INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2004

Questions 5: Due Nov 8th

1. Consider a Fermi gas containing Fermions with spin degeneracy $n_s = 2S+1 > 2$,and total density $n_s\rho$, where ρ is the density per spin component. Nucleons can be considered as the special case where $n_s = 4$, corresponding to spin and isospin quantum number.

   (a) Suppose the fermions interact via a repulsive three body interaction:
   $$V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) = \beta\delta^{(3)}(\mathbf{r}_i - \mathbf{r}_j)\delta^{(3)}(\mathbf{r}_j - \mathbf{r}_k), \tag{1}$$
   Write this interaction in second-quantized form.

   (b) Invent a symbol for a three body interaction and write down the Feynman rules.

   (c) Use your rules to calculate the interaction energy per unit volume, $V(n_s, \rho)$ to leading order in α. What happens when $n_s = 1$ or $n_s = 2$?

   (d) If we neglect Coulomb interactions, why is the case $n_s = 4$ relevant to nuclear matter?

2. The separation of electrons $R_e$in a Fermi gas is defined by
   $$\frac{4\pi R_e^3}{3} = \rho^{-1}$$
   where ρ is the density of electrons. The dimensionless separation r_s is defined as $r_s = R_e/a$ where $a = \frac{c\hbar^2}{me^2}$ is the Bohr radius.

   (a) Show that the Fermi wavevector is given by
   $$k_F = \frac{1}{\alpha r_s a}$$
   where $\alpha = \left(\frac{4}{9\pi}\right)^{\frac{1}{3}} \approx 0.521$.

   (b) Consider an electron plasma where the background charge density precisely cancels the charge density of the plasma. Show that the ground-state energy to leading order in the strength of the Coulomb interaction is given by
   $$\begin{aligned}\frac{E}{\rho V} &= \frac{3}{5}\frac{R_Y}{\alpha^2 r_s^2} - \frac{3}{4\pi}\frac{R_Y}{\alpha r_s} \\ &= \left(\frac{2.21}{r_s^2} - \frac{0.46}{r_S}\right) R_Y\end{aligned} \tag{2}$$
   where $R_Y = \frac{\hbar^2}{2ma^2}$ is the Rydberg energy. (Hint - in the electron gas with a constant charge background, the Hartree part of the energy vanishes. The Fock part is the second term in this expression. You may find it useful to use the integral
   $$\int_0^1 dx \int_0^1 dy xy \ln|\frac{x+y}{x-y}| = \frac{1}{2}$$

   (c) When can the interaction effects be ignored relative the kinetic energy?