**INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2003**

**Questions 6: Due Dec 8th**

If you get chance to get ahead on these questions during the break, I would be very grateful.

1. Consider a Fermi gas containing Fermions with spin degeneracy $n_s = 2S+1 > 2$,and total density $n_s\rho$, where $\rho$ is the density per spin component. Nucleons can be considered as the special case where $n_s = 4$, corresponding to spin and isospin quantum number.

   (a) Suppose the fermions interact via a repulsive three body interaction:
   $$V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) = \beta\delta^{(3)}(\mathbf{r}_i - \mathbf{r}_j)\delta^{(3)}(\mathbf{r}_j - \mathbf{r}_k), \tag{1}$$
   Write this interaction in second-quantized form.

   (b) Invent a symbol for a three body interaction and write down the Feynman rules.

   (c) Use your rules to calculate the interaction energy per unit volume, $V(n_s, \rho)$ to leading order in $\alpha$. What happens when $n_s = 1$ or $n_s = 2$?

2. (a) In a Fermi liquid, the density correlations of parallel spin electrons develop a "correlation hole", namely the equal-time density-density correlation function for parallel electrons is depressed relative to the equal-time density-density correlation function for antiparallel electrons. What is the origin of this "hole" in the density correlations between parallel spin electrons?

   (b) Suppose
   $$C_{\sigma\sigma'}(\mathbf{x}-\mathbf{y}) \quad = \quad \langle\phi_0| : \rho_\sigma(\mathbf{x})\rho_{\sigma'}(\mathbf{y}) : |\phi_0\rangle, \qquad (\sigma,\ \sigma' \in \{\downarrow, \uparrow\}) \tag{2}$$
   is the density correlation function of a non-interacting Fermi fluid with a Fermi momentum $k_F$. Show that the first order expectation value of the interaction energy is given by
   $$\Delta E = \frac{1}{2}\sum_{\sigma,\sigma'} \int d^3x d^3y V(\mathbf{x}-\mathbf{y}) C_{\sigma,\sigma'}(\mathbf{x}-\mathbf{y}) \tag{3}$$

   (c) By comparing the above expression with the Hartree Fock Diagrams for the first order interaction energy, or using any other method, write down the Feynman diagrams for $C_{\uparrow\uparrow}(\mathbf{x}-\mathbf{y}) = C_{\downarrow\downarrow}(\mathbf{x}-\mathbf{y})$ and $C_{\uparrow\downarrow}(\mathbf{x}-\mathbf{y}) = C_{\downarrow\uparrow}(\mathbf{x}-\mathbf{y})$.

   (d) Show that $C_{\uparrow\uparrow}(\mathbf{x}) = C_{\uparrow\downarrow}(\mathbf{x}) + C_{hole}(\mathbf{x})$, where
   $$C_{hole}(\mathbf{x}) = G(\mathbf{x}, 0^-)G(-\mathbf{x}, 0^-) \tag{4}$$
   determines the "correlation hole" and where $G(x-y, 0^-)$ is the electron propagator evaluated for a small negative time interval.

   (e) In Fourier space, $G(\mathbf{p}, 0^-) = i\theta(p_F - |\mathbf{p}|)$. Transform this expression back into real-space, i.e, calculate $G(x, 0^-) = i\int_{p<p_F} \frac{d^3p}{(2\pi)^3} \; e^{i\mathbf{p}.\mathbf{x}}$ to obtain a closed form expression for $G(x, 0^-)$. Write your answer in terms of the density per spin $\rho$ and hence derive an expression for the correlation hole.

   (f) Sketch, or plot, as accurately as you can, $C_{\uparrow\uparrow}(\mathbf{x}-\mathbf{y})$, explaining any key features . What value is this function when $\mathbf{x}-\mathbf{y} = 0$?