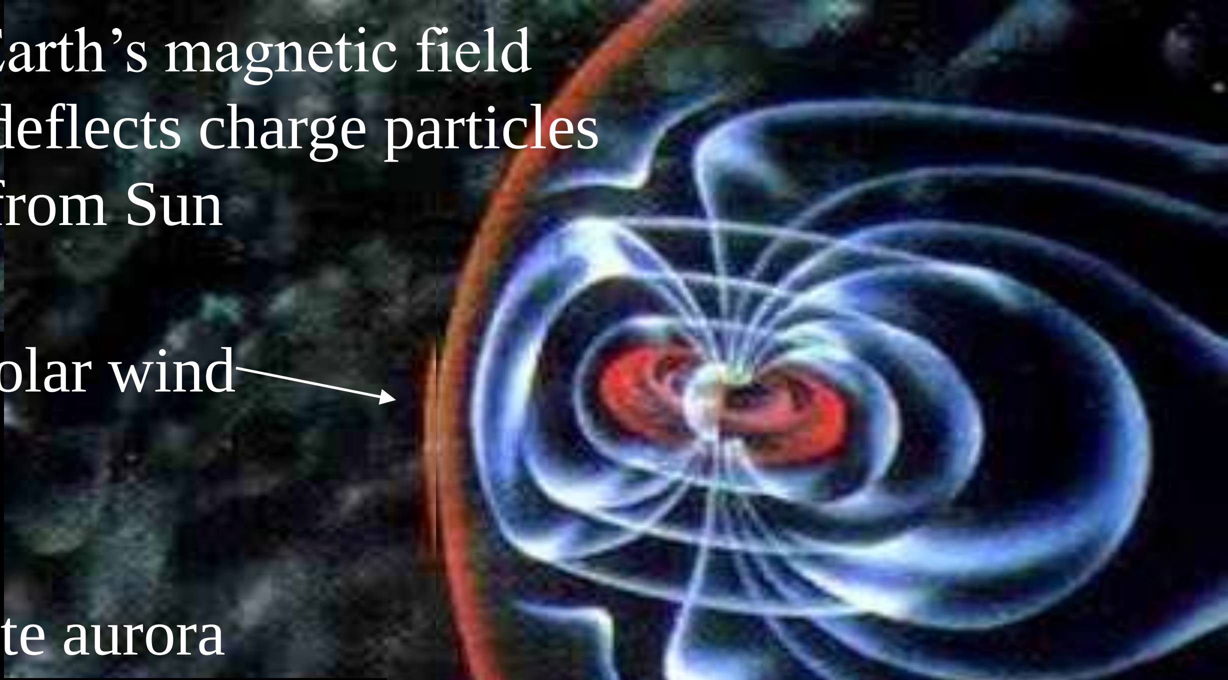
speed = $|v_{\perp}|$



(b)

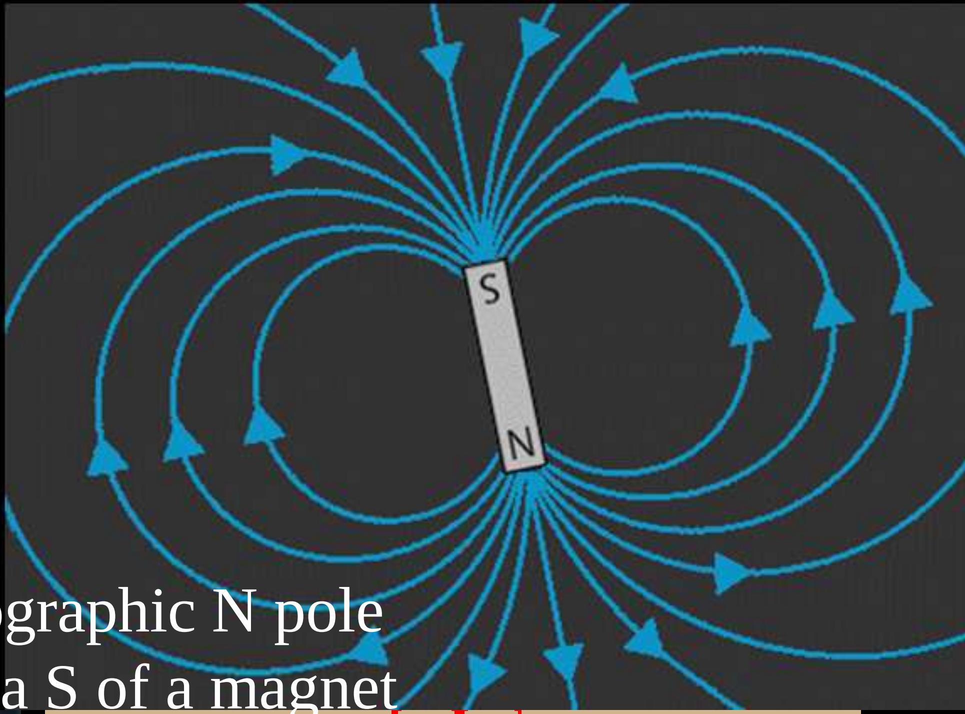
helical-motion

Earth’s magnetic field
deflects charge particles
from Sun



solar wind

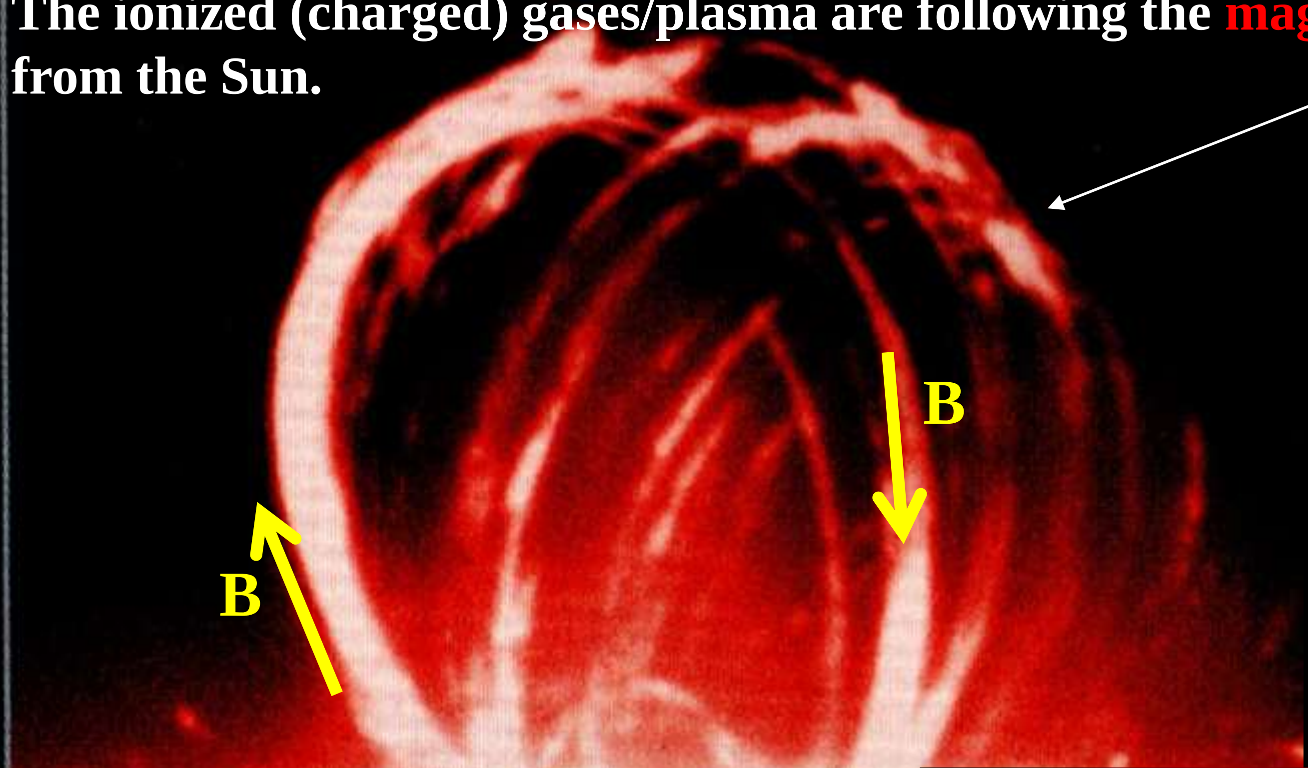
charged particles
spiral down
magnetic field lines
near magnetic poles create aurora



Note: Geographic N pole
is actually a S of a magnet

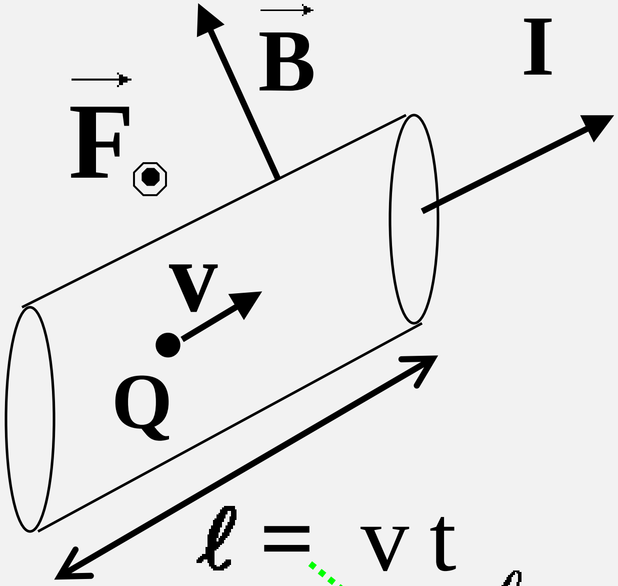
*** Caravaglio (2011)**

The ionized (charged) gases/plasma are following the magnetic field line protruding from the Sun.



PROMINENCES: Surging clouds of ionized gas are best visible along the limb; loop prominences can tower tens of thousands of miles high.

Force on a straight wire with I due to B



Note: here $v \perp B \perp F$

$$\boxed{F = Q v B}$$

but $I = \frac{Q}{t}$

$\ell = v t$

or $t = \frac{\ell}{v}$

so $I = \frac{Q}{\ell / v} = \frac{Q v}{\ell}$

and so $\boxed{I \ell = Q v}$

$$\Rightarrow F = (Q v) B = (I \ell) B \qquad \boxed{F = (I \ell) B}$$

units

$N = A m T$

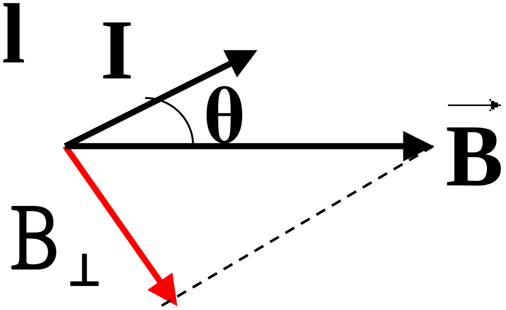
$T = \frac{N}{A m}$

$T = \frac{\frac{kg\, m}{s^2}}{\frac{C}{s} m}$

$T = \frac{kg}{C s}$

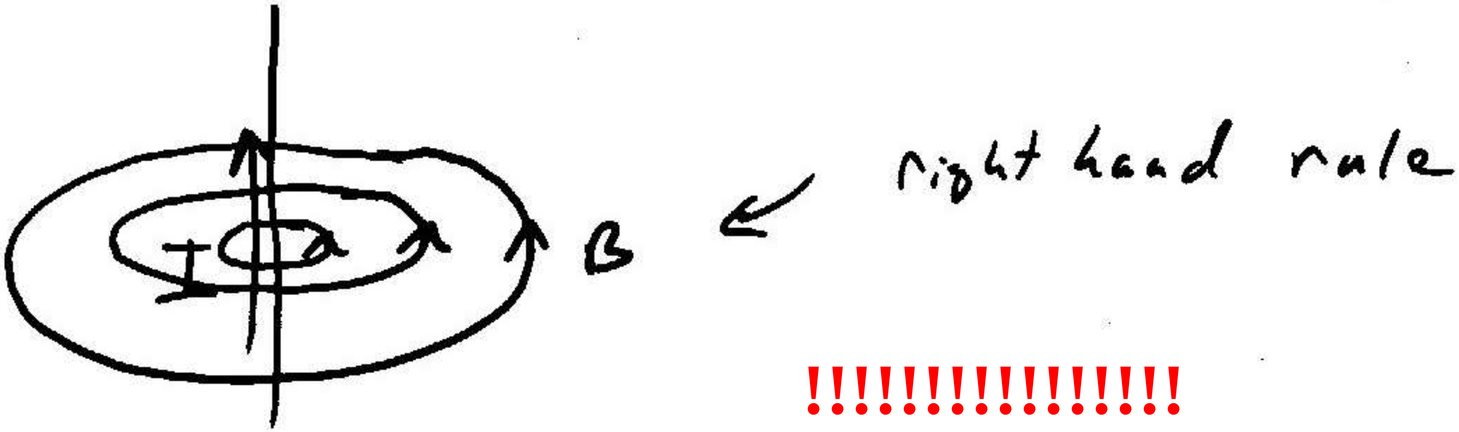
ok

In general



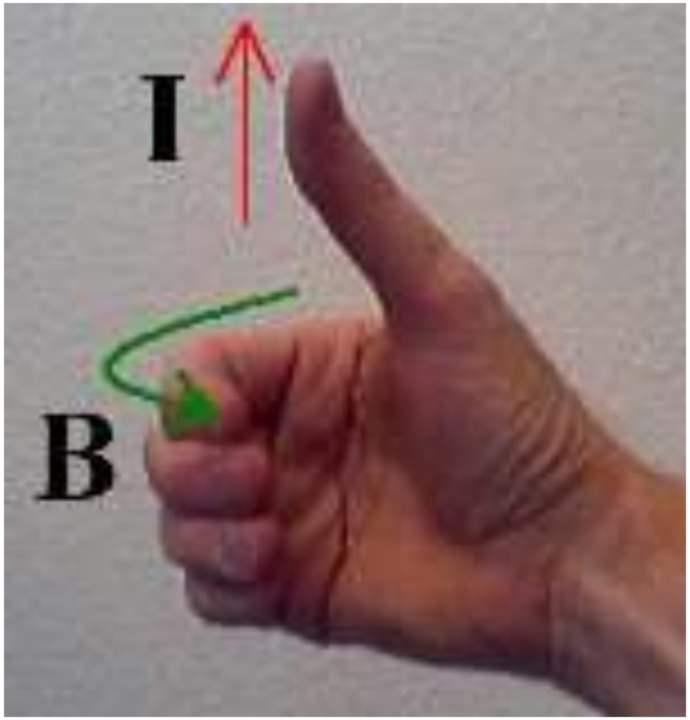
$$\boxed{F = I \ell B_\perp = I \ell B \sin(\theta)}$$

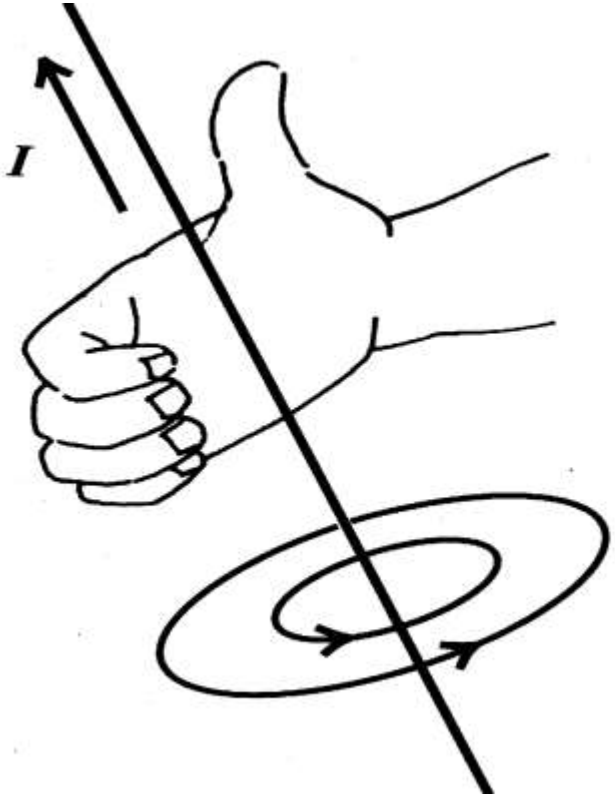
Flowing charge (current) creates magnetic field.



!!!!!!!!!!!!!!!!

2'nd right hand rule: RHR for B direction created by current

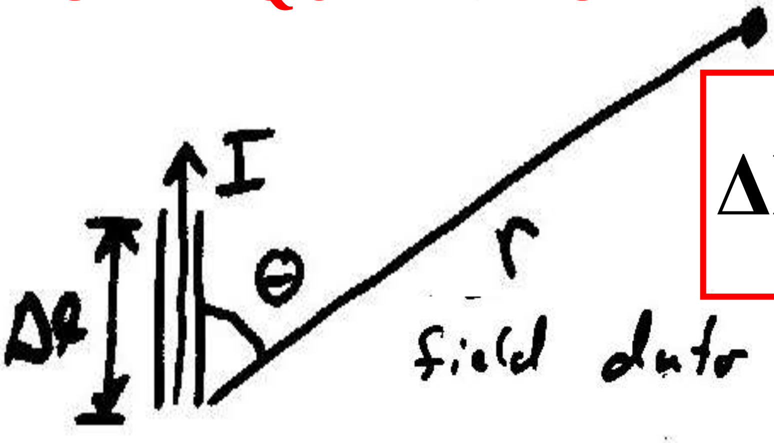




Magnitude of field (Biot & Savart Law)

{Like point charge electric field law but for current}

{**NOT REQUIRED FOR THIS COURSE : BUT GOOD TO KNOW**}



$$\boxed{\Delta B = \frac{\mu_0}{4\pi}\frac{I\,\Delta \ell}{r^2}\sin(\theta)}$$

field due to I in length Δℓ

To calculate tot field add (*as vectors) up all ΔB from each $\Delta \ell$

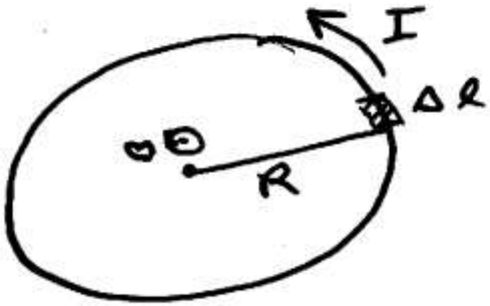
[B&S Law- magnetic field equivalent of electric field point charge]

$$\Delta B = k'\frac{I\,\Delta \ell}{r^2}\sin(\theta)$$

$\underbrace{k' = (10)^{-7}\frac{N}{A^2}}$ also $\underbrace{k' = \frac{\mu_0}{4\pi}}$

$\vec{B}$ field – simple example **(using Biot & Savart Law)**

circular loop – field at center



$$\Delta B = \frac{kI}{R^2}\,\Delta \ell$$

divide circle into N parts of Δℓ each

$$\Delta \ell = \frac{2\pi R}{N} \Longrightarrow N\Delta\ell = 2\pi R$$

note every ΔB same ∴

$$B = N\,\Delta B = N k \frac{I}{R^2}\,\Delta \ell$$

$$= \frac{kI}{R^2}(N\Delta\ell)$$

$$= \frac{kI}{R^2}\,2\pi R$$

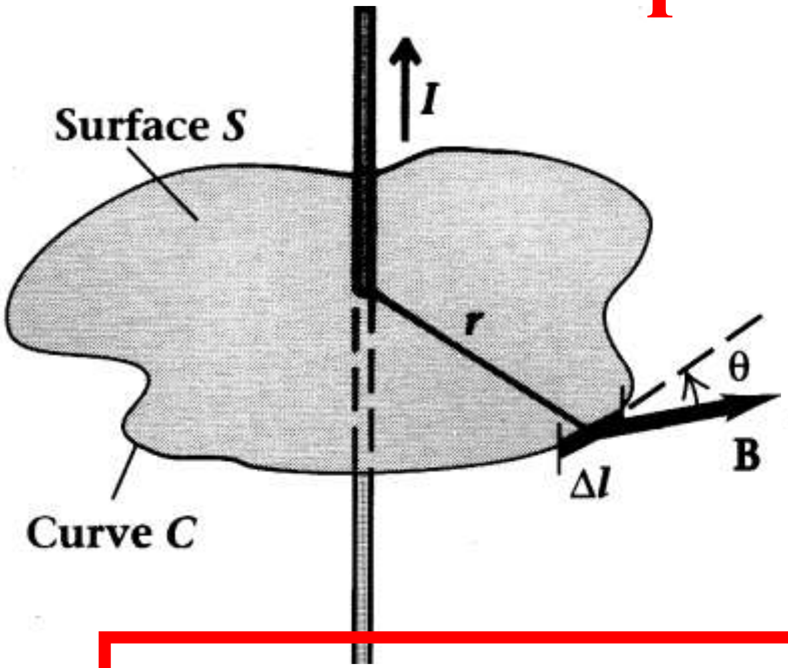
{NOT REQUIRED FOR THIS COURSE : BUT GOOD TO KNOW}

$$B = \frac{2\pi\mu_0}{4\pi}\,\frac{I}{R}$$

last slide 9/27/17 lec

$$\boxed{B = \frac{2\pi k I}{R}} \quad \text{or} \quad \boxed{B = \frac{\mu_0 I}{2R}}$$

Ampere's Law



$$\underbrace{\mu_0 I} = [\underbrace{B_{\|}} \Delta l]_{\text{add up around surface edge}}$$

I=curr. through surface

B along Δl

$B_{\|}$=B cos (θ)
here

closed edge- line integral form.

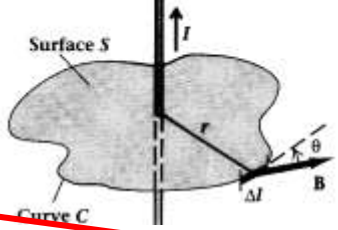
$$\mu_0 \, I = \sum_{\text{edge}} B_{\|} \, \Delta l \qquad \mu_0 \, I = \oint_{\text{edge}} B_{\|} \, dl$$

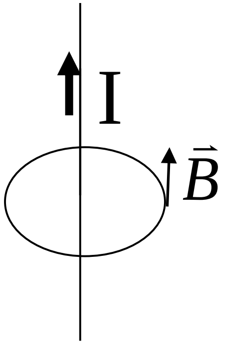
I= current
through surface

B along Δ*l*

Add up around surface edge.

recall Ampere's Law $\mu_0 \mathrm{I} = \sum_{\text{edge}} [\mathrm{B}_{\parallel} \Delta l]$





B field around an ∞ wire with I

$\vec{B}$ along circle & B always constant on circle.

[circumference]

$\mu_o \mathrm{I} = \mathbf{B}\ [2\pi \mathbf{R}]$

From Ampere's Law

$k' = \frac{\mu_0}{4\pi}$

$\Rightarrow \boxed{\mathbf{B} = \frac{\mu_o\ \mathrm{I}}{2\pi \mathbf{R}}}$ ⟵ For a wire

or $\boxed{\mathrm{B} = \frac{2k'I}{R}}$

example

Suppose I = 1A : R = 1m : B=?

$$\mathrm{B} = \frac{[4\pi(10)^{-7} \frac{N}{A^2}]\ 1\mathrm{A}}{2\pi\ \ 1\mathrm{m}}$$

$$= 2(10)^{-7} \frac{\mathrm{NA}}{\mathrm{A}^2 m} \quad \mathrm{B} = 2(10)^{-7} \frac{\mathrm{N}}{\mathrm{A\ m}}$$

B= $2(10)^{-7}$ T very small!

Units check

Recall: F=qVB F=IℓB

$\mathrm{B} = \frac{\mathrm{F}}{\mathrm{I}\ell} \quad \Rightarrow \quad \mathrm{T} = \frac{\mathrm{N}}{\mathrm{Am}}$

- Two long magnetic wires carry currents of 2.0 A and 6.0 A. The wires are 1.0 m apart and the currents flow in opposite directions. Determine the magnetic field strength and its direction at a point (P) 2.0 m away from the right long straight wire.

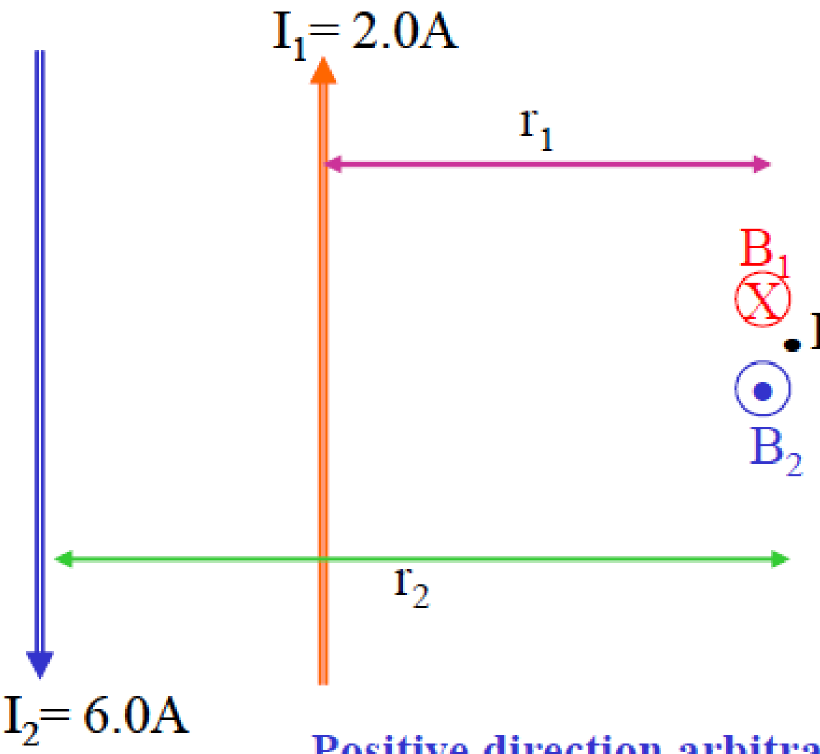
$$\mathbf{B}_1 = \frac{\mu_0 \mathbf{I}_1}{2\pi \mathbf{r}_1} = 2.0\ (10^{-7})\ \mathbf{T}$$

$$\mathbf{B}_2 = \frac{\mu_0 \mathbf{I}_2}{2\pi \mathbf{r}_2} = 4.0\ (10^{-7})\ \mathbf{T}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2$$

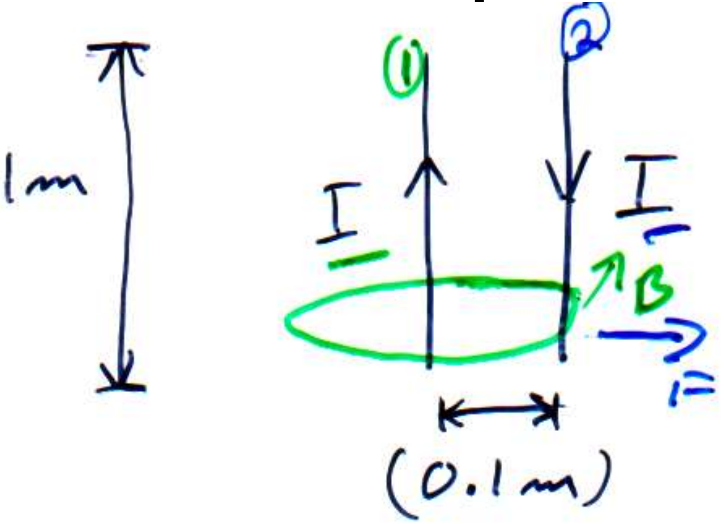
Out of the page

$$\mathbf{B} = +\mathbf{B}_1 - \mathbf{B}_2 = -2.0\ (10^{-7})\ \mathbf{T}$$



Positive direction arbitrary chosen to be into the page(+)

Force between 2 parallel wires L=1 m, at 0.1m, I = ±50 A



B due to 1

$$B = \frac{k 2 I}{R}$$

$$F_{on\,2} = l I B$$

$$= l I \frac{2kI}{R}$$

$$\boxed{F = \frac{l}{R} I^2 2k}$$

$$F = \frac{1\,m}{(0.1\,m)} (50)^2 A^2 \; 2 \; 10^{-7} \frac{N}{A^2}$$

$$= (10) \cdot 2500 \cdot 2 (10)^{-7} \quad N$$

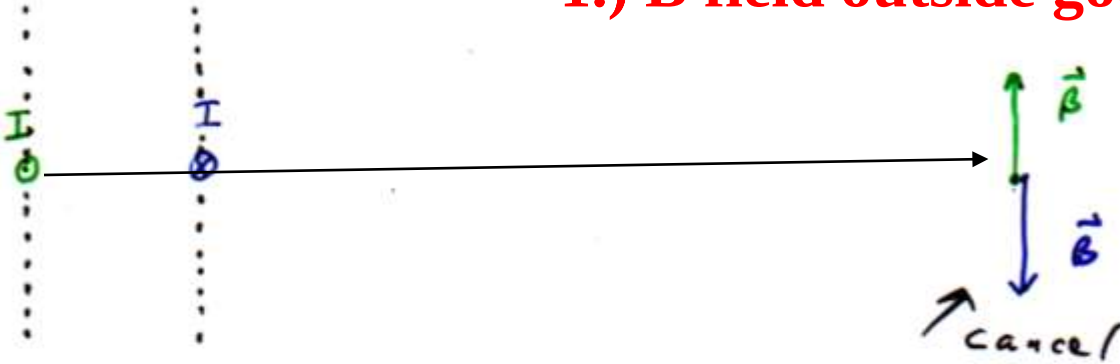
$$= 50{,}000 (10)^{-7} \; N$$

$$\boxed{F = .005\,N}$$

.01 m ⇒ .05 N
.001 m ⇒ .5 N

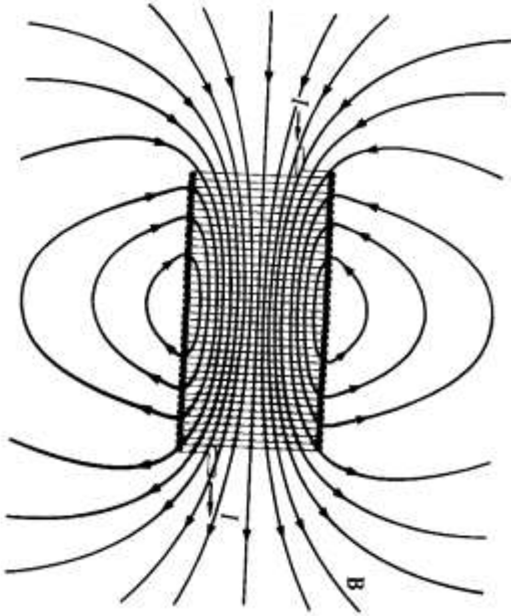
Magnetic field of long solenoid

1.) B field outside goes to 0



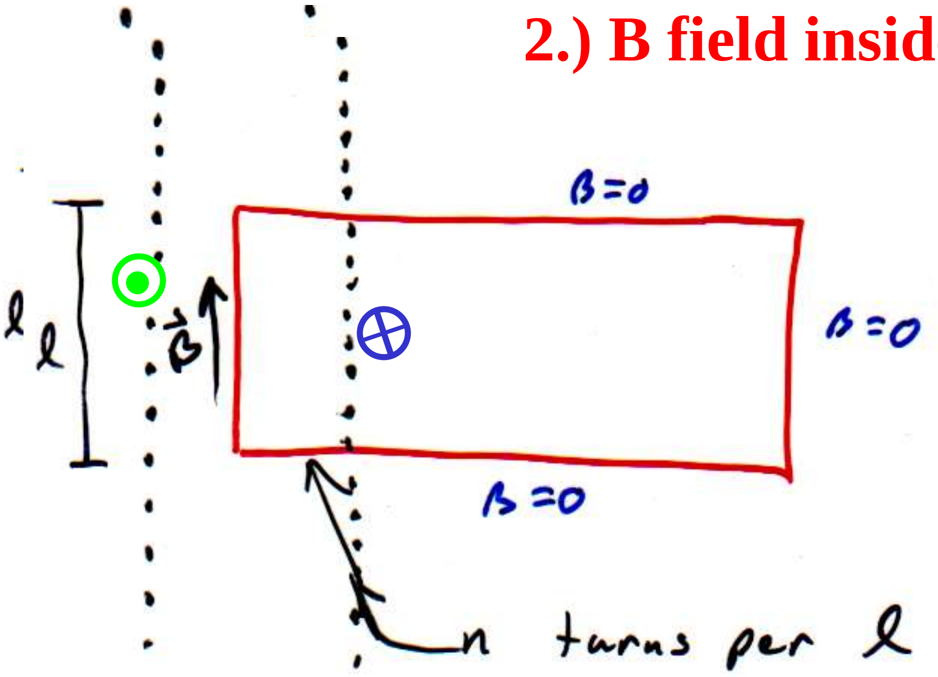
cancel
$\vec{B}$ field outside
solenoid is 0

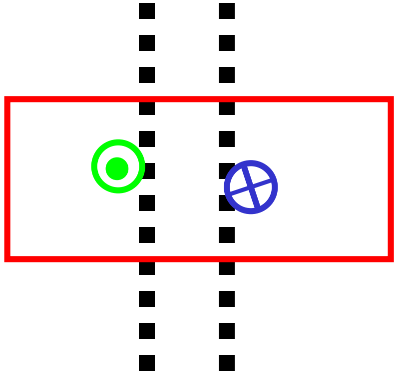
(= 0 if ∞)



Magnetic field of long solenoid

2.) B field inside





$0 = +I\,n\,L - I\,n\,L$

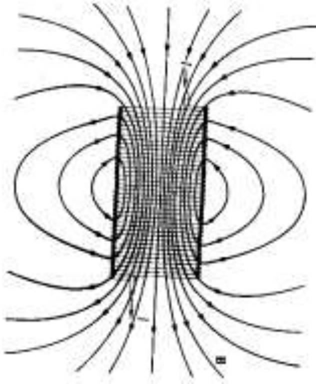
$B_{out} = 0$

$I \cdot n\ell$ = tot curr through surface

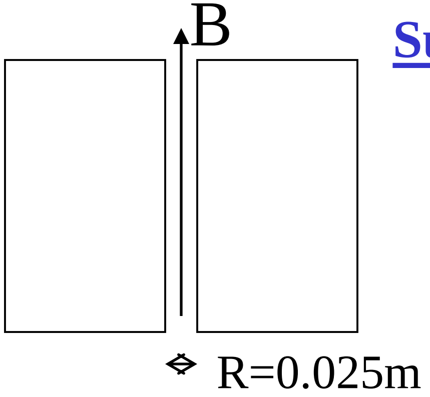
B around loop

$$B \cdot \ell + 0 = I n \ell \mu_0$$

$$\therefore \boxed{B = I n \mu_0}$$ ∞ solenoid



Superconducting solenoid



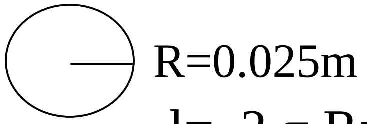
$$B = I\, n\, \mu_0$$

$$= \; 100A\, [100{,}000\, \frac{\text{turns}}{\text{m}}]\, [\, 4\pi(10)^{-7}]\frac{N}{A^2}$$

$$=(10)^2\;\, 10^5\;\, 4\pi\; (10)^{-7}\, \frac{NA}{mA^2} = 4\; [3.14]\;\; T$$

$$\boxed{B = 12T}$$

Q: What is outward force on inside of such a solenoid?



l= 2 π R= 2 π 0.025m

$$F = I\; l\; B= (100A)\; (.025m\;\; 2\pi)\; 12T$$

$$= (100)\; (.025)\; (6.28)12\; \frac{\cancel{A}N\cancel{m}}{\cancel{A}\cancel{m}}$$

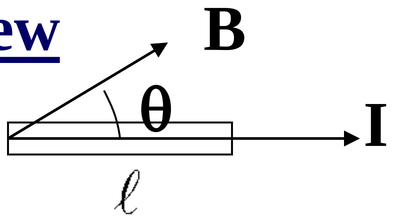
$$=(2.5)(6.28)\; 12N$$

$$=150N$$

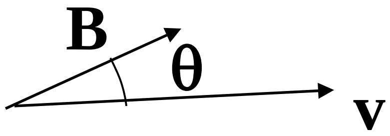
~33.7 lbs.

Magnetic Field Review

$F = qv_{\perp}B$ or $F = qvB \sin\theta$



Right Hand Rule



$F = I \ell B_{\perp}$ or $F = I \ell B \sin\theta$

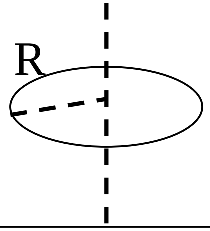
$$\Delta B = k' \frac{I\Delta l}{r^2} \sin\theta$$

Not required:

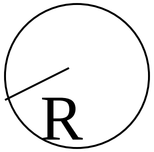
$$k' = \frac{\mu_0}{4\pi}$$

Long wire:

$$B = k' \frac{2I}{R} \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

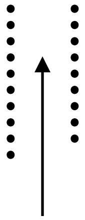


Center of Loop (circular):



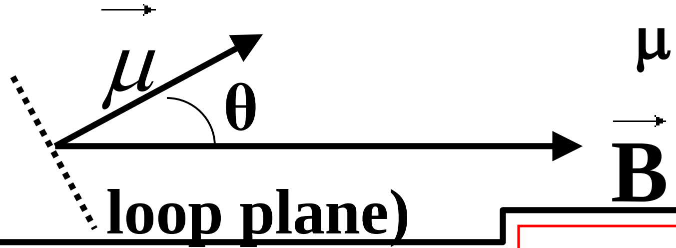
$$B_{\text{center}} = \frac{2\pi k' I}{R} = \frac{\mu_0 I}{2R}$$

Solenoid:



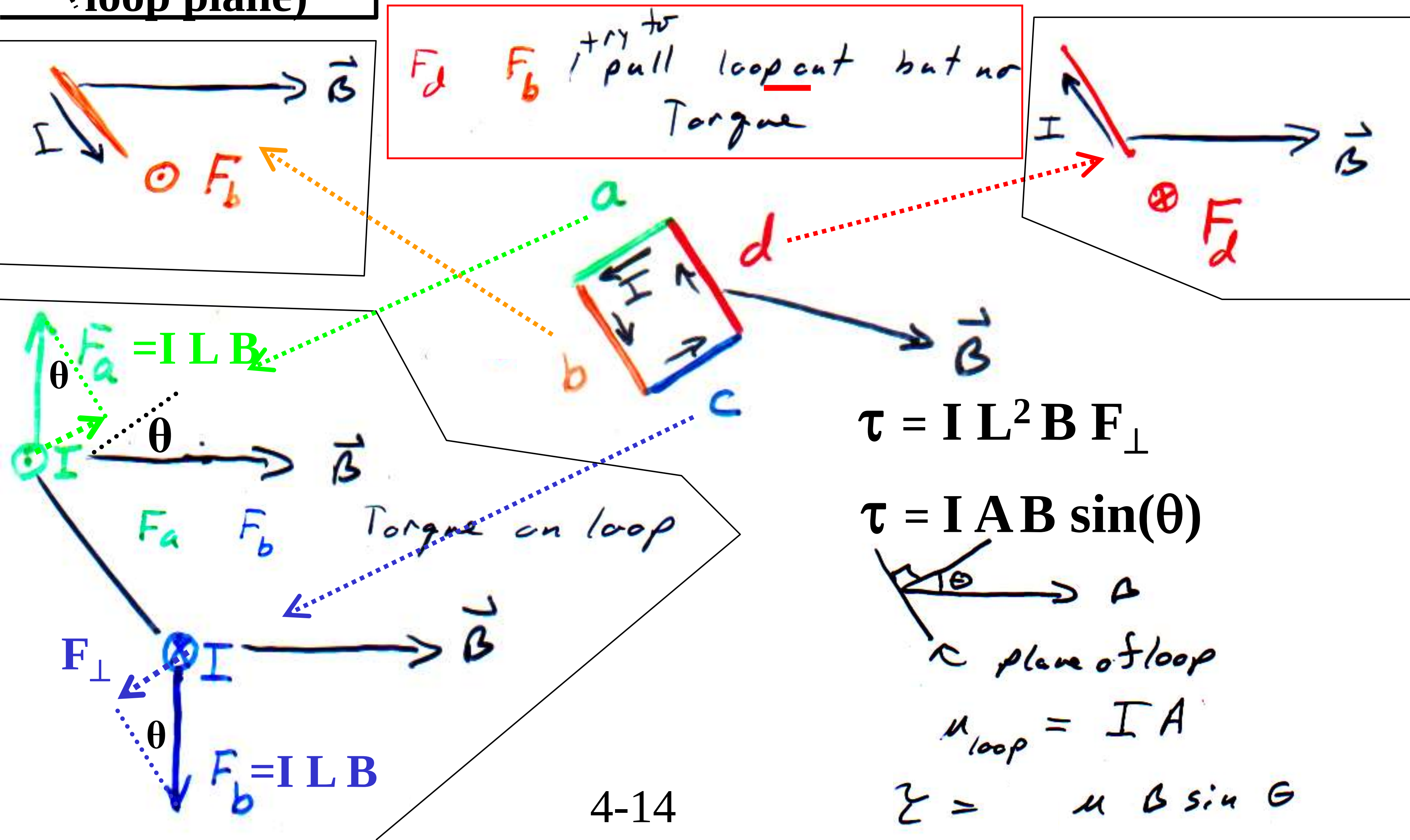
$$B = \mu_0 n I = 4\pi k' n I$$

Torque, τ, on current loop (assume square) in B field



μ = magnetic moment (like electric dipole)

$$\tau = \mu\, B \sin(\theta)$$



$$\tau = I\, L^2\, B\, F_\perp$$

$$\tau = I\, A\, B \sin(\theta)$$

$$\mu_{loop} = I\, A$$

$$\tau = \mu\, B \sin\theta$$