INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2004

Questions 3. (Due Mon , Oct 11th)

1. (a) Show that for a general system of conserved particles at chemical potential, the total particle number in thermal equilibrium can be written as

$$N = -\partial F/\partial \mu \tag{1}$$

where

$$\begin{aligned} F &= -k_B T \ln Z \\ Z &= \mathrm{Tr}[e^{-\beta(\hat{H}-\mu N)}]. \end{aligned} \tag{2}$$

(b) Apply this to a single bosonic energy level, where

$$H - \mu N = (\epsilon - \mu) a^\dagger a \tag{3}$$

and $\hat{a}^\dagger$ creates either a Fermion, or a boson, to show that

$$\begin{aligned} F &= \pm k_B T \ln[1 \mp e^{-\beta(\epsilon-\mu)}] \\ \langle \hat{n} \rangle &= \frac{1}{e^{\beta(\epsilon-\mu)} \mp 1} \end{aligned} \tag{4}$$

where the upper sign refers to bosons, the lower, to fermions. Sketch the occupancy as a function of ϵ for the case of fermions and bosons. Why does μ have to be negative for bosons?

2. Bose Einstein condensates created inside optical atom traps contain alkali atoms at densities of about $10^{14} - 10^{15} cm^{-3}$.

(a) What is the Bose Einstein transition temperature of a gas of Sodium atoms at a density $10^{15} cm^{-3}$? (Give your answer in micro-Kelvin.) How are such temperatures attained in practice? (See http://cua.mit.edu/ketterle_group for more information.)

(b) Liquid Helium has a density of $122 g/litre$ at its boiling point. Compare its theoretical Bose Einstein condensation temperature with its superfluid transition temperature (2.21 K). numbers not the same?

3. Consider a system of fermions or bosons, created by the field $\psi^\dagger(\vec{r})$ interacting under the potential

$$V(r) = \begin{cases} U, & (r < R), \\ 0, & (r > R), \end{cases} \tag{5}$$

(a) Write the interaction in second quantized form.

(b) Switch to the momentum basis, where $\psi(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} c_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$. Verify that $[c_{\vec{k}},\ c^\dagger_{\vec{k}'}]_\pm = (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}')$, and write the interaction in this new basis. Please sketch the form of the interaction in momentum space.