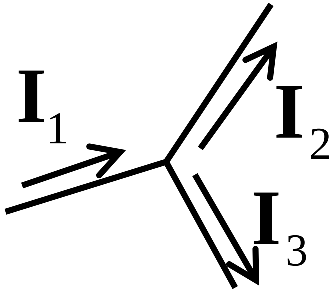
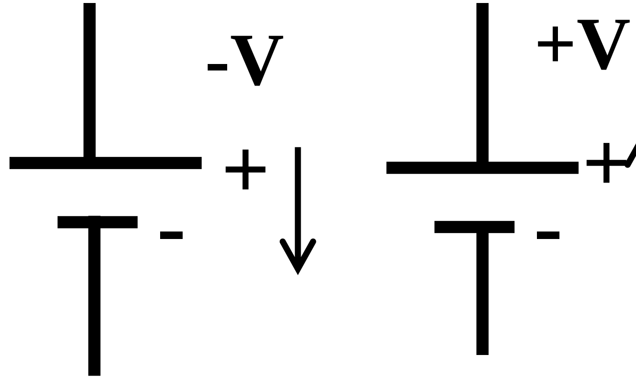
$$V = I\,R \qquad R = \rho\left[\frac{L}{A}\right]$$

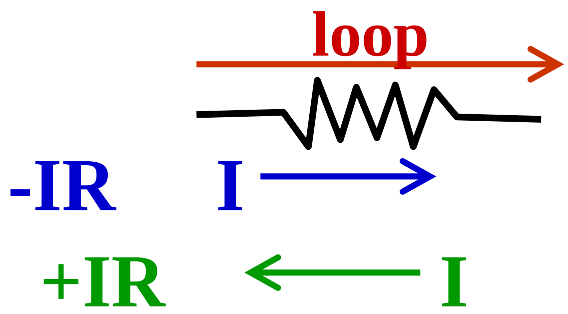
Kirchhoff's Laws

$$\sum_{junc} I_j = 0$$



$$\sum_{loop} V_j = 0$$



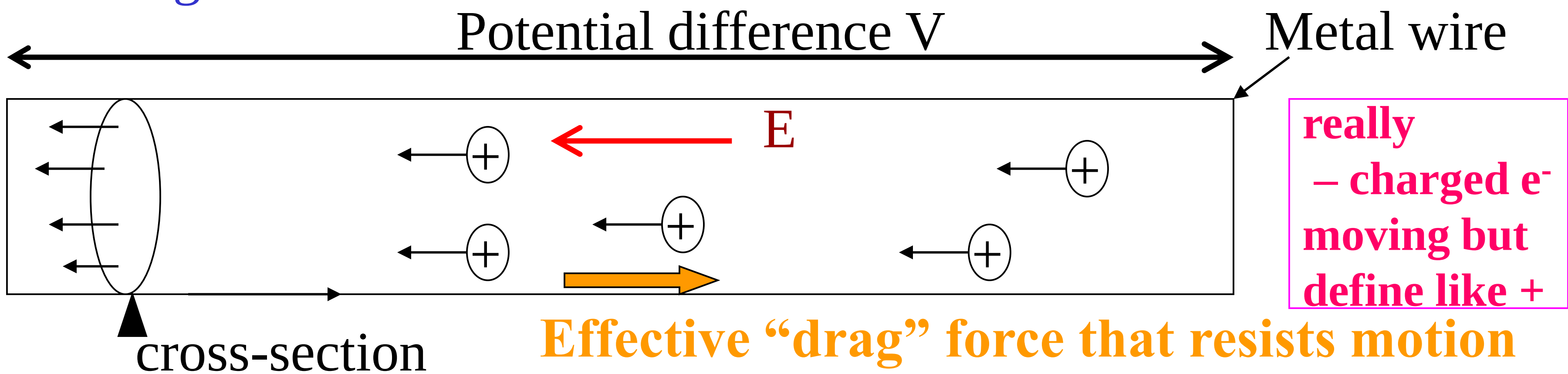


$$P = I\,V = I^2\,R = \frac{V^2}{R}$$

$$R_{eff} = R_1 + R_2 \qquad \frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad C_{eff} = C_1 + C_2$$

Charges in motion



charge Δq flows through in Δt

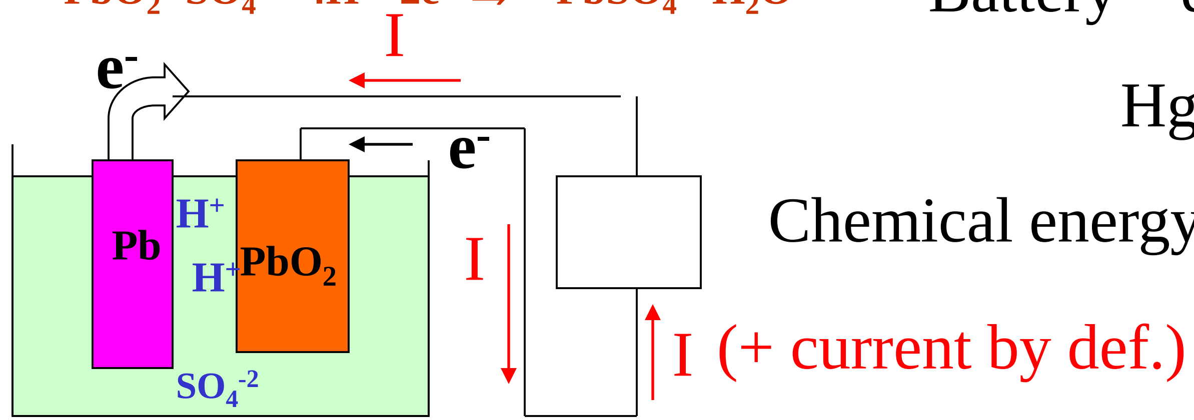
$$I = \frac{\Delta q}{\Delta t}$$

$$\text{current} = \frac{\text{coulombs}}{\text{sec}} = \text{ampere}$$

Electric Potential Source

$Pb + SO_4^{-2} \Rightarrow PbSO_4 + 2\,e^-$

$PbO_2 + SO_4^{-2} + 4H^+ + 2e^- \Rightarrow PbSO_4 + H_2O$



Battery – dry cell 1.5V

Hg cell 1.35V

Chemical energy ⟶ electrical energy

$$EL = \left[\frac{m}{n\,q\,\tau}\right]\left[\frac{L}{A}\right] I$$

$$V = \rho \left[\frac{L}{A}\right] I$$

electrical resistivity

$$\rho = \left[\frac{m}{n\,q\,\tau}\right]$$

marerial's reluctance to carry current

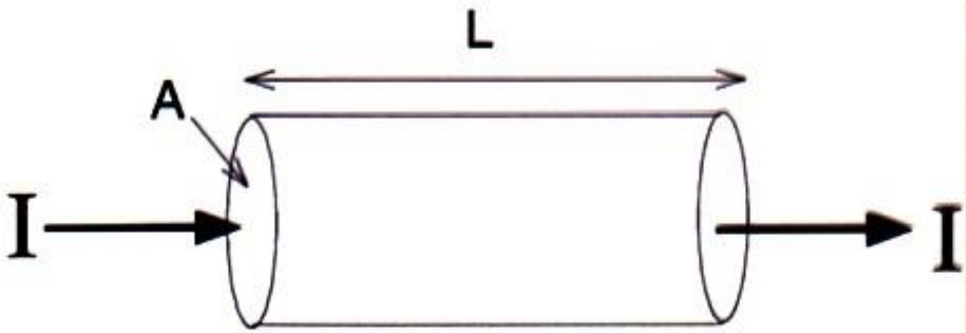
Ohm's Law

$$V = R\,I$$

applied voltage — resistance — current

$$R = \rho \left[\frac{L}{A}\right]$$

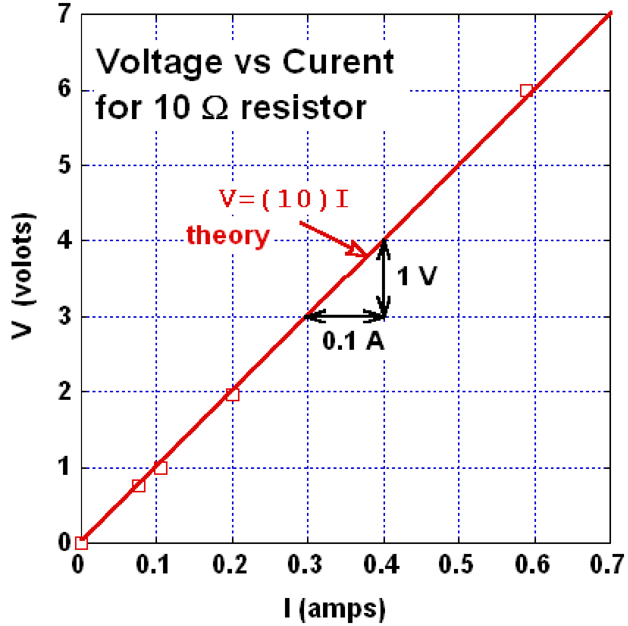
material property — geometrical factor



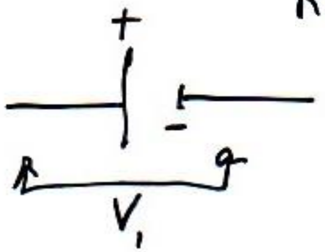
Example: $10\ \Omega$ resistor

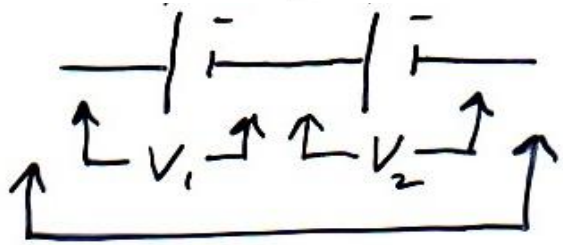
$V = IR = I(10)$

$V = 10\,I$ theory.



Represent battery with picture

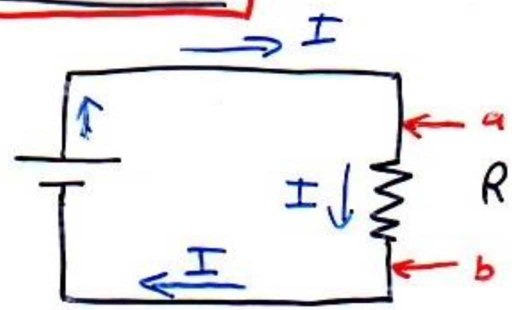




$V = V_1 + V_2$

add batteries to make larger Voltage

Ohm's law



R = resistance (ohms, Ω)
resistor

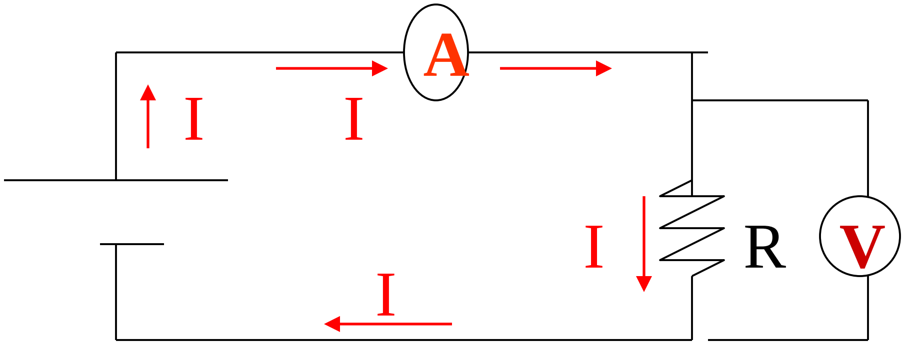
$V_{ab} = I\,R$

units

$$\Omega = \frac{V}{A} = \frac{J/C}{C/\text{sec}} = \frac{J\ \text{sec}}{C^2} = \frac{\left(\text{kg}\ \frac{m^2}{\text{sec}^2}\right)\text{sec}}{C^2} = \frac{\text{kg}\ m^2}{\text{sec}\ C^2}$$

Electrical Measurements

"Amp" meter (very little resistance

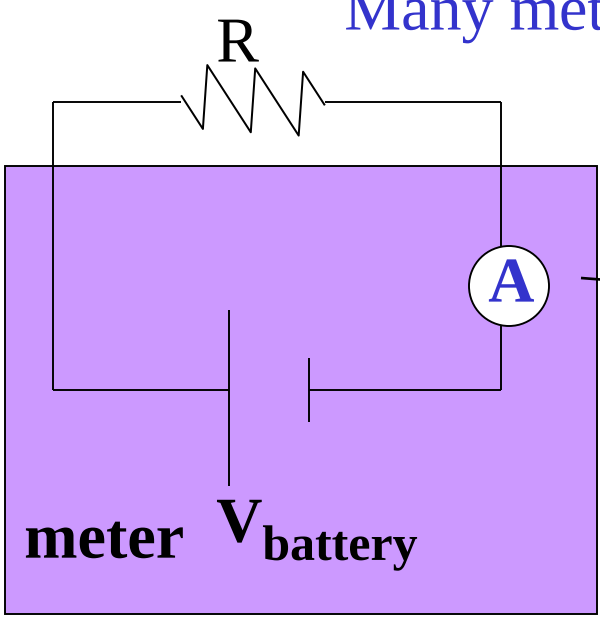


Volt meter (very large resistance)
(draws ~NO current.)

DC (direct current) multi-meters measures volts & amps.

Don't make a mistake on settings! [especially don't try to measure V on amp setting]

Many meters also measures R (Ω)

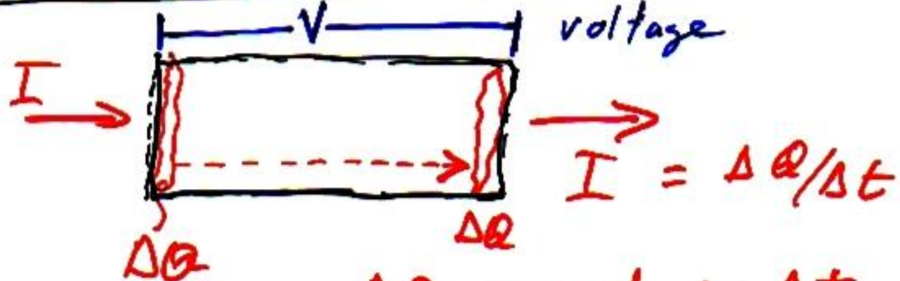


$$V_{battery} = IR$$

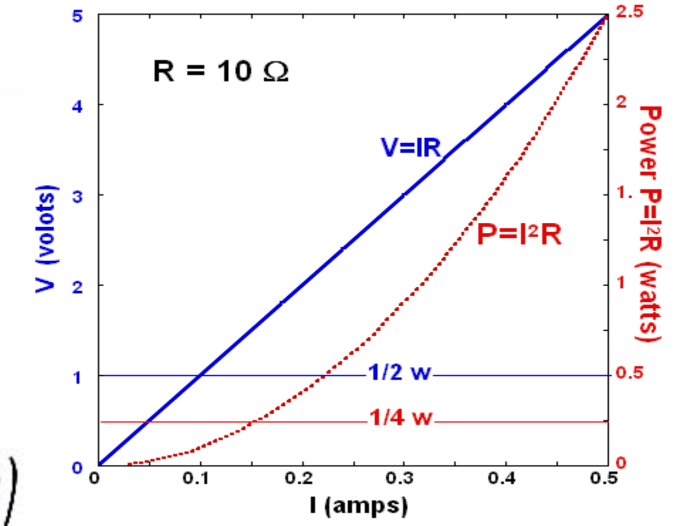
Measures I for known V_{bat} so..

$$R = \frac{V_{bat}}{I}$$

Power disipated to heat in resistor



ΔQ moved in Δt



work done in moving ΔQ across V:

$$\Delta W \equiv \Delta Q \, V$$

(this W done in Δt)

power:

$$P \equiv \frac{\Delta W}{\Delta t} = \frac{\Delta Q}{\Delta t} V \Rightarrow \boxed{P = IV}$$

work/time; $\frac{\Delta Q}{\Delta t} = I$

(J/sec) = watts = w

but $V = IR$ so $P = I(IR) \Rightarrow$ also $\boxed{I^2 R = P}$

also $I = \frac{V}{R}$ so $P = \left(\frac{V}{R}\right) V \Rightarrow$ also $\boxed{\frac{V^2}{R} = P}$

Units of power

P=IV

$$A\,V = \left(\frac{C}{sec}\right)\left(\frac{J}{C}\right) = \frac{J}{sec}$$

=watts

Power Example

10 Ω [$\frac{1}{2}$ W resistor]
Limit where it's OK

$$P = I^2 R$$

Limit

$$0.5\ (W) = I^2\ (10\ \Omega)$$

$$I^2 = 0.05\ \left[\frac{W}{\Omega}\right]$$

$$I = \sqrt{0.05}\ \sqrt{\left[\frac{J/s}{V/A}\right]}$$

$$I_{limit} \approx 0.22\ \sqrt{\frac{J/s}{\frac{J}{C}\frac{1}{C/s}}}$$

$$I_{limit} \approx 0.22\ \left(\frac{C}{s}\right)$$

A

or $P = \frac{V^2}{R}$

$$P = \frac{V^2}{10}\ \frac{1}{(\Omega)}$$

limit ($V = IR$)

$$0.5\ W = \frac{V^2}{10}\ \frac{1}{(\Omega)}$$

$$5\ (W\ \Omega) = V^2$$

$$\left(\frac{J}{s}\ \frac{V}{A}\right)$$

$$\left(\frac{J}{s}\ \frac{J}{C\ \frac{C}{s}}\right)$$

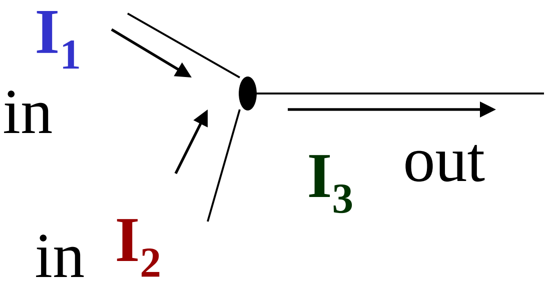
$$2.2\ \frac{J}{C} = V$$

$$2.2\ (V) = V$$

volts

Circuit Analysis Kirchhoff's Laws

Conservation of current charge

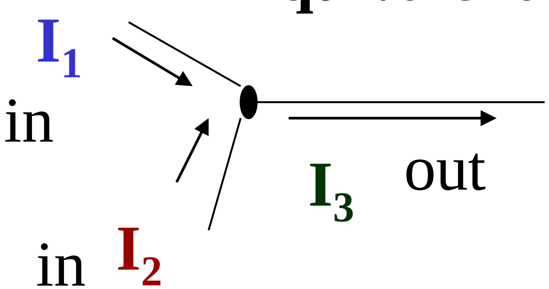


$$I_1 + I_2 - I_3 = 0$$

in in out

The sum of the currents into any junction must equal zero.
(charge does not build up). (+=in -=out)
You can choose any direction for I, just stay with your choice throughout.

Equivalently

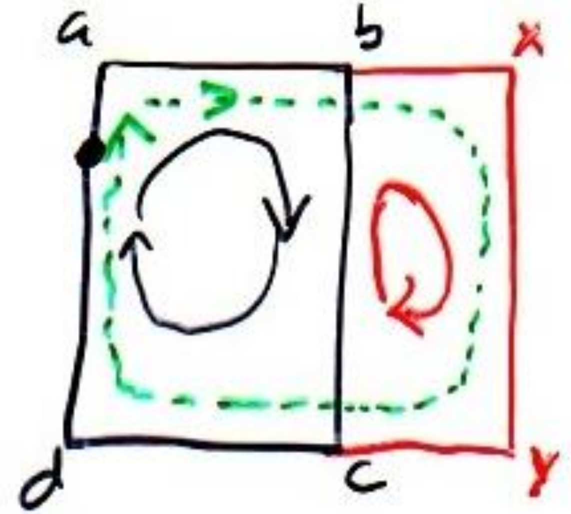


$$I_1 + I_2 = I_3$$

in in out

Electric potential energy conservation

- The sum of the potential difference around any closed loop is zero!

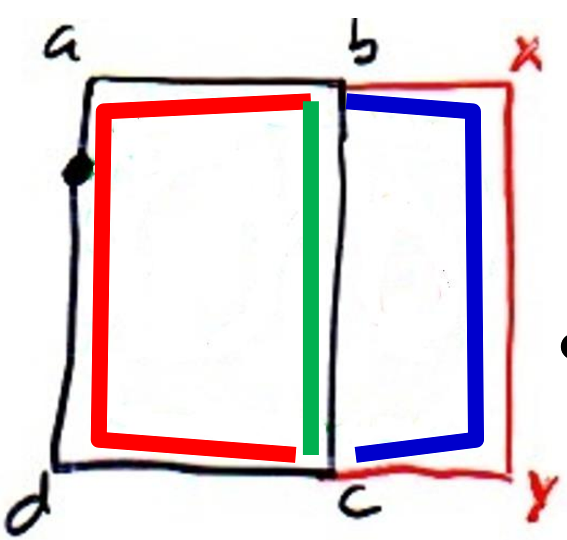


$$V_{ab} + V_{bc} + V_{cd} + V_{da} = 0$$

$$V_{bx} + V_{xy} + V_{yc} + V_{cb} = 0$$

$$V_{ax} + V_{xy} + V_{yd} + V_{da} = 0$$

$$V_{ab} + V_{bx}$$

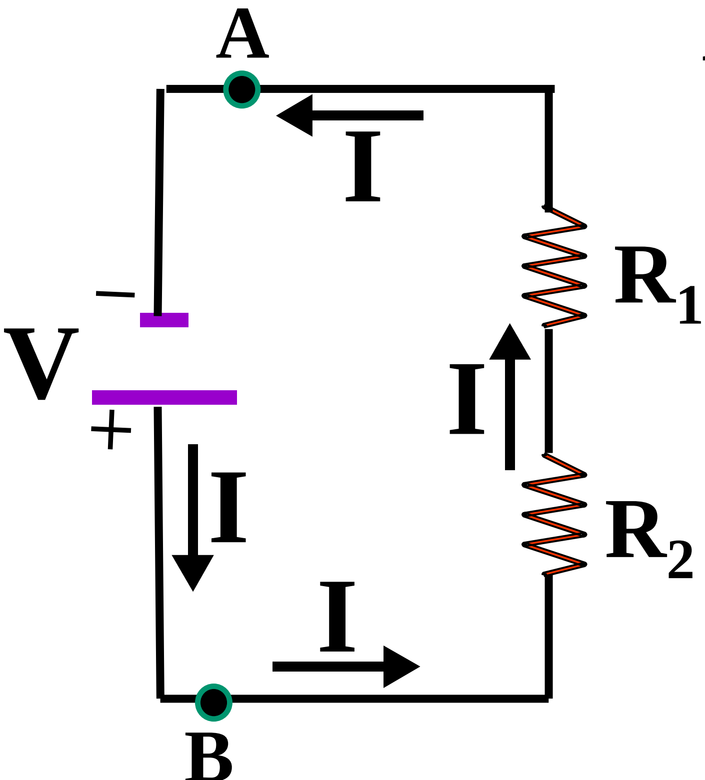


Equivalently

$$V_{cdab} = V_{cyxb} = V_{cb}$$

- Potential difference between 2 points same for all possible paths !

Resistors in Series



I same everywhere

$V_1 = I R_1$

$V_2 = I R_2$

$V = V_1 + V_2$

$V = I R_1 + I R_2$

$V = I [R_1 + R_2]$

$V = I R_{eff}$

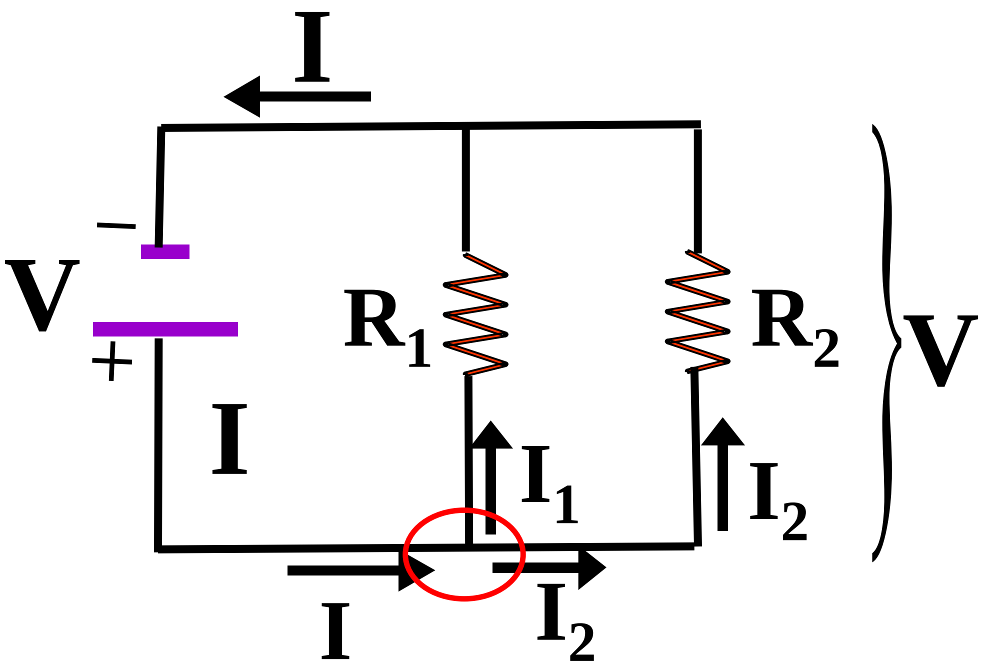
$R_{eff} = R_1 + R_2$

in general

$V = V_1 + V_2 + \ldots + V_n = I R_{eff}$

$R_{eff} = R_1 + R_2 + \ldots + R_n$

Resistors in Parallel



$$V = I_1 R_1 = I_2 R_2$$

$$\Downarrow$$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$I_1 = \frac{V}{R_1} \qquad I_2 = \frac{V}{R_2}$$

$$I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V}{R_{eff}}$$

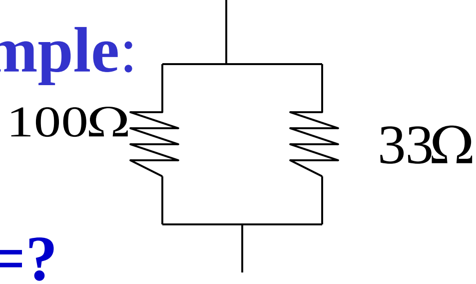
$$\frac{1}{R_{eff}} = \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

in general

$$V = I R_{eff} \qquad I = V / R_{eff}$$

$$\frac{1}{R_{eff}} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots + \frac{1}{R_n}\right]$$

Example:



R_{reff}=?

$$\frac{1}{R_e} = \left[\frac{1}{100} + \frac{1}{33}\right]\left\{\frac{1}{\Omega}\right\}$$

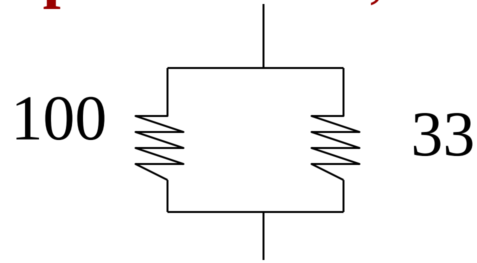
$$= [.01 + .03]\left\{\frac{1}{\Omega}\right\}$$

$$\frac{1}{R_e} = .04\frac{1}{\Omega}$$

$$R_e = \frac{1}{.04} = \frac{1}{4}(10)^2 = .25(10)^2$$

$$R_e = 25\Omega$$

Example: If 2A=I, what is I_{33} & I_{100}?



What is I in each leg?

$$V = I_{100}(100) = I_{33}(33) \Rightarrow I_{100} = I_{33}\frac{33}{100} = .33 I_{33}$$

$$I = I_{100} + I_{33}$$

$$I = I_{33}(.33) + I_{33}$$

$$I = I_{33}(1.33)$$

$$\frac{2}{1.33} = I_{33} \qquad I_{33} = \frac{2}{\frac{4}{3}} = \frac{3}{2} = 1.5\text{A}$$

$$I_{100} = 2\text{A} - 1.5\text{A} = 0.5\text{A}$$

Symmetry approach for = parallel R's

n resistors of R in //

$R_{eff} = ?$

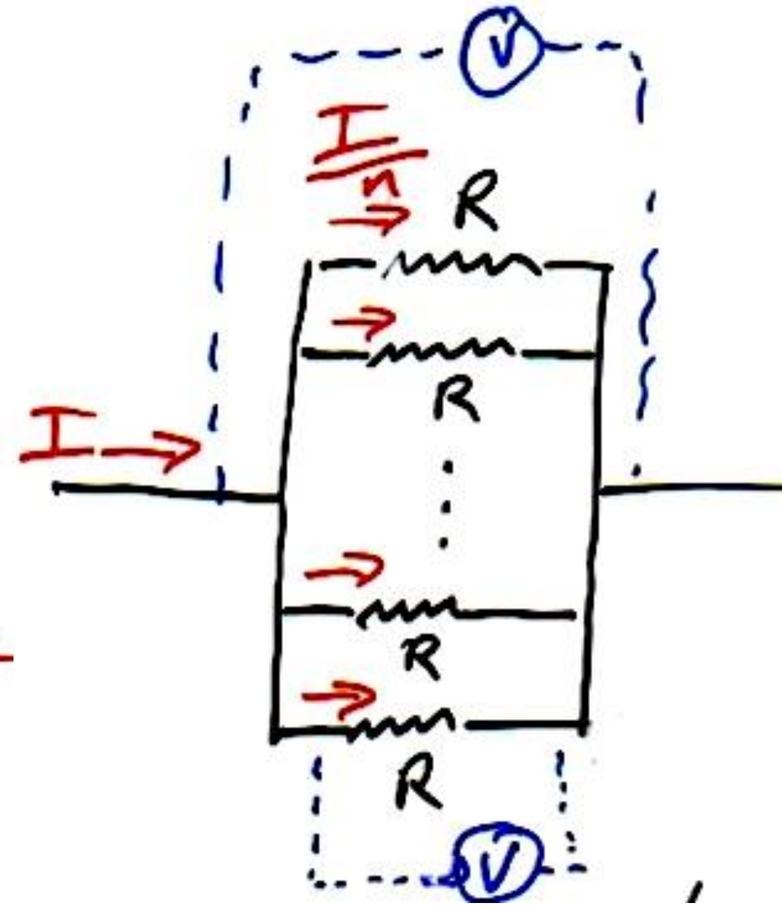
I sees n paths all the same

so $\frac{I}{n}$ in each path

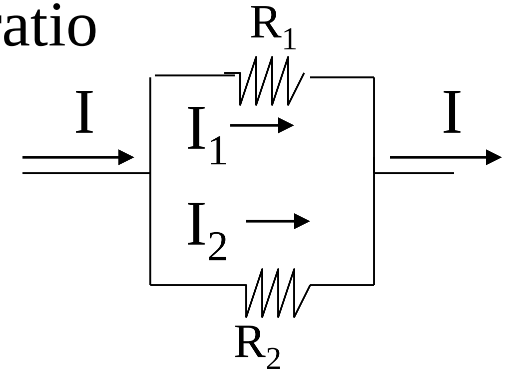
$V = \frac{I}{n} R$

or $V = I \left(\frac{R}{n}\right)$

$$R_{eff} = \frac{R}{n}$$

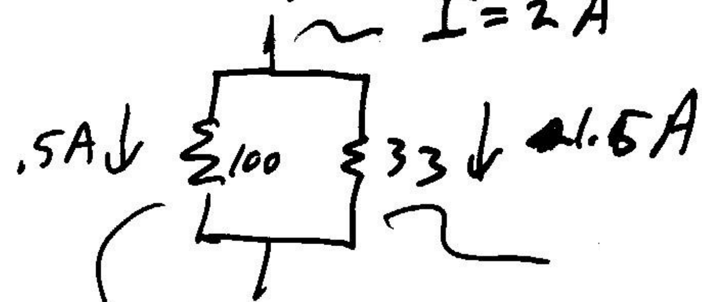


In general current ratio



small R larger power dissipated

our example



$$R_1 I_1 = I_2 R_2 \Rightarrow \frac{I_2}{I_1} = \frac{R_1}{R_2} \Rightarrow I_2 = I_1 \frac{R_1}{R_2}$$

$$\frac{I_2}{I_1} = \frac{R_1}{R_2}$$ **Ratio of current inverse ratio of R.**

$I_{33}^2 R_{33} = (1.5)^2 33$

$= 2.25 (33)$

$= 74.25$ Watts

$I_{100}^2 R_{100}$

$(0.5 A)^2 100$

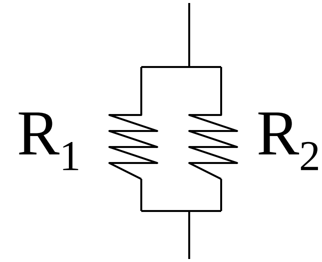
$.25 \; 100$

25 watts

$$\frac{P_2}{P_1} = \frac{I_2^{\,2} R_2}{I_1^{\,2} R_1} \qquad \frac{P_2}{P_1} = \frac{R_2}{R_1}\left(\frac{R_1}{R_2}\right)^2$$

$$\frac{P_2}{P_1} = \frac{R_1}{R_2}$$ **Ratio of power inverse ratio of R.**

R_{eff} in parallel-general observation



$$R_1 = 1000\Omega \qquad R_2 = 100\Omega$$

$$\frac{1}{R_{eff.}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eff.}} = \frac{1}{1000} + \frac{1}{100} = \frac{11}{1000}$$

$$R_{eff.} = \frac{1000}{11} = 91\Omega$$

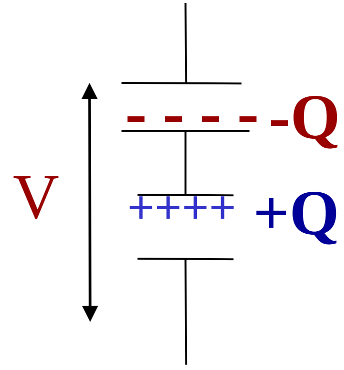
R_1 big

R_2 small → $R_{eff.}$ A little less than R_2 (small).

Capacitors **Series**

$$V_1 = \frac{Q_1}{C_1}$$

$$V_2 = \frac{Q_2}{C_2}$$



equal charge Q !!

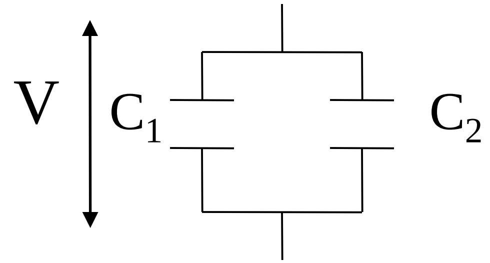
$$V = V_1 + V_2 \rightarrow V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\therefore V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = Q\frac{1}{C_{eff.}}$$

$$\boxed{\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

In general: $$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Capacitors Parallel



equal voltages Vs !!!

$$V\,C_{eff.} = Q_{tot.} = Q_1 + Q_2$$

$$V\,C_{eff.} = VC_1 + VC_2$$

$$\boxed{C_{eff} = C_1 + C_2}$$

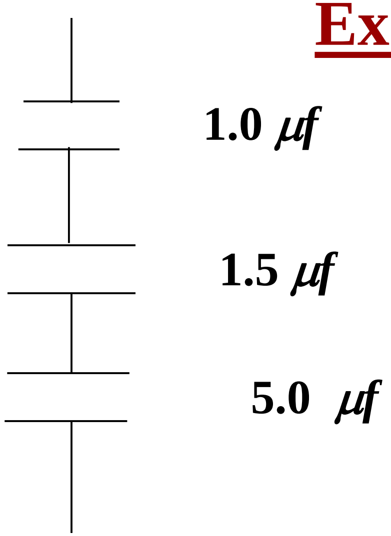
Parallel capacitors add in general

$$C_{eff} = C_1 + C_2 + C_3 \ldots\ldots$$

(opposite from resistors)

like just increasing area of C

Example:



$$\frac{1}{C_{eff}} = \left(\frac{1}{1.0}+\frac{1}{1.5}+\frac{1}{5.0}\right)\left(\frac{1}{\mu f}\right)$$

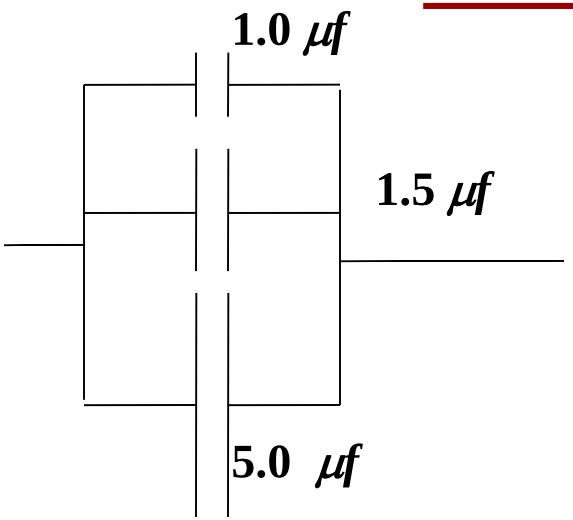
$$= \left(1+\frac{1}{\frac{3}{2}}+.2\right)\frac{1}{\mu f} = \left(1+\frac{2}{3}+.2\right)\frac{1}{\mu f}$$

$$= (1+.66+.2)\frac{1}{\mu f}$$

$$\frac{1}{C_{eff}} = 1.86\frac{1}{\mu f}$$

$$C_{eff} = 0.54\mu f$$

Example:



$$C_{eff} = (1.0+1.5+5.0)(\mu f) = 7.5\ \mu f$$

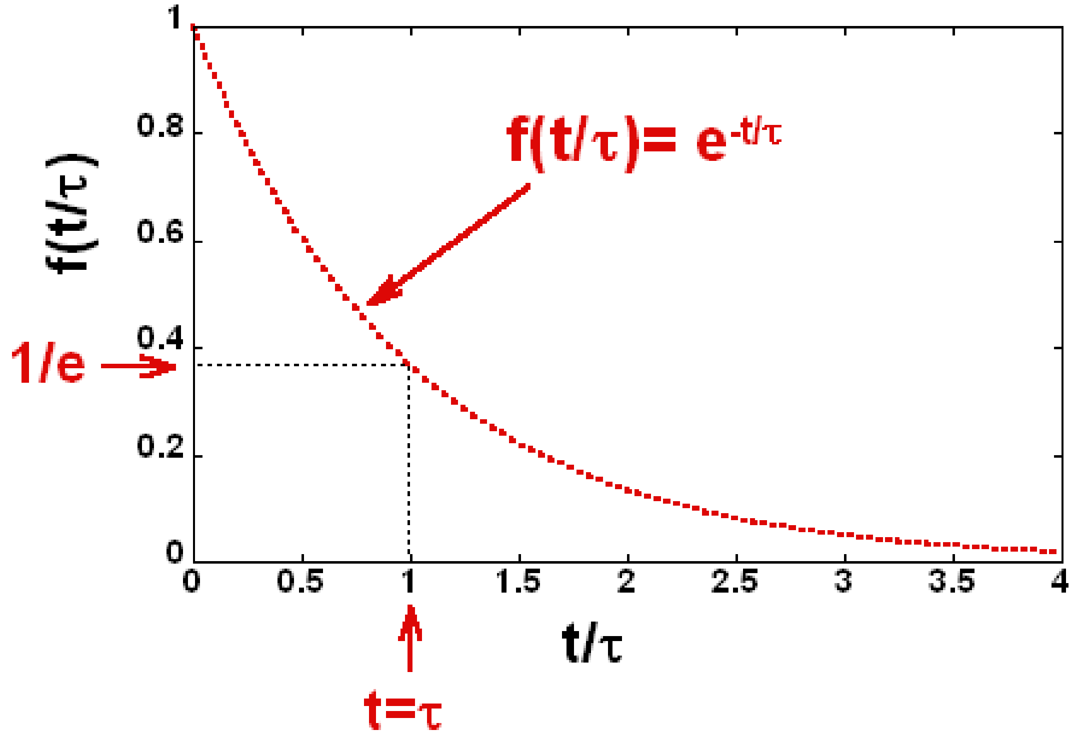
Exponential Function

$$f(t) = e^{-\frac{t}{\tau}}$$

= time constant

τ

$f(t=\tau) = e^{-1} = 1/e = 1/2.718$



$$\frac{df}{dt} = -\frac{1}{\tau} f$$

$$\frac{df}{dt} + \frac{1}{\tau} f = 0$$

or $$\frac{\Delta f}{\Delta t} = -\frac{1}{\tau} f$$

$$\frac{df}{f} = -\frac{1}{\tau} dt$$

$$\int \frac{df}{f} = -\frac{1}{\tau} \int dt$$

$$\ln(f) = -\frac{t}{\tau}$$

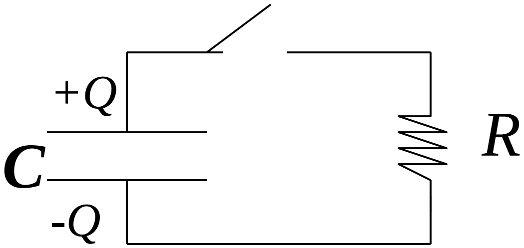
$$f(t) = e^{-\frac{t}{\tau}}$$

Note: gen. soln. $f(t) = f_o \, e^{-\frac{t}{\tau}}$ (f_0= constant)

Time varying electrical current

Example: Capacitor discharge

t=0 close switch



$$0 = V_c + V_R$$

$$t=0 \quad Q = Q_0$$

$$0 = \frac{Q}{C} + IR$$

$$Q \to 0$$
$$t = \infty$$

$$I = \frac{dQ}{dt}$$

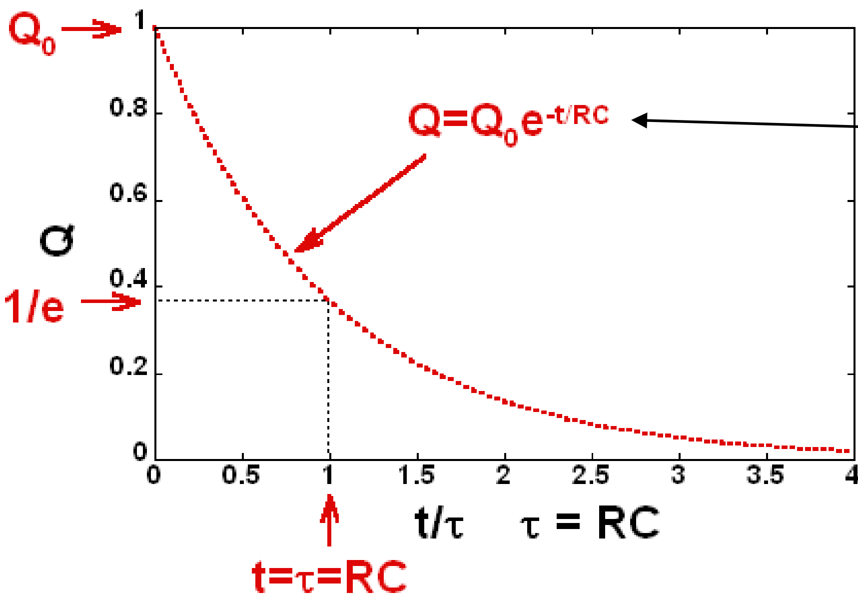
$$\frac{dQ}{dt} + \frac{Q}{[RC]} = 0$$

τ=RC=time constant

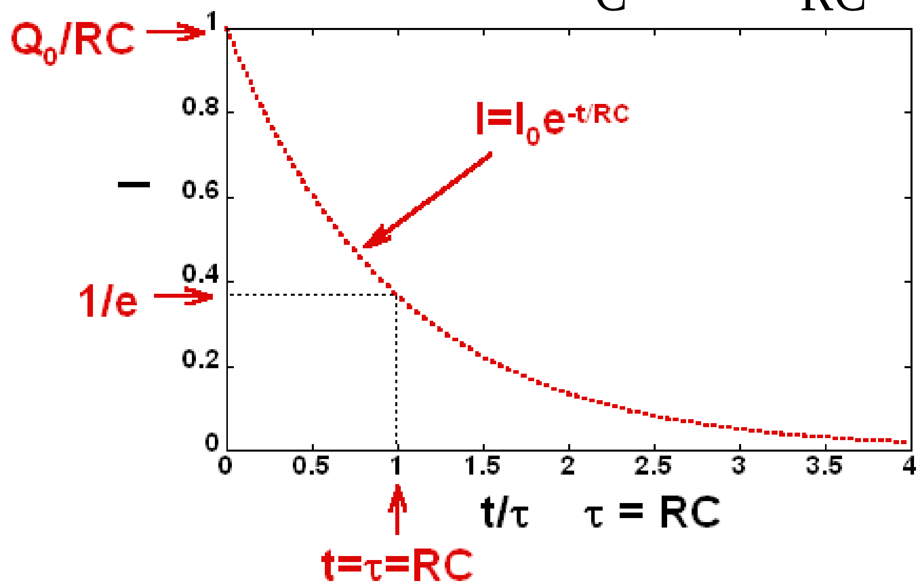
$$Q = [?] \; e^{-\frac{t}{Rc}}$$

$$t=0 \qquad Q = Q_0 \Rightarrow [?] = Q_0$$

$$\therefore Q = Q_0 e^{-\frac{t}{RC}}$$



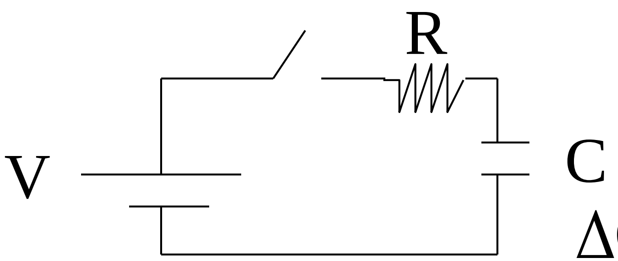
Recall $IR = \frac{Q}{C} \to I = -\frac{Q}{RC}$



$$\mathbf{Q = Q_0 e^{-\frac{t}{RC}}}$$

Capacitor Charging

t=0 close switch.



$$-V + IR + \frac{Q}{C} = 0$$

Again: $I = \frac{\Delta Q}{\Delta t}$

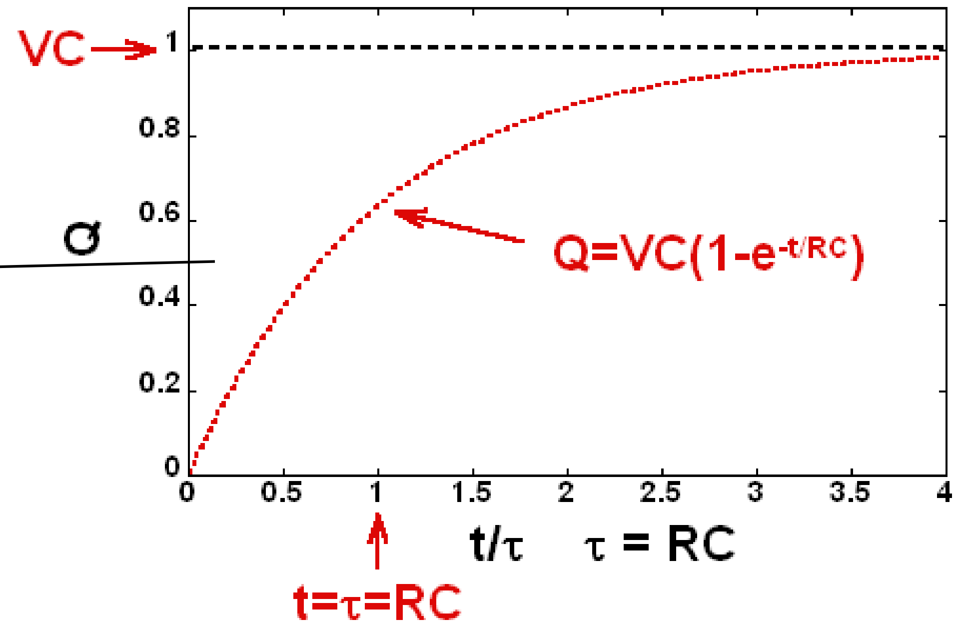
$$\frac{\Delta Q}{\Delta t} R + \frac{Q}{C} = V \Rightarrow \frac{\Delta Q}{\Delta t} + \frac{Q}{RC} = \frac{V}{R} \Rightarrow \frac{dQ}{dt} + \frac{Q}{RC} = \frac{V}{R}$$

Very similar to before but not identical

t=0 Q=0: t=∞ I=0

ie. $Q_o = VC$

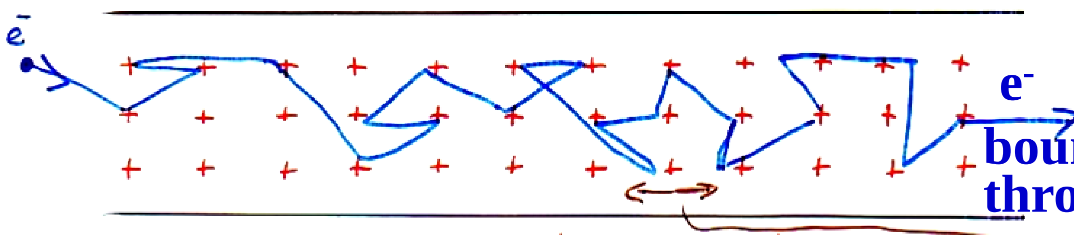
$$Q = VC\,(1 - e^{-\frac{t}{RC}})$$



$$I = -\frac{Q}{RC} + \frac{V}{R} = -\frac{V}{R}(1 - e^{-\frac{t}{RC}}) + \frac{V}{R}$$

$$I = \frac{V}{R} e^{-\frac{t}{RC}}$$

Appendix I: Temperature dependence of resistivity & superconductivity



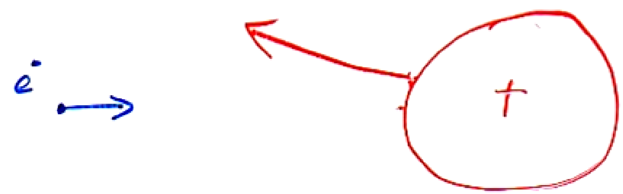
e⁻ bounces through

+ atomic position (+ ions) vibrate)

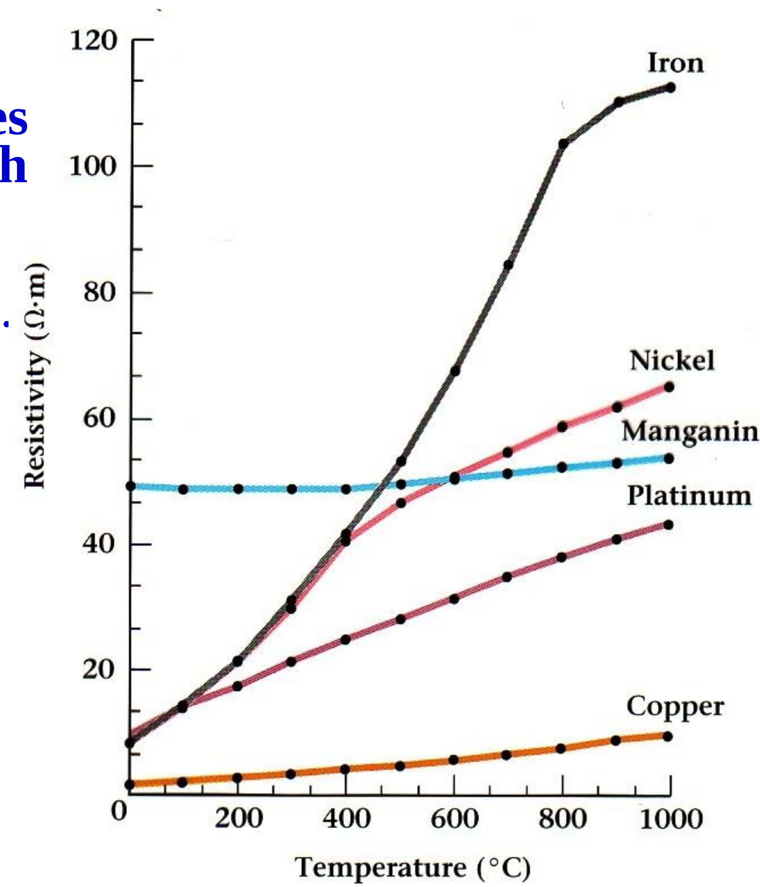
- Scattering of e⁻ creates <u>resistance</u> to forward motion.

- **Resistance is <u>energy loss</u> mechanism**

  motion ⟶ heat



vibrating



Heavy atoms scatter little e⁻ strongly when atoms are vibrating (at finite T).

- **Atoms vibrate less at low T**

⟶ Less electron scattering

⟶ Lower resistivity

Super conductivity R $\Rightarrow$ 0 at critical temp. T_c in some materials

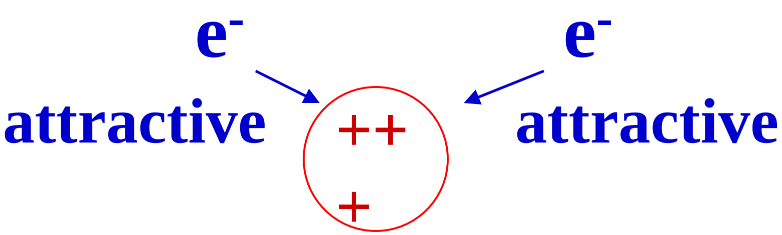
1911-Hg T_c=4.2K

1950-1970 Nb_3Sn T_c=23K

1989 $Y_1Ba_2Cu_3O_7$ T_c=95K

T_c =121 K highest yet!!!!

1) 2 e^- attracted to + ion
2) effective attraction between e^-
3) e^--e^- pairs form
4) pairs don't scatter so no resistance



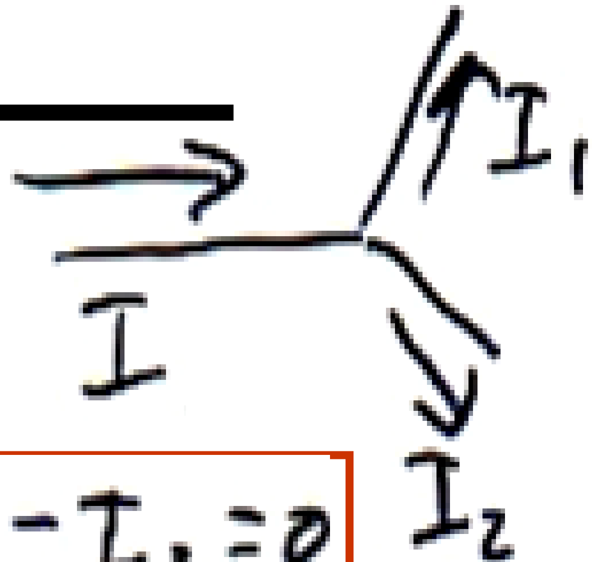
3-I-2

An infinite number of mathematicians walk into a bar.
The first one tells the bartender he wants a beer.
The second one says he wants half a beer.
The third one says he wants a fourth of a beer.
The bartender puts two beers on the bar and says "You guys need to learn your limits."

Appendix II: Formal Kirchhoff's Laws approach/concepts

Rules

junctions



$$I - I_1 - I_2 = 0$$

junction sign convention
+ in :: - out

(or opposite your choice but stick to it)

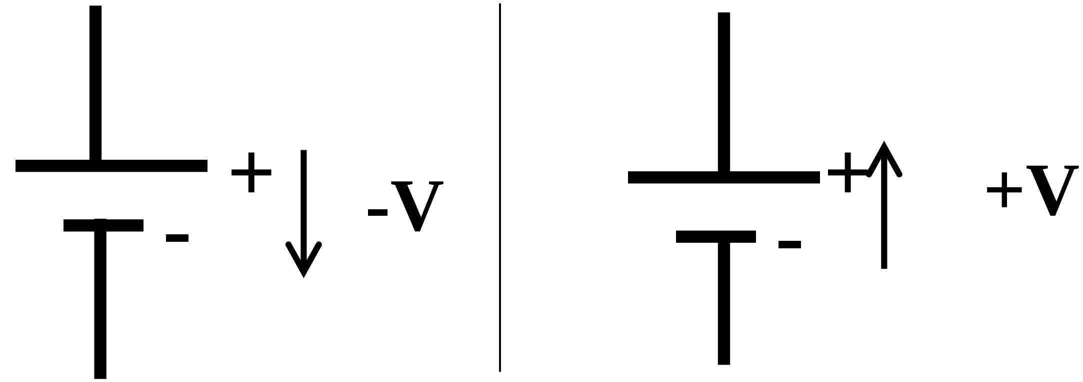
OR $I = I_1 + I_2$

current conservation !!
= no charge build-up

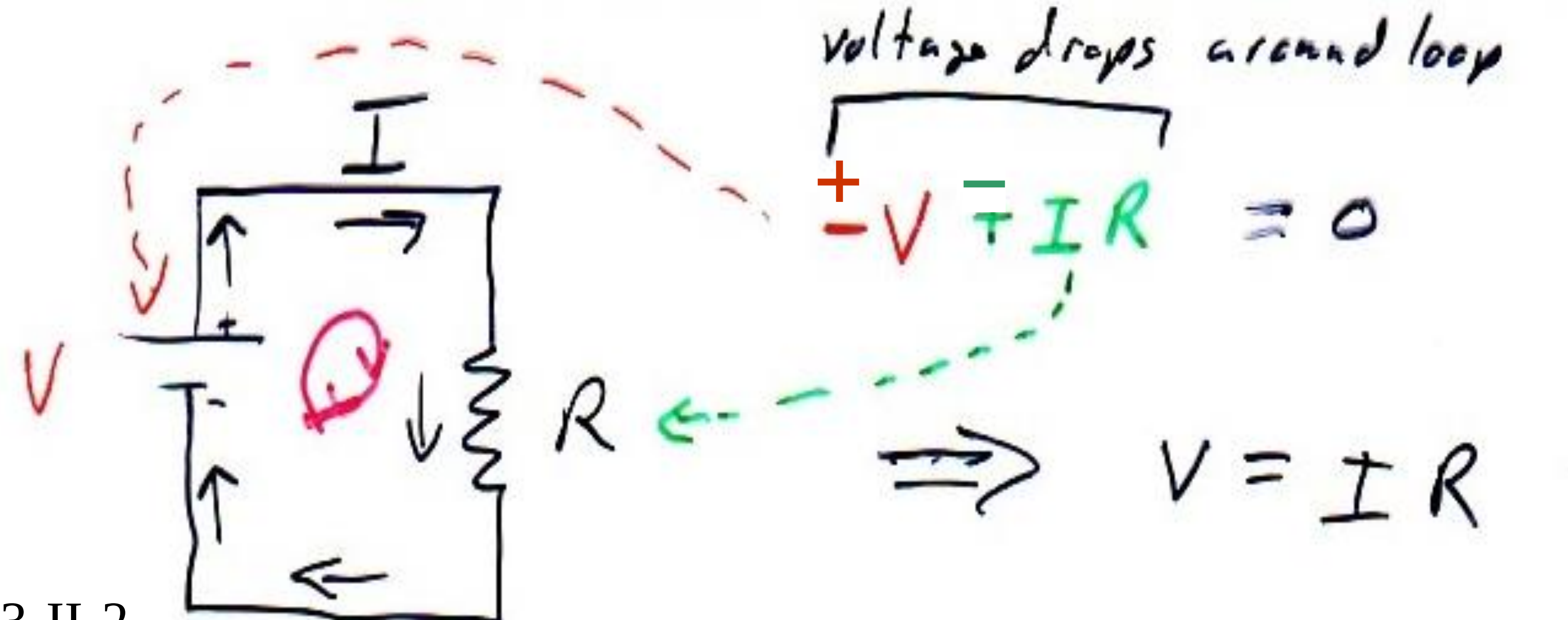
Resistors: magnitude and sign convention

Rules

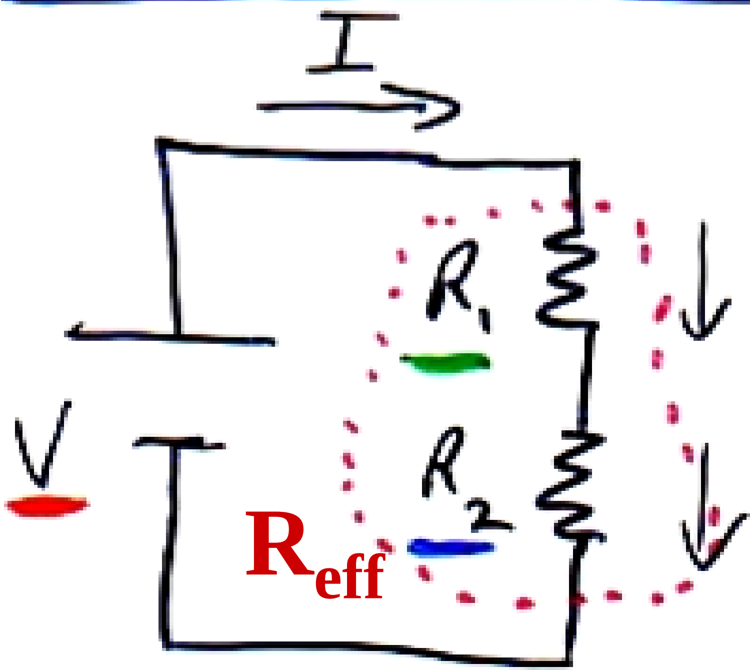
Battery sign convention



Voltage drop around loop = 0 (Energy conservation !!)



Series Resistors



$V - IR_1 - IR_2 = 0$

$V = IR_1 + R_2 I$

$V = I(R_1 + R_2) = I R_{eff}$

R_{eff} – for series

In General $R_{eff} = R_1 + R_2 + R_3 + \ldots$

for series resistors

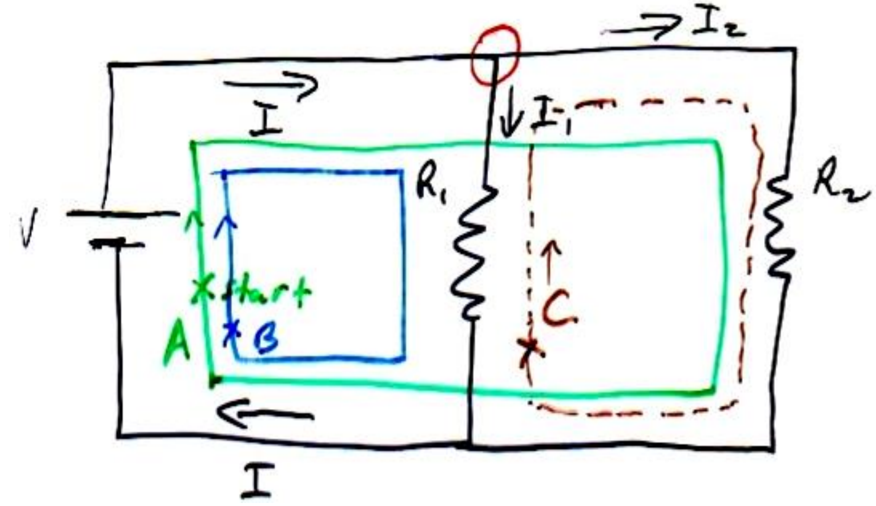
Resistors in parallel

Curr. Conserv. at ⊕

$$+I - I_1 - I_2 = 0$$

in (+I); out ($-I_1$, $-I_2$)

$$I = I_1 + I_2 \quad [1]$$



Loop A

$$+V - I_2 R_2 = 0 \longrightarrow I_2 = \frac{V}{R_2} \quad [2]$$

Loop B

$$+V - I_1 R_1 = 0 \longrightarrow I_1 = \frac{V}{R_1} \quad [3]$$

Loop C

$$+I_1 R_1 - I_2 R_2 = 0 \longrightarrow I_1 R_1 = I_2 R_2$$

going through R backwards ($+I_1 R_1$); forwards ($-I_2 R_2$)

$$\text{or} \quad \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$I_x = \frac{V}{R_2} \quad [2]$$

$$I_1 = \frac{V}{R_1} \quad [3]$$

[2] and [3] in [1]

$$[1] \quad I = I_x + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\Rightarrow \quad I = V \underbrace{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}_{1/R_{eff} \text{ for } \parallel} = \frac{V}{R_{eff}}$$

$$\boxed{V = R_{eff} I}$$

$$\boxed{\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

in general

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \cdots = \frac{1}{R_{eff}}$$