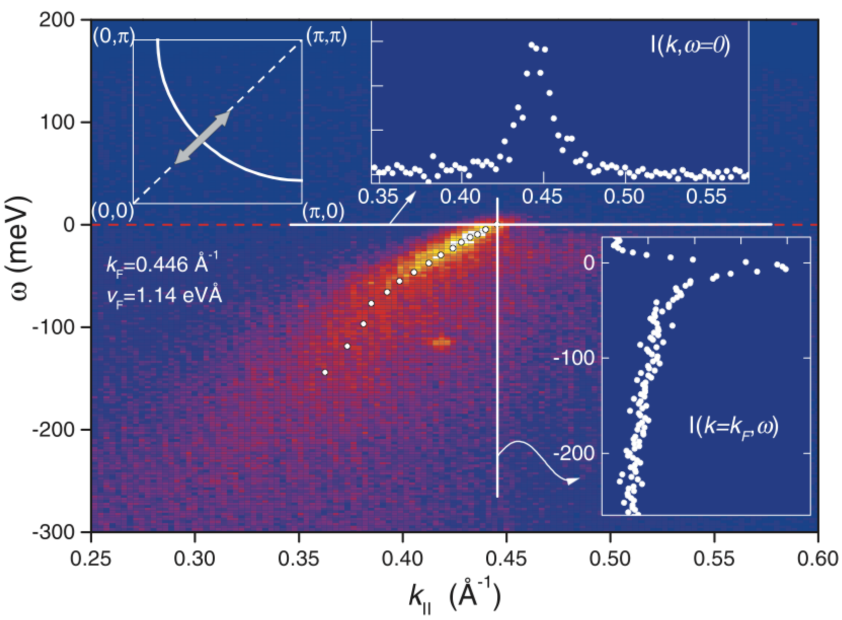
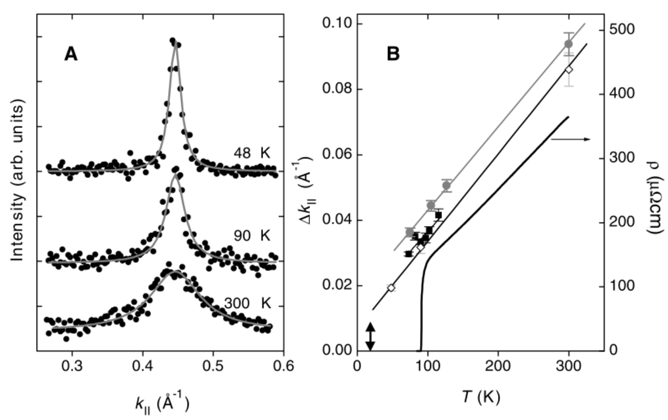
Today we will do a survey of various theories of Strange metals. Before we do so, I would like to show you some of the ARPES measurements that display the linear scattering rate.



Evidence for Quantum Critical Behavior in the Optimally Doped Cuprate $Bi_2Sr_2CaCu_2O_{8+\delta}$

T. Valla,[1] A. V. Fedorov,[1] P. D. Johnson,[1] B. O. Wells,[1,4] S. L. Hulbert,[2] Q. Li,[3] G. D. Gu,[5] N. Koshizuka[6]

24 SEPTEMBER 1999 VOL 285 SCIENCE www.sciencemag.org



$$\hbar v_k \Delta k = \frac{\hbar v_k}{\ell} \approx |2\mathrm{Im}\Sigma(\mathbf{k},\omega)|$$

$$A(\mathbf{k},\omega) \propto \frac{1}{\pi}$$

$$\frac{\mathrm{Im}\Sigma(\mathbf{k},\omega)}{[\omega - \varepsilon_k - \mathrm{Re}\Sigma(\mathbf{k},\omega)]^2 + [\mathrm{Im}\Sigma(\mathbf{k},\omega)]^2} \quad (1)$$

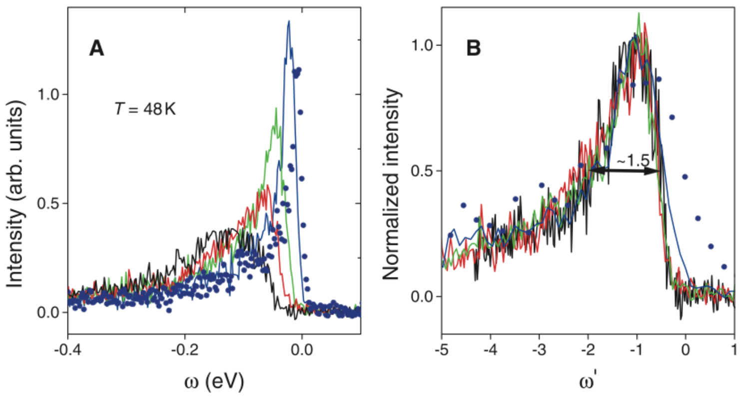


Fig. 3. (**A**) EDCs obtained in the (0,0) $\rightarrow$ (π,π) direction after background subtraction. (**B**) EDCs scaled to the same peak position, showing that the overall shape scales linearly with binding energy. The peak width is approximately 1.5 times the binding energy in all spectra (double-headed arrow).

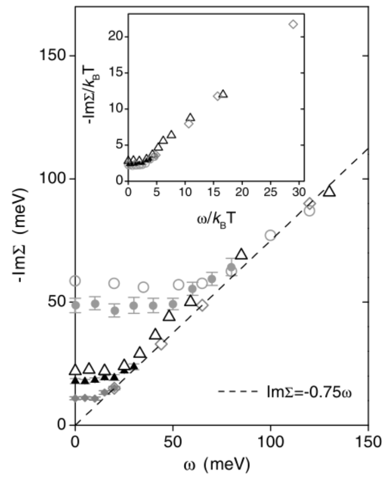
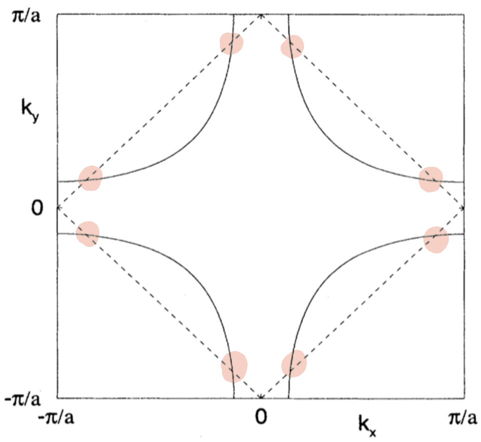


Fig. 4. A compilation of ImΣ obtained from Δk cuts or MDCs (solid symbols) and from peak widths in EDCs (open symbols) as a function of binding energy for 48 temperatures of K (diamonds), 90 K (triangles), and 300 K (circles). The inset shows the same data plotted in dimensionless units confirming scaling behavior. Error bars indicate uncertainties from the fits of MDCs to Lorentzian line shapes.

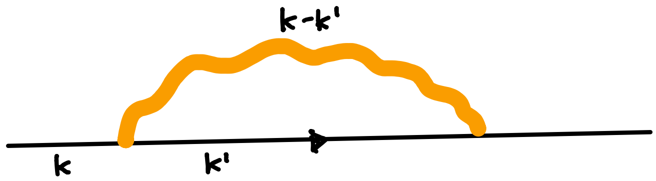
Resistivity as a function of temperature for models with hot spots on the Fermi surface

R. Hlubina* and T. M. Rice
Theoretische Physik, Eidgenössische Technische Hochschule Hönggerberg, CH-8093 Zürich, Switzerland
(Received 2 December 1994)

"Hot Spots".



$$\frac{1}{\tau_{\mathbf{k}}} = 2g^2 \sum_{\mathbf{k}'} \int_0^{\varepsilon_{\mathbf{k}}} d\omega\ \mathrm{Im}\chi(\mathbf{k}-\mathbf{k}',\omega)\,\delta(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}-\omega),$$



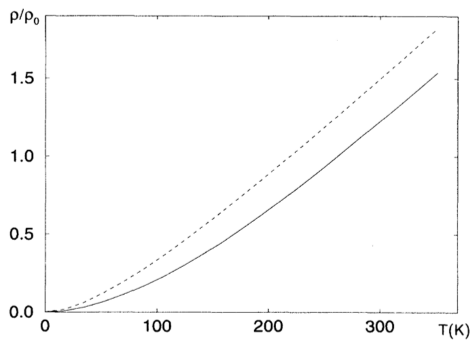


FIG. 3. Resistivity due to spin fluctuations as a function of temperature for the spin-fluctuation parameters $T^* = 110$ K, $\alpha = 0.55$, $\omega_D = 1760$ K, the electron parameters $t = 0.25$ eV, $t' = 0.45t$, $n = 0.75$, and the coupling constant $g = 0.64$ eV. Solid line: calculation with the improved ansatz Eq. (2.11). Dashed line: calculation with the standard ansatz Eq. (2.10).

The weakly scattered regions of the FS short-circuit the hot spots.

$$\sigma \propto \sum -\frac{\partial f}{\partial \epsilon_k} \frac{1}{\Gamma_{tr}(\vec{k})}$$

How should we interpret the *two* transport relaxation times in the cuprates?

P Coleman†, A J Schofield† and A M Tsvelik‡

† Serin Laboratory, Rutgers University, PO Box 849, Piscataway, NJ 08855-0849, USA

‡ Department of Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Received 20 June 1996, in final form 20 August 1996

J. Phys.: Condens. Matter **8** (1996) 9985–10015. Printed in the UK

$$\sigma_{xy} \propto \frac{H}{\Gamma_{\mathrm{tr}}\Gamma_H} \qquad (\sim T^{-3})$$

$$\Delta\sigma_{xx} \propto \frac{H^2}{\Gamma_{\mathrm{tr}}\Gamma_H^2} \qquad (\sim T^{-5}).$$

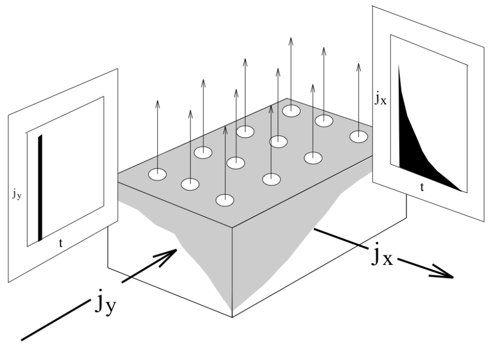


Figure 1. Illustrating the Hall response $j_x(t) = j_0\Theta_H(t)$ to an input current pulse $j_y(t) = j_0\,\delta(t)$.

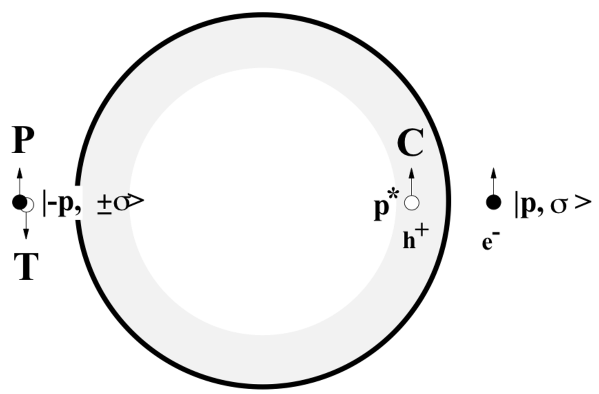
$$j_x(t) = \int_{-\infty}^{t} \sigma_{xx}(t-t')E_x(t')\,\mathrm{d}t'$$

$$\sigma_{xx}(t-t') = \frac{ne^2}{m}\mathrm{e}^{-\Gamma_{\mathrm{tr}}(t-t')}.$$

$$j_x(t) = \int_{-\infty}^{t} \Theta_H(t-t')j_y(t')\,\mathrm{d}t'$$

$$\Theta_H(t-t') = \omega_c \mathrm{e}^{-\Gamma_H(t-t')}$$

$$\Gamma_{tr} \propto k_B T \qquad\qquad \Gamma_H \propto T^2/\omega.$$



	$\mathcal{J}_E \sim \boldsymbol{A}$	$\mathcal{J}_H \sim \boldsymbol{E}\times\boldsymbol{H}$
C	$-$	$+$
P	$-$	$-$
T	$-$	$-$

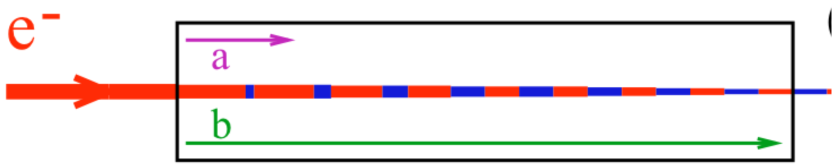
$$\underline{\Sigma} = \Sigma_1 + \Sigma_2 \tilde{C}$$

$$a_{p\sigma} = \frac{1}{\sqrt{2}}[\psi_{p\sigma} + \sigma\psi^{\dagger}_{p^*-\sigma}] \qquad (C = +1)$$

$$b_{p\sigma} = \frac{1}{\mathrm{i}\sqrt{2}}[\psi_{p\sigma} - \sigma\psi^{\dagger}_{p^*-\sigma}] \qquad (C = -1).$$

$\Gamma_a \sim T$ "Short"

$\Gamma_b \sim T^2$ "Long"

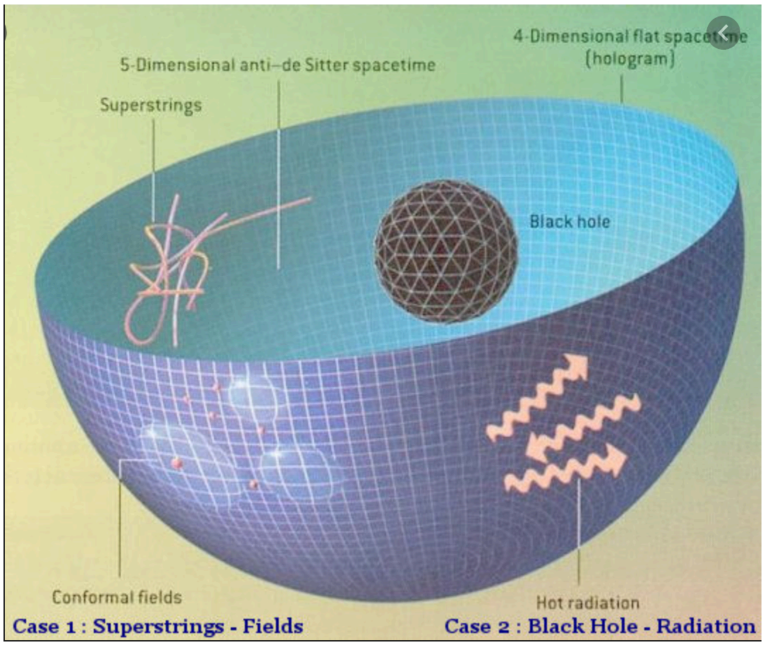


AdS CFT

$$\left\langle e^{-\int d^d x\ j(x)\mathcal{O}(x)}\right\rangle_{\text{CFT}} = \int D[\phi]\, e^{-\int dr \int d^d x \mathcal{L}_{\text{grav}}[\phi]}.$$

$\boxed{\text{MALDACENA CONJECTURE}}$

$$\lim_{r\to\infty} \phi(x,r) = j(x)$$



Key idea:

Condensed matter near a quantum critical point will acquire a simpler description when rewritten as a gravity dual.

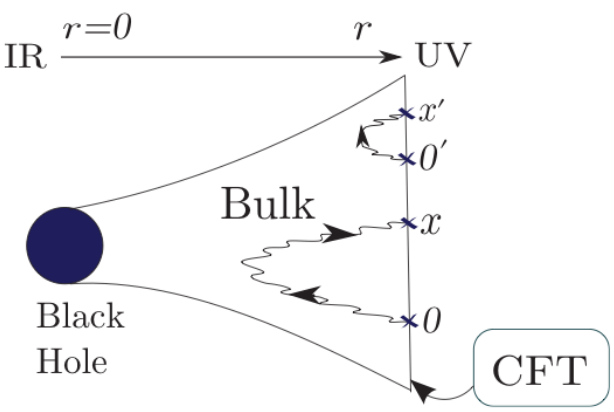
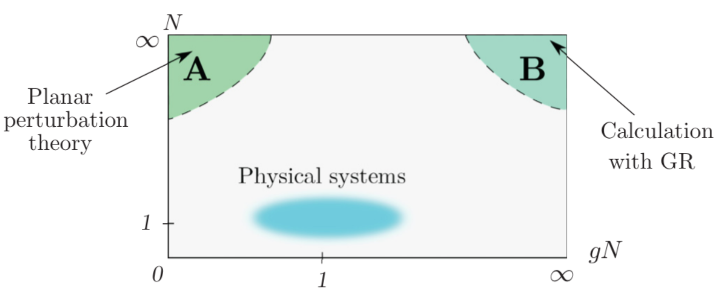


FIG. 1. (Color online) Illustrating the surface excitations "propagating" into the bulk. The vertical axis is the physical coordinate of the critical theory (CFT), while the horizontal axis is the AdS coordinate r. A physical picture for r is obtained as follows. Consider the injection and removal of a particle on the boundary of the AdS space, separated by a distance x. When the point of injection and removal are nearby, the Feynman paths connecting them will cluster near the boundary, probing *large* values of r. By contrast, when the two points are far apart, the Feynman paths connecting them will pass deep within the gravity well of the anti–de Sitter space, probing *small* values of r close to the black hole. In this way, the additional dimension tracks the evolution of the physics from the infrared to the ultraviolet.

Alexandrov + Coleman, PRB 86, 125145 '12.



SU(N) supersymmetric QCD is equivalent to classical gravity in 5 dimensional AdS space

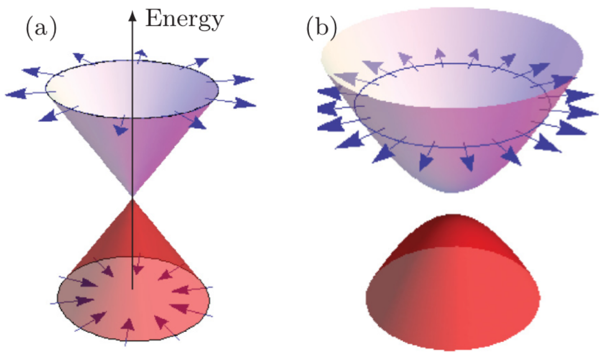


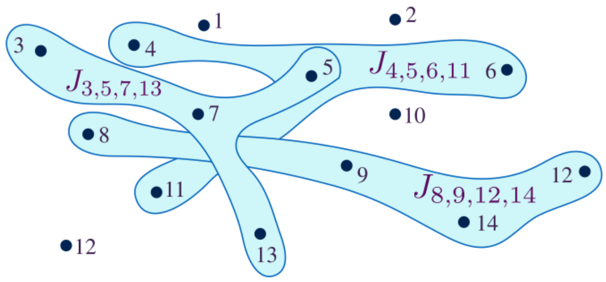
FIG. 4. (Color online) Typical dispersion and Fermi surface of Dirac fermion in $2+1$ (a); and the holographic metal from Fig. 6(a) (b).

$$G = \frac{z}{\omega - v_F(\mathbf{k}\cdot\boldsymbol{\sigma} - k_F) + c_1\omega^{2\nu}} + G_{\rm incoh}.$$

SYK MODELS

$$\boxed{H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell}$$

SACHDEV - YE (1992)



$$G(\tau) = -\langle c(\tau) c^\dagger(0)\rangle \sim \frac{1}{\sqrt{\tau}} \Rightarrow G(\omega) \sim \frac{1}{\sqrt{\omega}} \Longleftrightarrow \Sigma(\omega) \propto \sqrt{\omega}$$

Strongly Correlated Metal Built from Sachdev-Ye-Kitaev Models

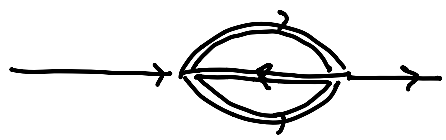
Xue-Yang Song,[1,2] Chao-Ming Jian,[2,3] and Leon Balents[2]
[1] *International Center for Quantum Materials, School of Physics, Peking University, Beijing 100871, China*
[2] *Kavli Institute of Theoretical Physics, University of California, Santa Barbara, California 93106, USA*
[3] *Station Q, Microsoft Research, Santa Barbara, California 93106-6105, USA*
(Received 23 May 2017; published 20 November 2017)

$$\overline{|U_{ijkl,x}|^2} = 2U_0^2/N^3$$

$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx'\rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'},$$

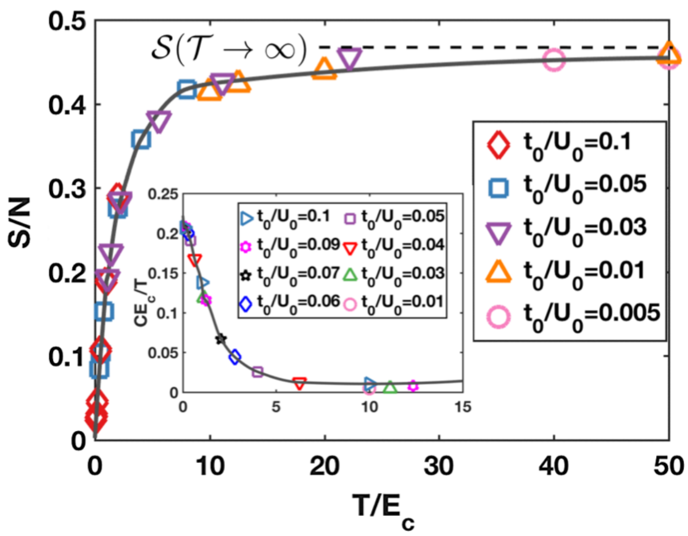
$$\overline{|t_{ij,x,x'}|^2} = t_0^2/N.$$

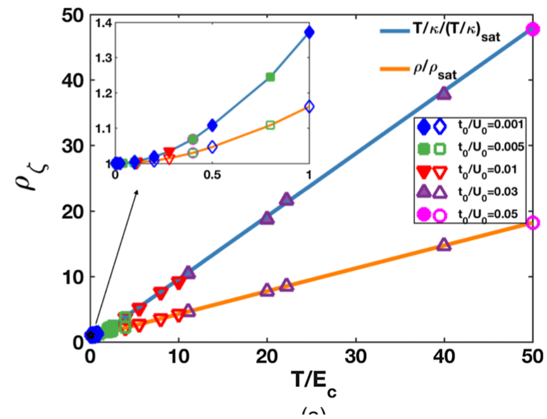
QUENCHED DISORDER



$$\boxed{\begin{aligned}\Sigma &= -U_0^2\, G(\tau)^2 G(-\tau)\\ G^{-1} &= i\omega_n + \mu - \Sigma - z t_0^2\, G(i\omega_n)\end{aligned}}$$

$E_c \sim t_0^2/U_0$





$$\rho_\zeta(t_0, T \ll U_0) = \frac{1}{N} R_\zeta\left(\frac{T}{E_c}\right), \qquad \zeta \in \{\varphi, \varepsilon\},$$

- Crossover from Strange to Fermi liquid Metal for $T \lesssim E_0$
- Not clear why $\rho \propto T$ if $\Sigma \sim \sqrt{\omega}$.
- No fractionalization — just local criticality.
- Beautiful model, but lacks certain features seen in strange metals — no crossover to FL & Two relaxation times.