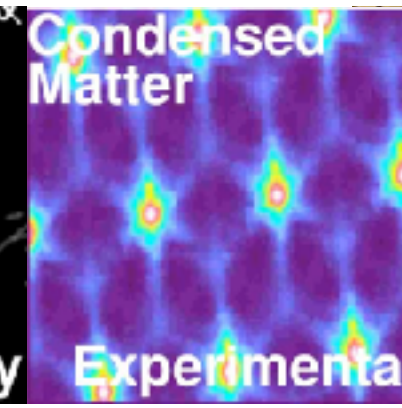
Heavy Fermion Physics: a 21st Century perspective

Piers Coleman: Rutgers Center for Materials Theory, USA

Julich,
21 Sept, 2015



Royal Holloway
University of London

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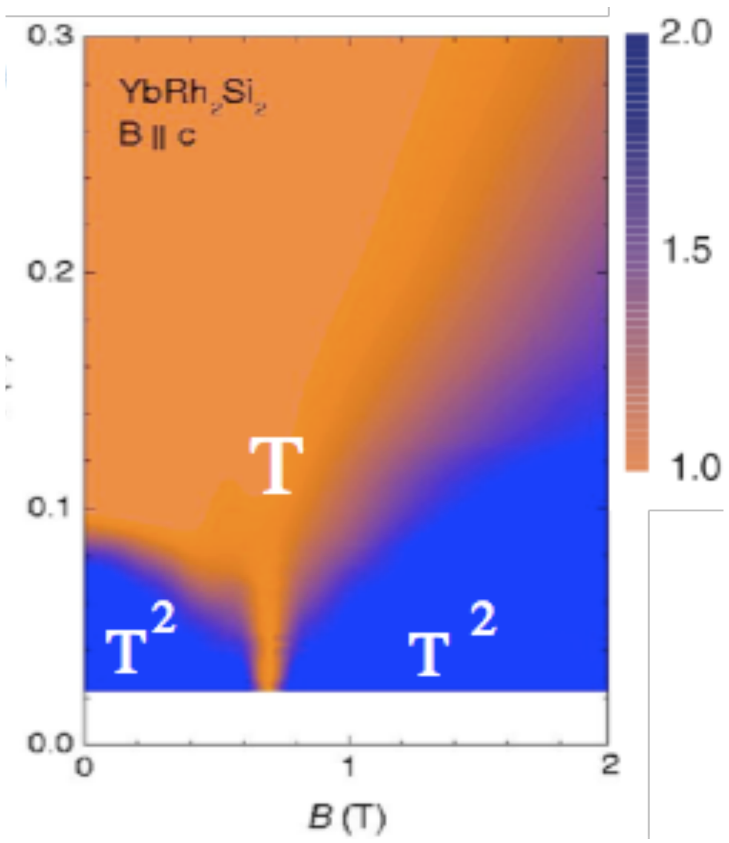
Quantum Criticality & Strange Metals



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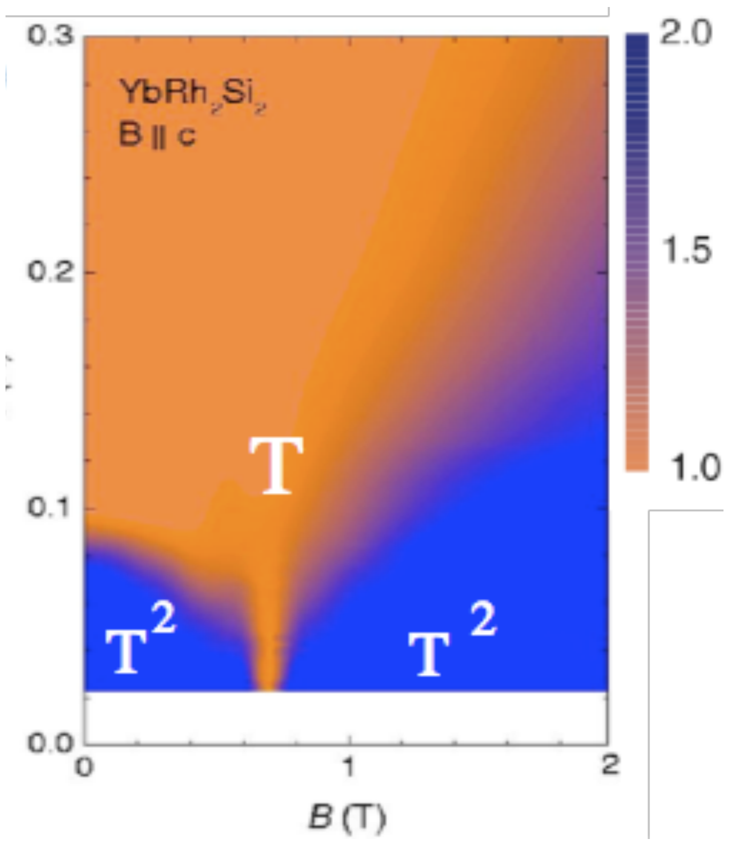

Heavy Fermion Superconductivity

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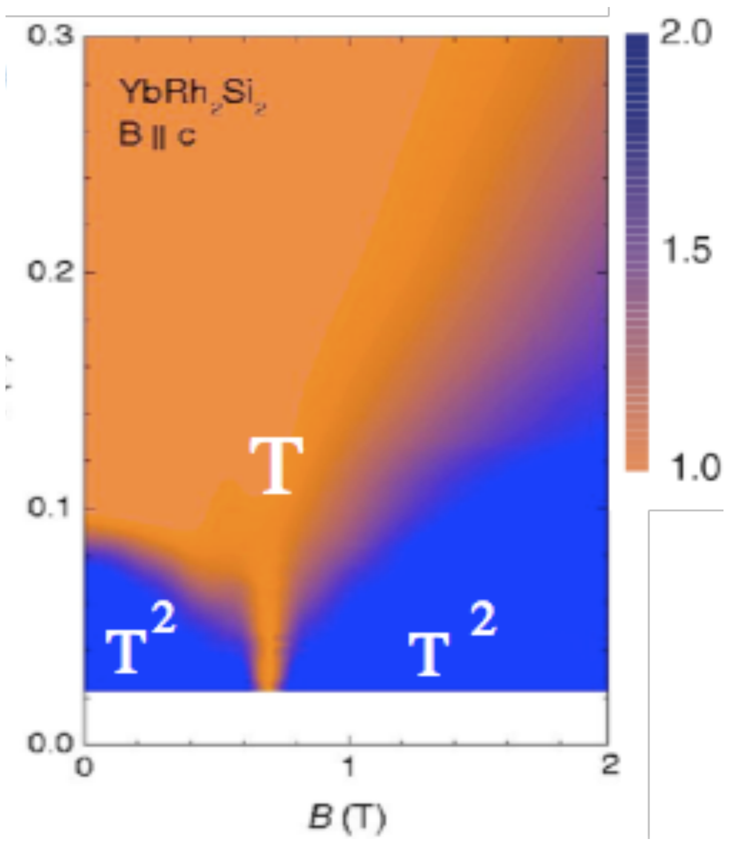
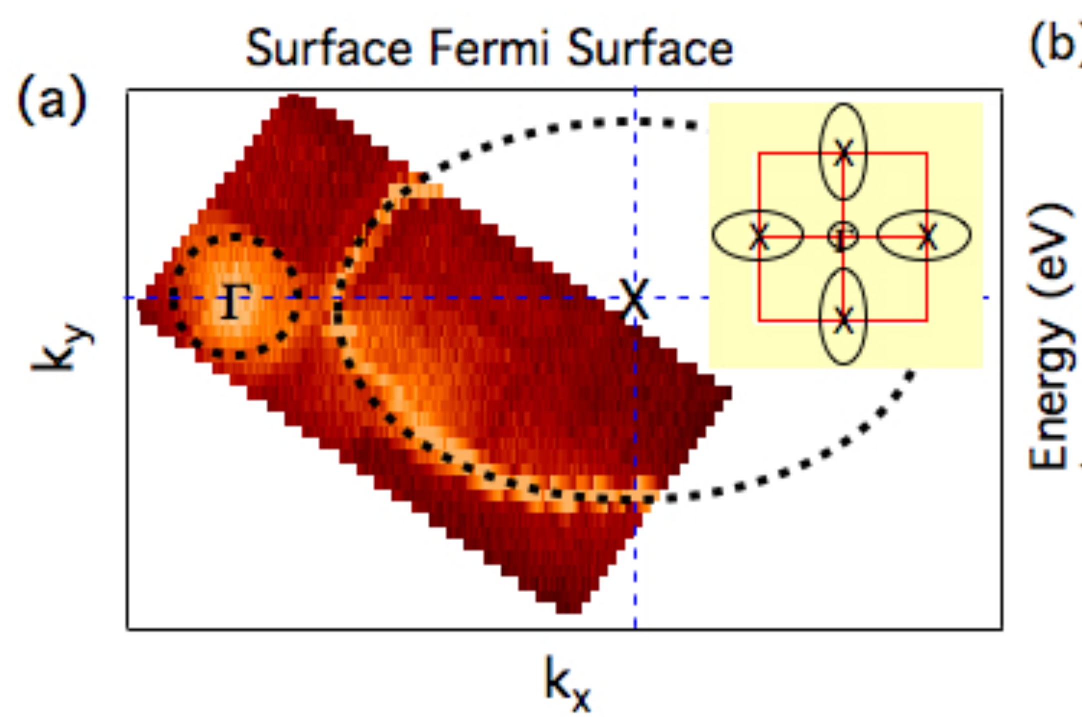


Heavy Fermion Superconductivity

Topological Kondo Insulators



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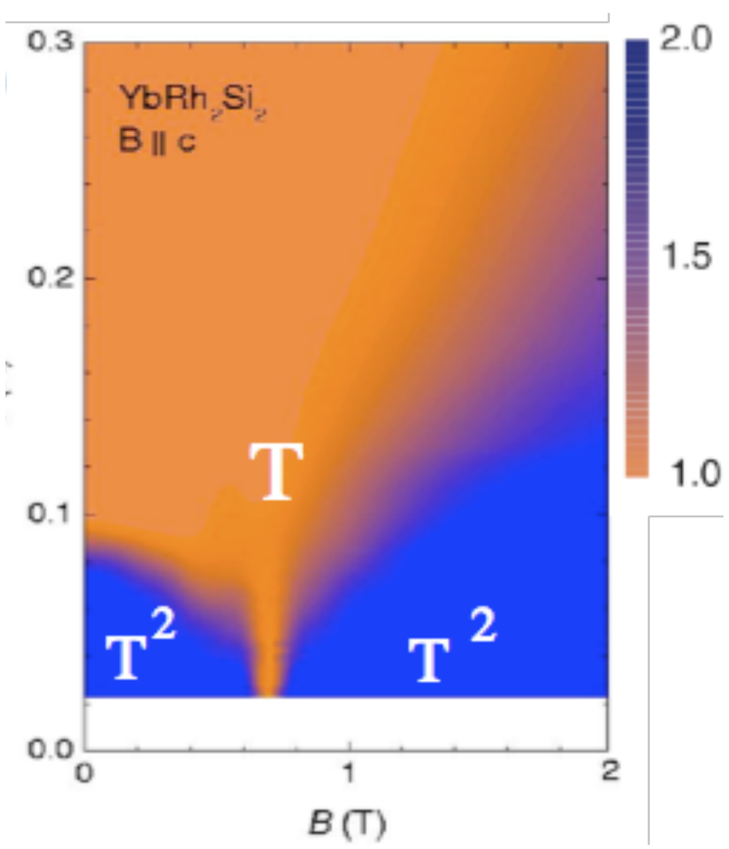

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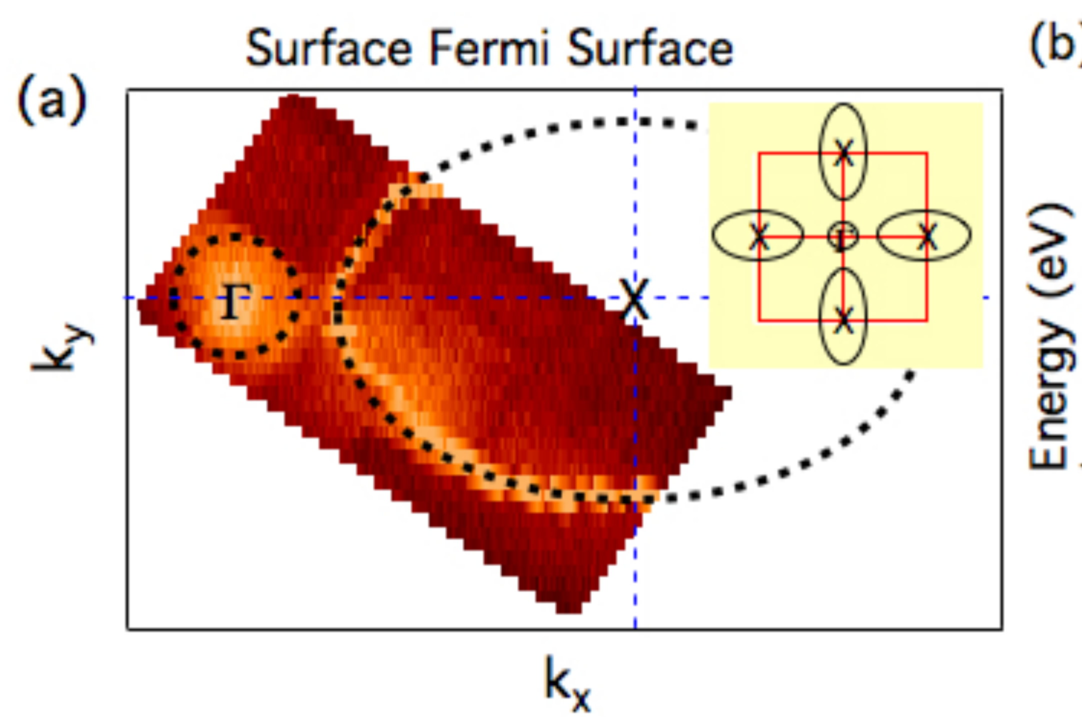
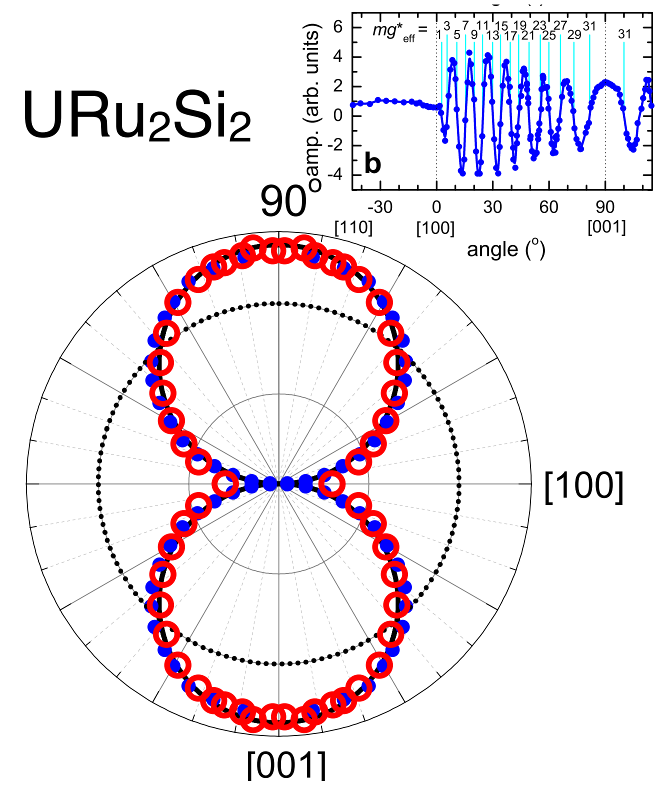


Hidden Order

URu$_2$Si$_2$



Collaborators.

Y Komijani	U. Cincinatti
Q. Si	Rice
R. Ramazashvili	CNRS, Toulouse
C. Pepin	CEA, Saclay
Aline Ramires	PSI, Zurich
Jerome Rech	CNRS, Marseille
Rebecca Flint	Iowa State
Premi Chandra	Rutgers
Andriy Nevidomskyy	Rice
Alexei Tsvelik	Brookhaven NL
Hai-Young Kee	U. Toronto
Natan Andrei	CMT, Rutgers
Onur Erten	ASU, Tempe
Tamaghna Hazra	CMT, Rutgers
Eduardo Miranda	Campinas
Maxim Dzero	Kent State
Victor Galitski	U. Maryland
Kai Sun	U. Michigan
Gergely Zarand	TU, Budapest

Experimentalists:

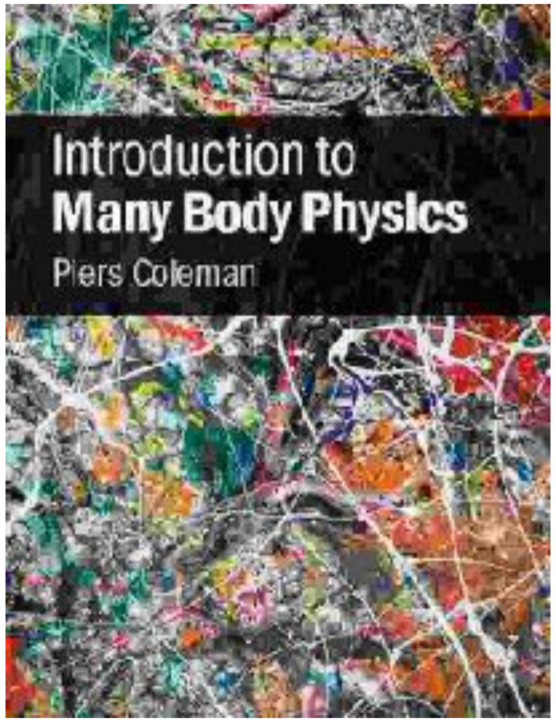
H. von Lohneysen	Karlsruhe
G. Aeppli	ETH, Zurich
A. Schröder	Kent State
S. Nakatsuji	ISSP
G. Lonzarich	Cambridge
S. Paschen	Vienna
J. Thompson	Los Alamos
J. Allen	U. Michigan
Z. Fisk	UC Irvine
F Steglich	Dresden/Zhejiang
H Yuan	Zhejiang

U.S. DEPARTMENT OF ENERGY

NSF

Notes:

"Introduction to Many Body Physics", Ch 8,15-16", PC, CUP to be published (2015).



"Heavy Fermions: electrons at the edge of magnetism." Wiley encyclopedia of magnetism. PC. cond-mat/0612006.

"I2CAM-FAPERJ Lectures on Heavy Fermion Physics", (X=I, II, III) http://physics.rutgers.edu/~coleman/talks/RIO13_X.pdf

"Julich lectures: Heavy Fermion Physics: A 21st Century Perspective" arXiv:1509.05769

General reading:

A. Hewson, "Kondo effect to heavy fermions", CUP, (1993).

"The Theory of Quantum Liquids", Nozieres and Pines (Perseus 1999).

Outline of the Topics

1. **<u>Trends in the periodic table.</u>**
2. Introduction: Heavy Fermions and the Kondo Lattice.
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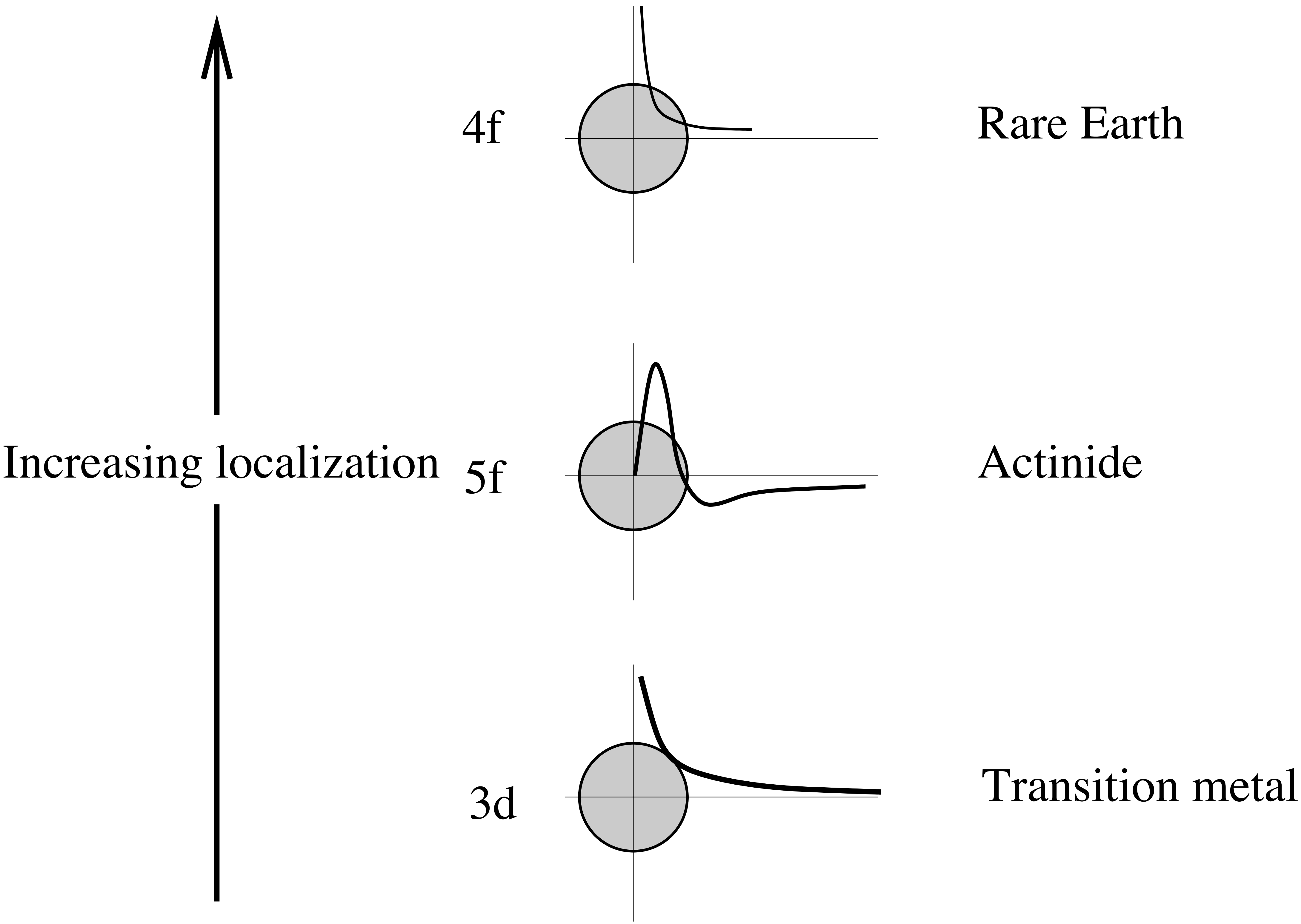
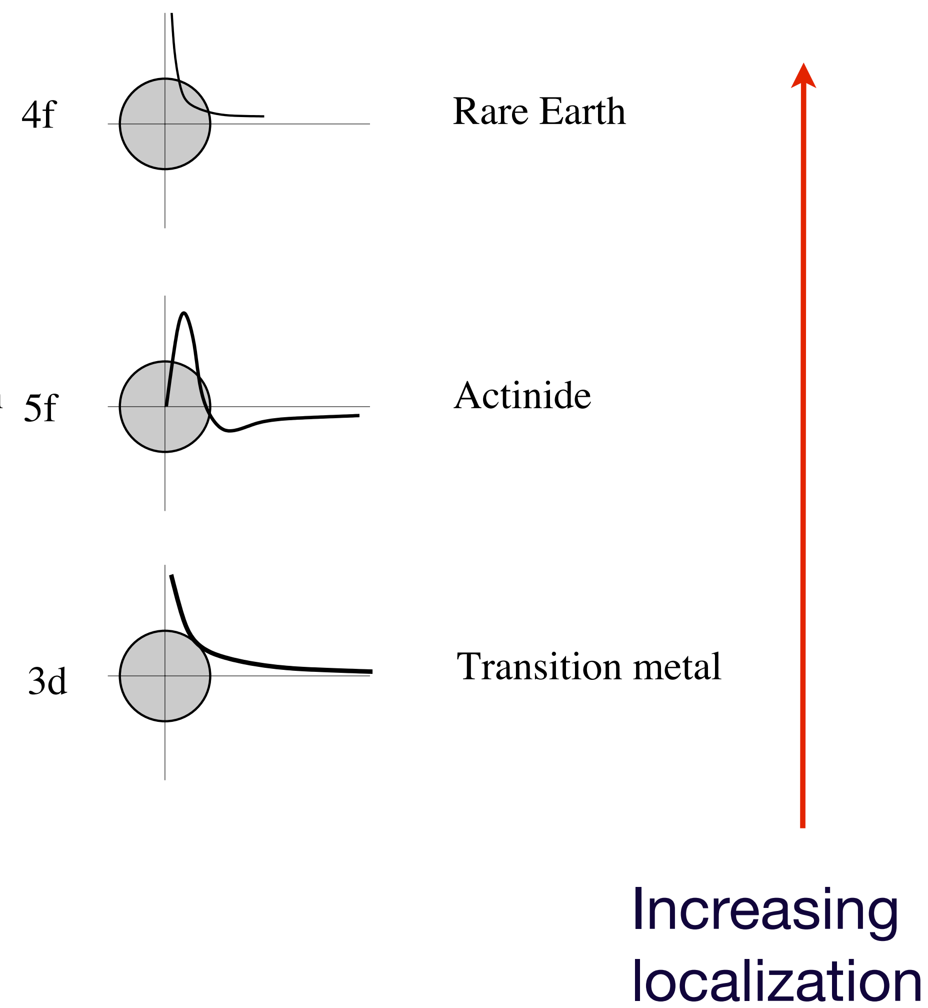
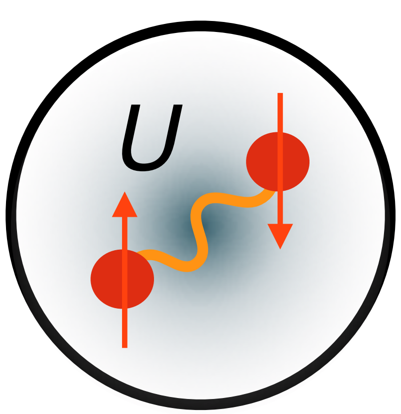
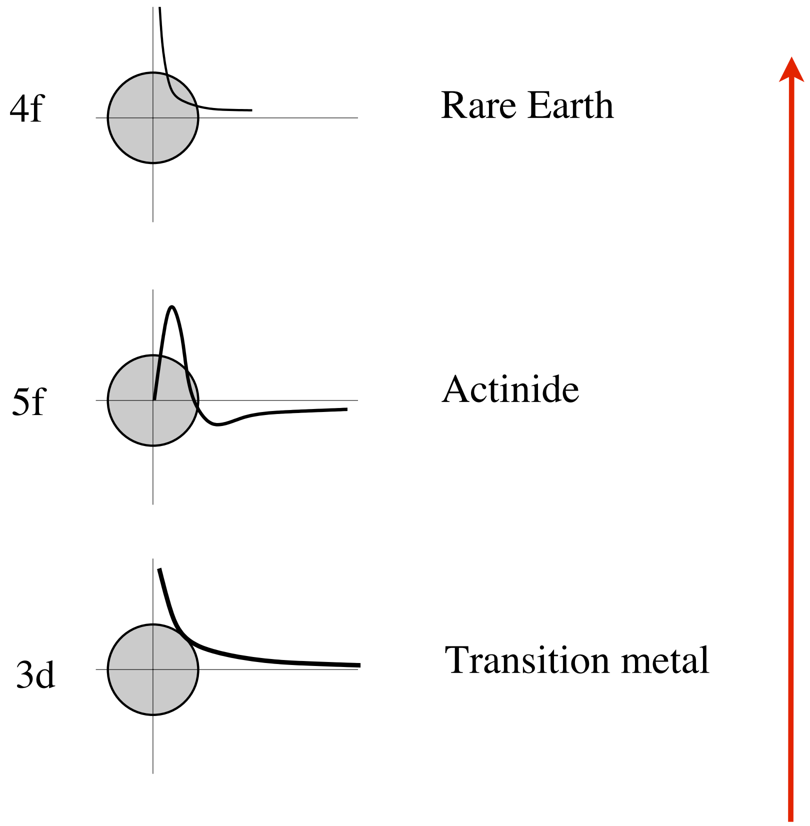

Increasing localization
4f
Rare Earth
5f
Actinide
3d
Transition metal





Mott Mechanism.

Anderson U (Anderson 1959)



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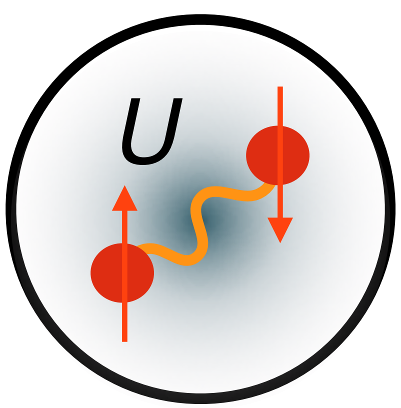

- No double occupancy: strongly correlated

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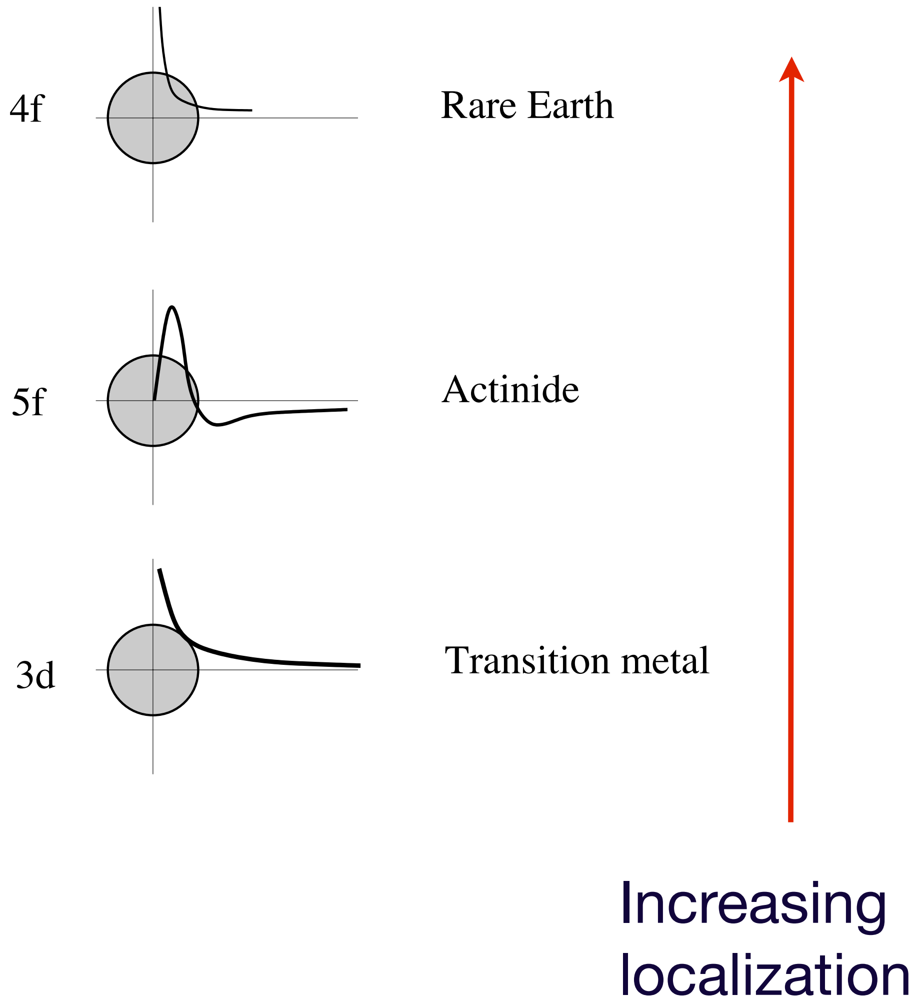

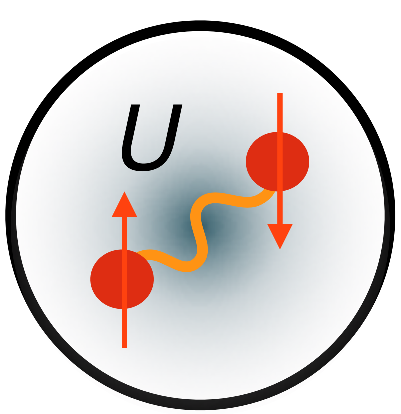

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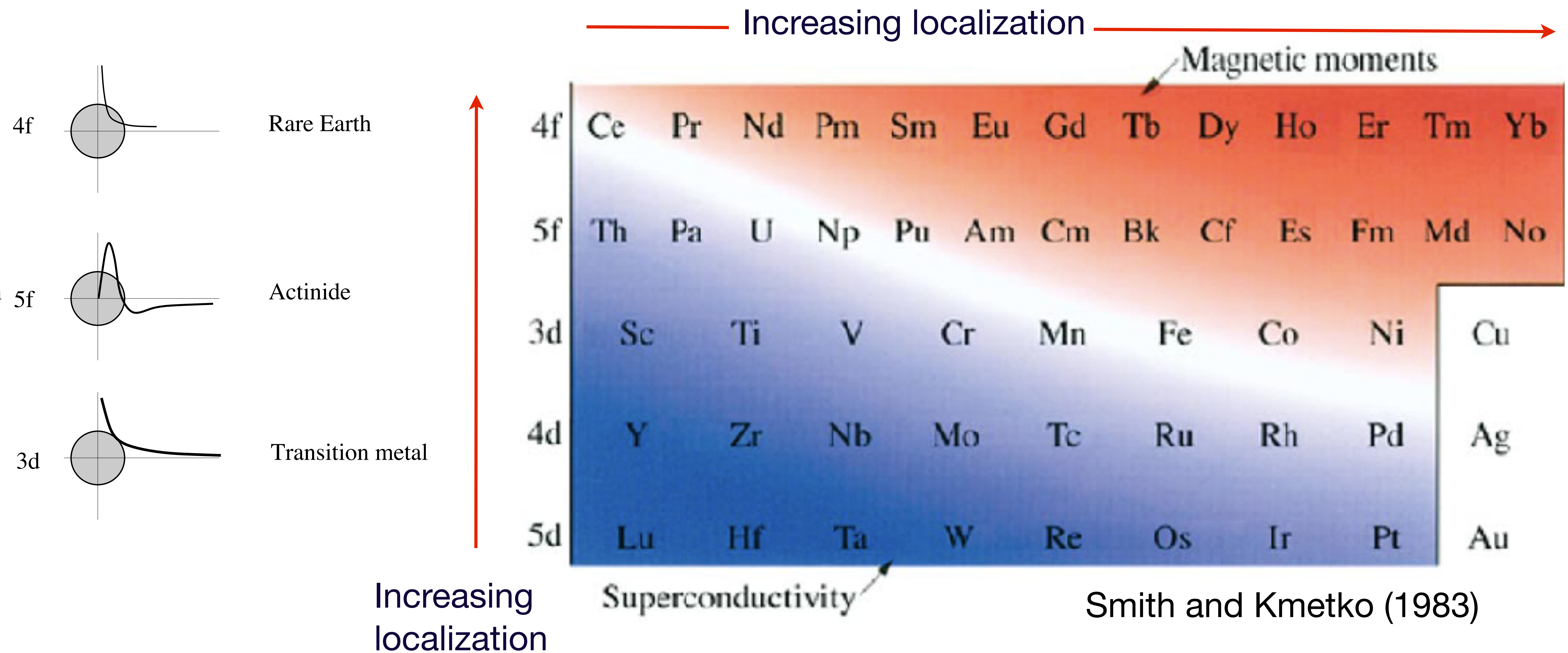
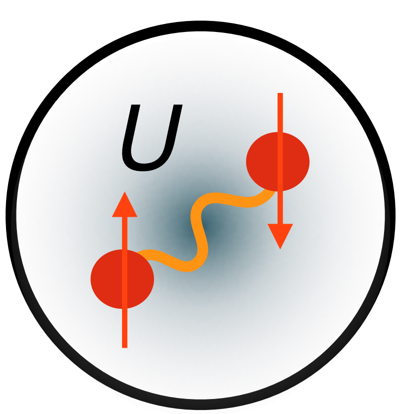


Smith and Kmetko (1983)



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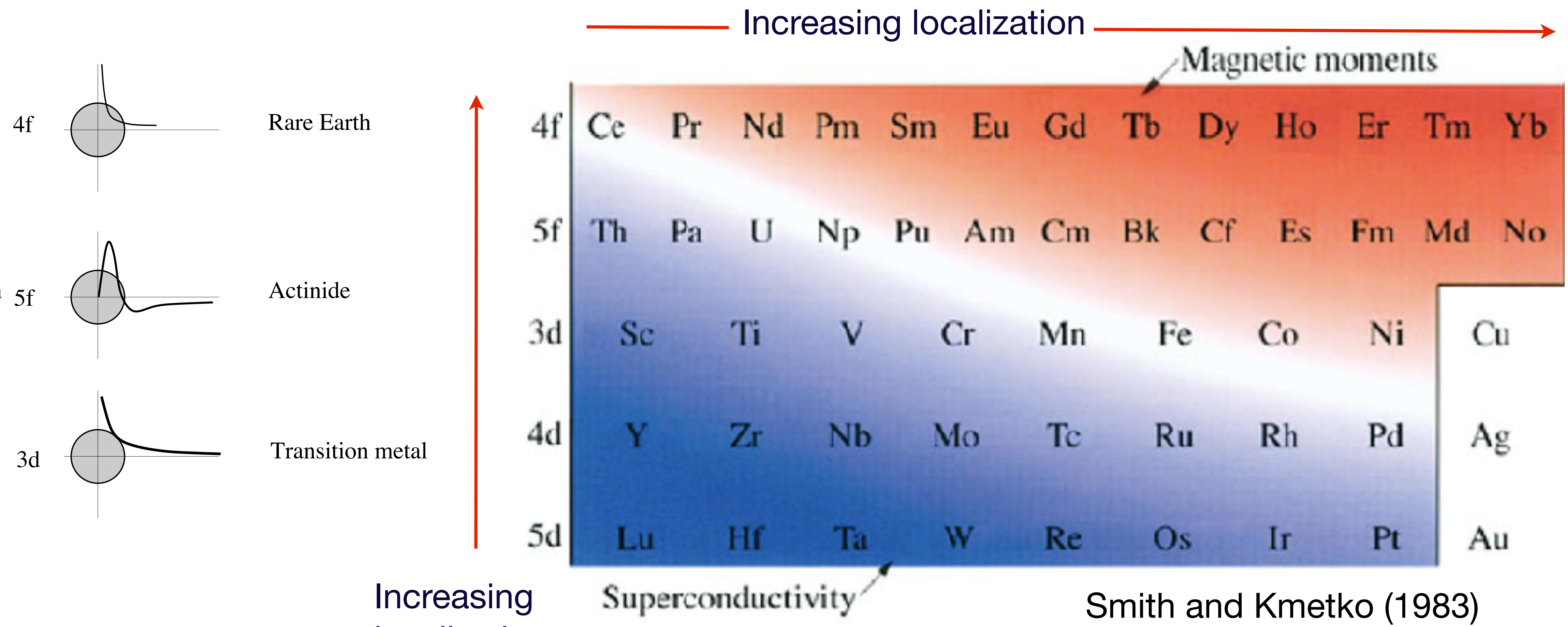

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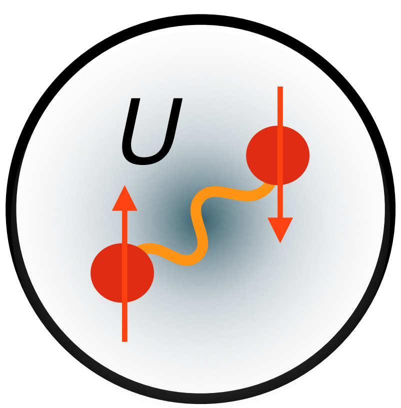

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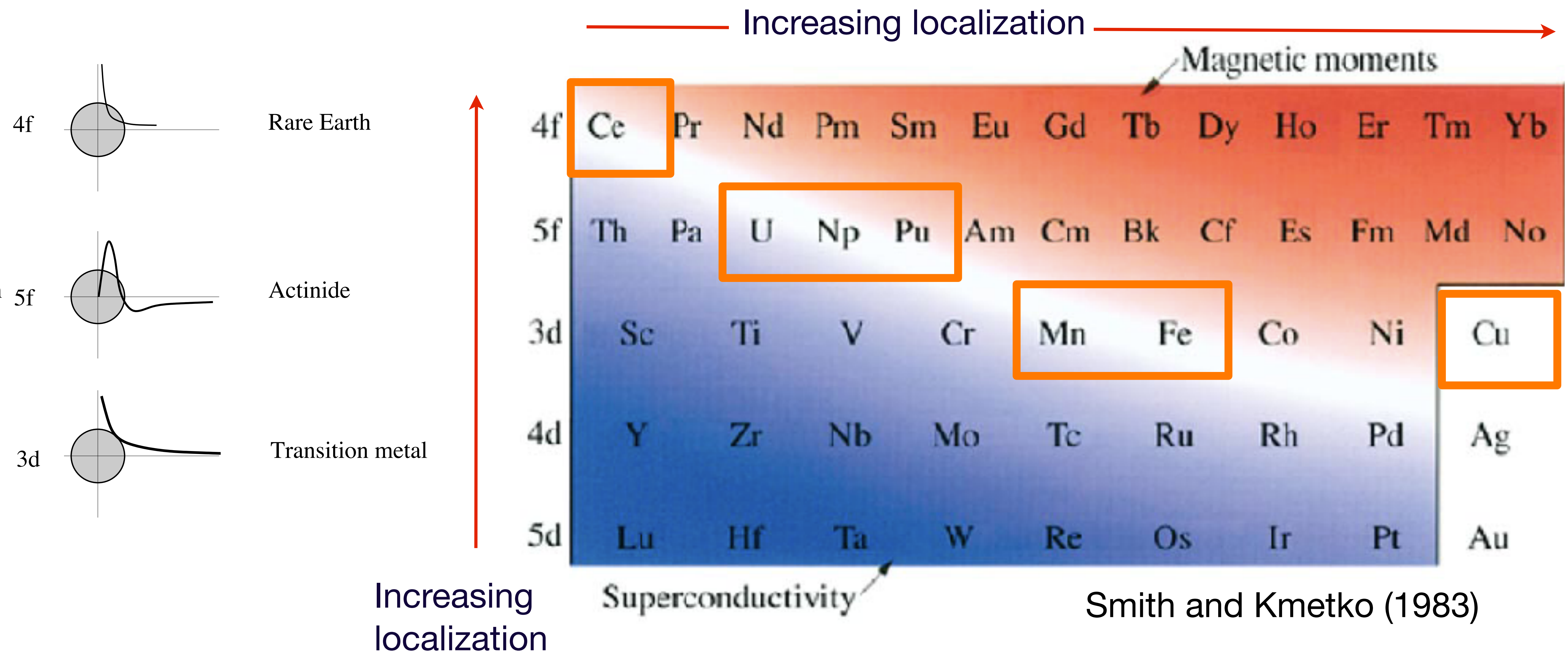

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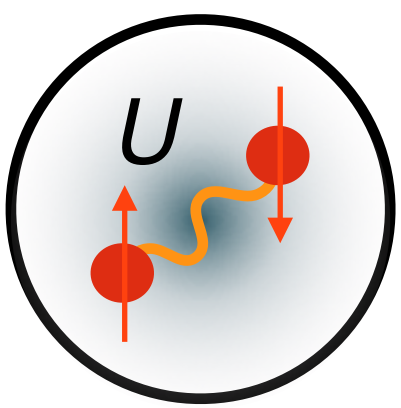

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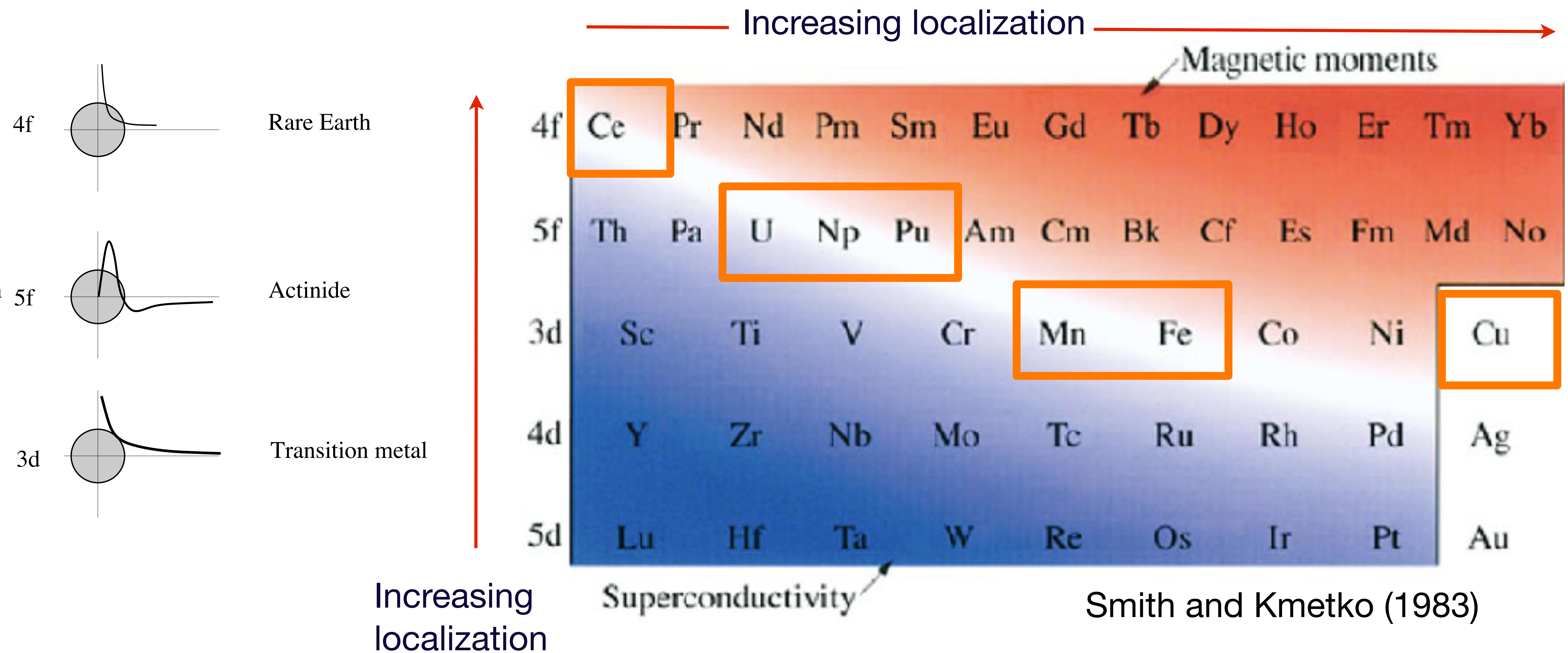

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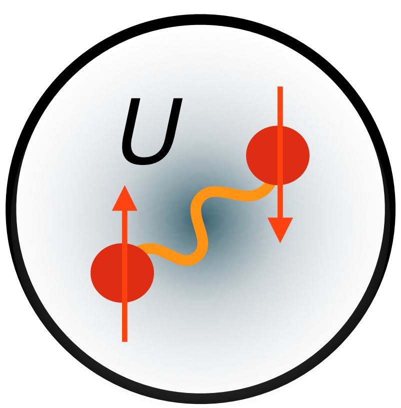

Many things are possible at the brink of magnetism.

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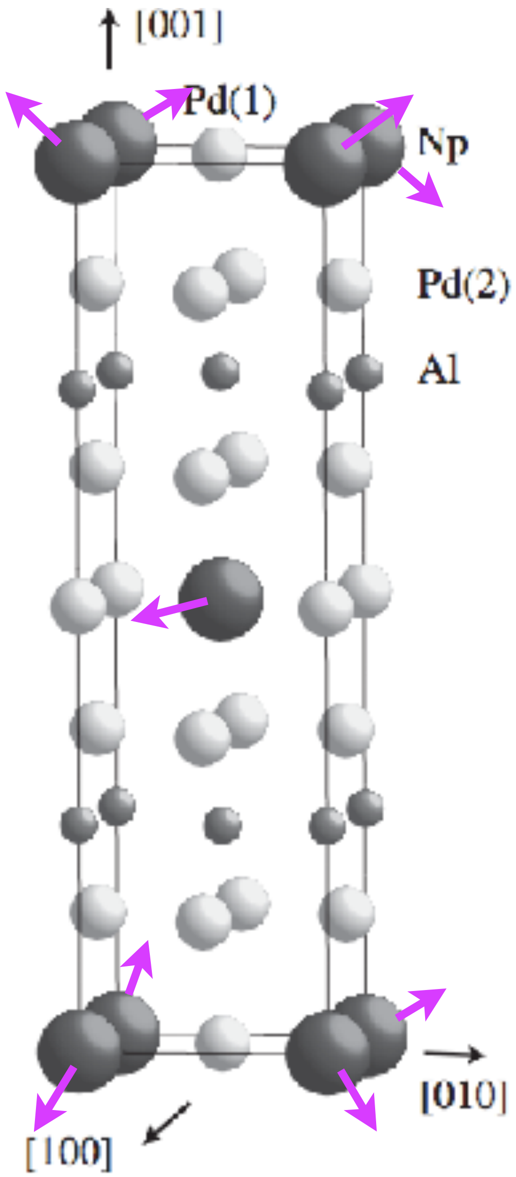
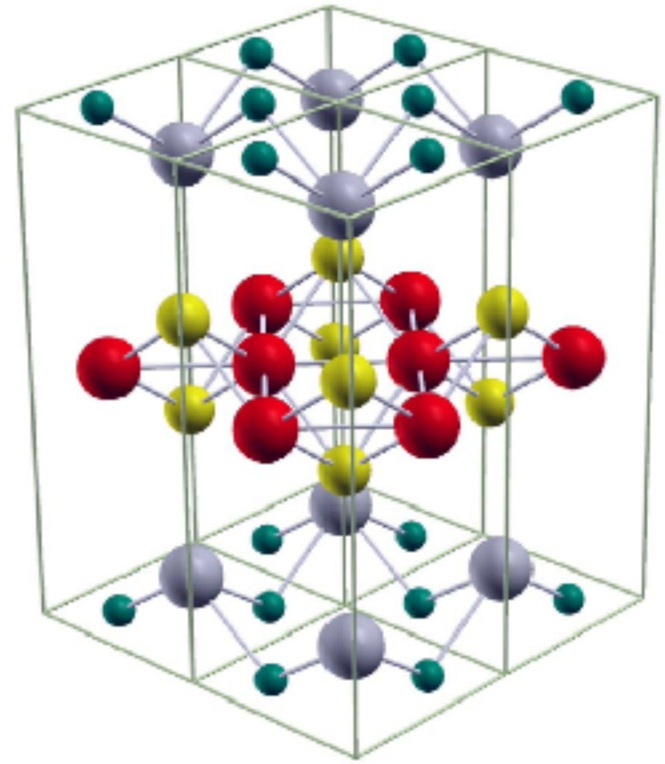
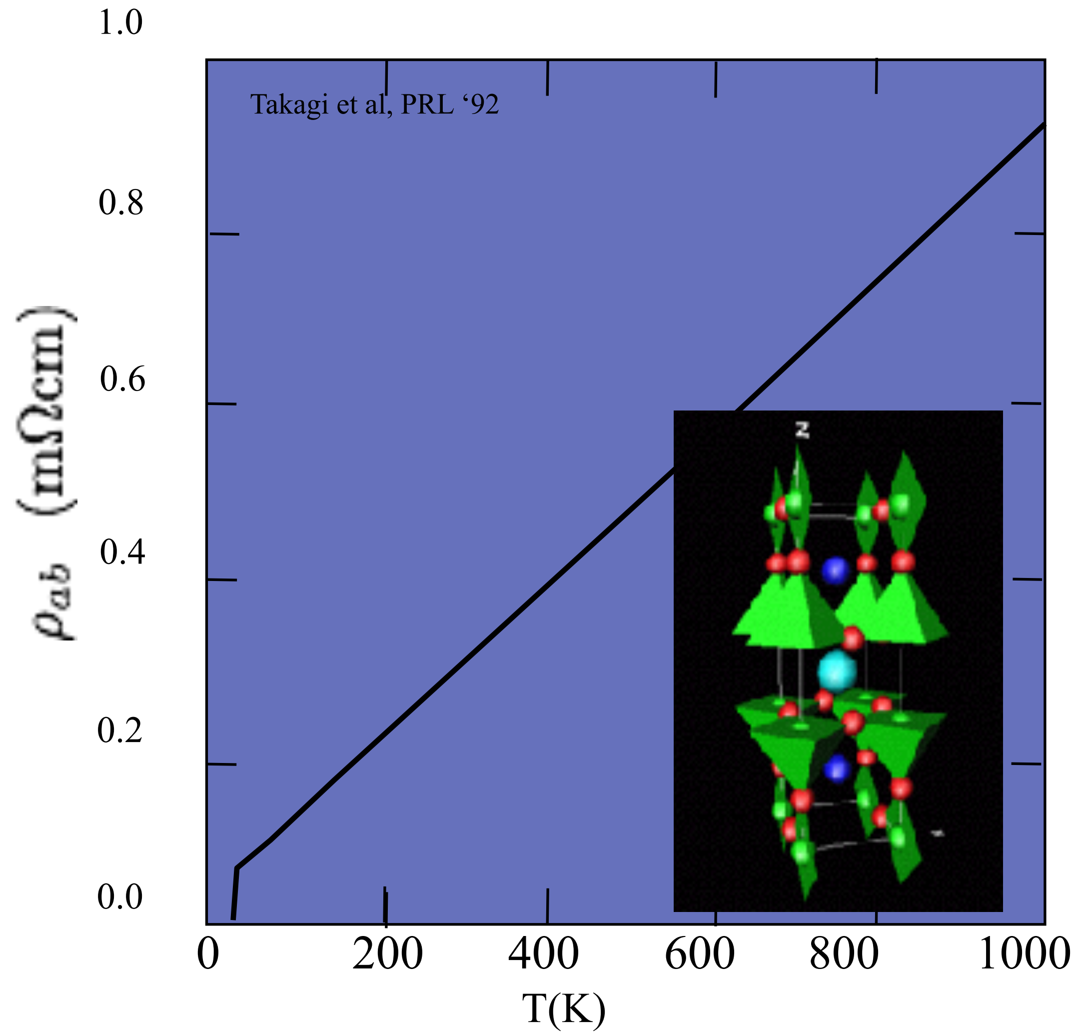


$NpAl_2Pd_5$



$PrF_xO_{1-x}FeAs$

Z.A. Ren et.al, Beijing, (08)



HF 115s
T_c=0.2 -18.5 K

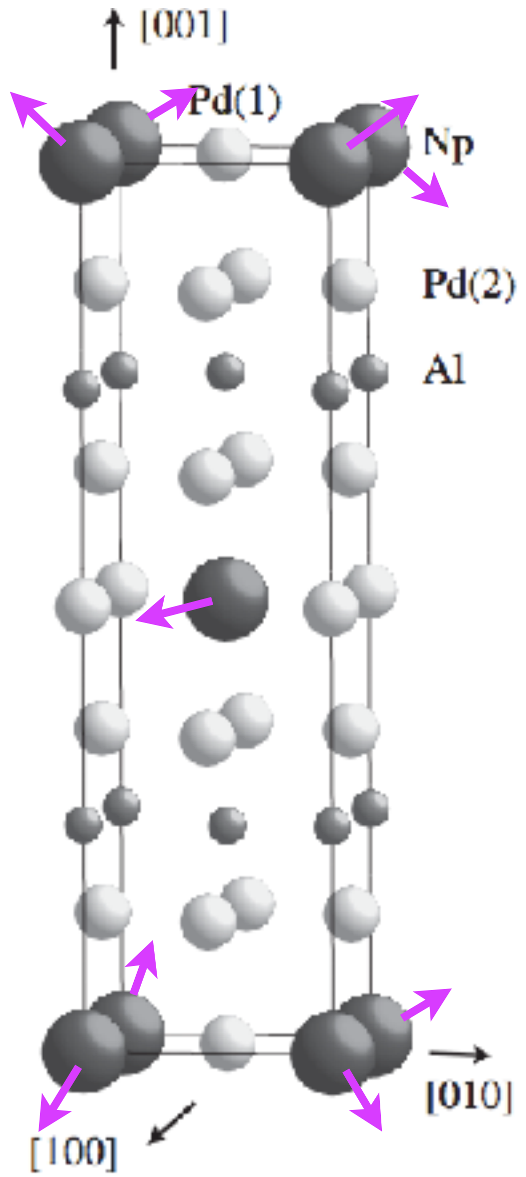
Iron based sc
T_c= 6 - 53 ++ ? K

Cuprates T_c=11-92K

Diversity of new ground-states on the brink of localization.

f-electron systems: 4f Ce, Yb systems 5f U, Np, Pu systems.

d-electron systems: e.g Pnictides, Cuprate SC.



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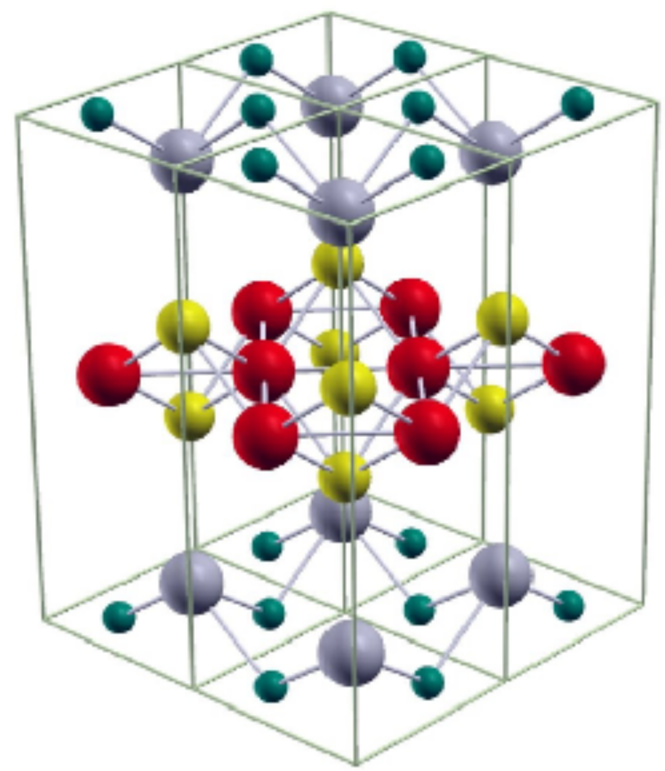

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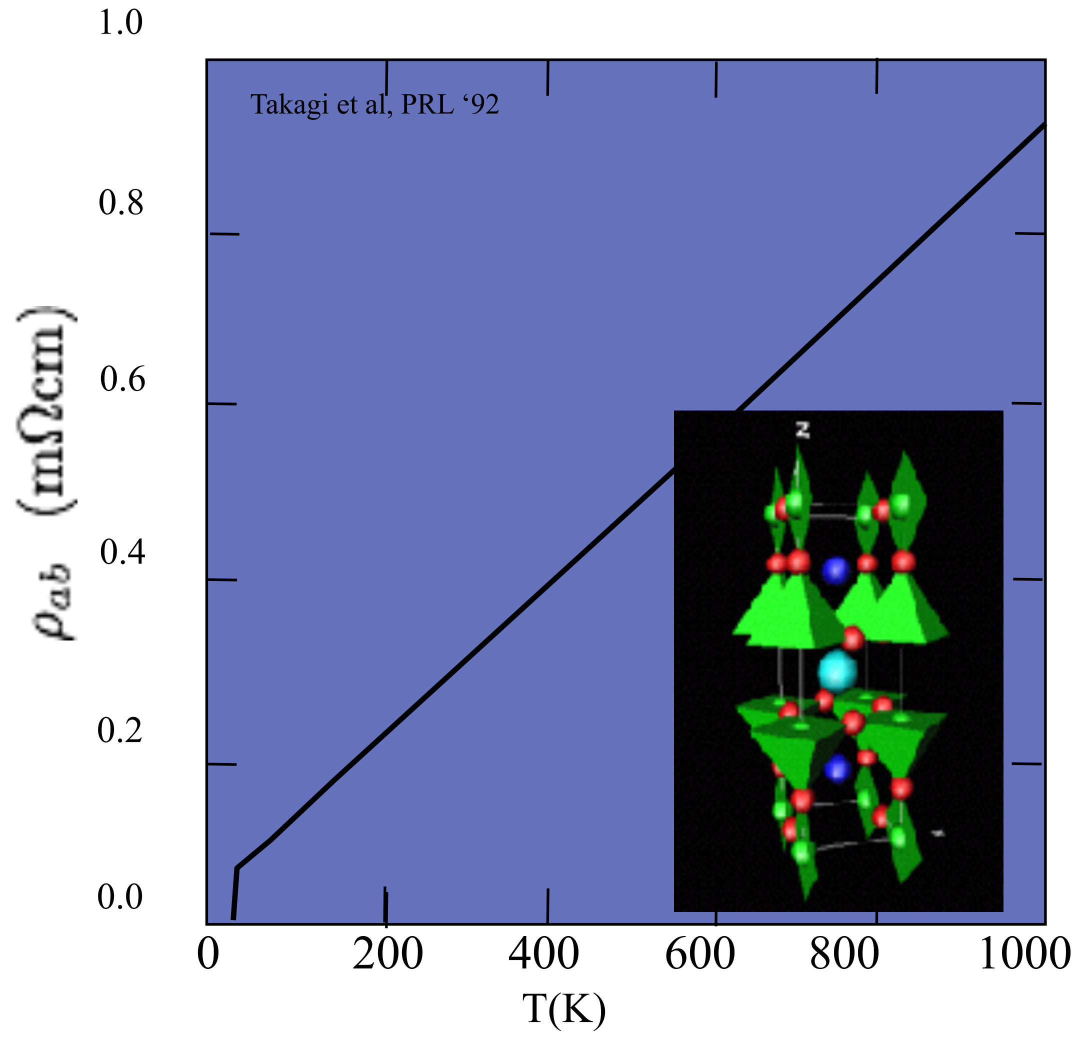

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A new era of mysteries

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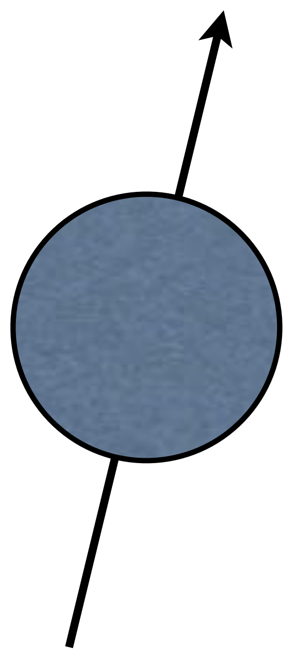
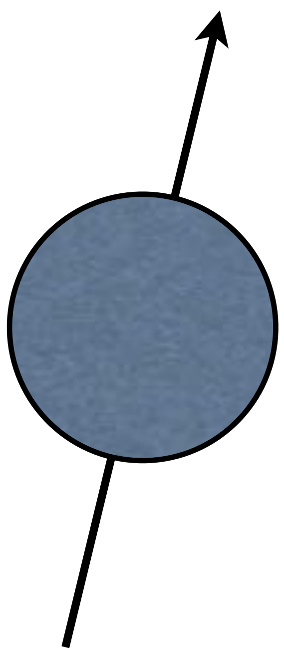
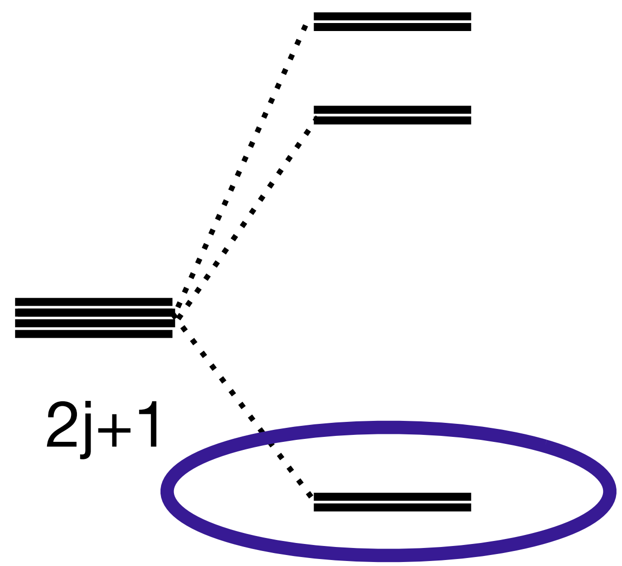
Heavy Fermions + Kondo



Spin (4f,5f):
"quark" of heavy
electron physics.

Heavy Fermions + Kondo





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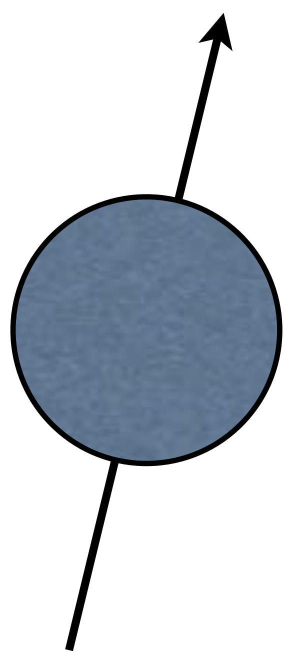

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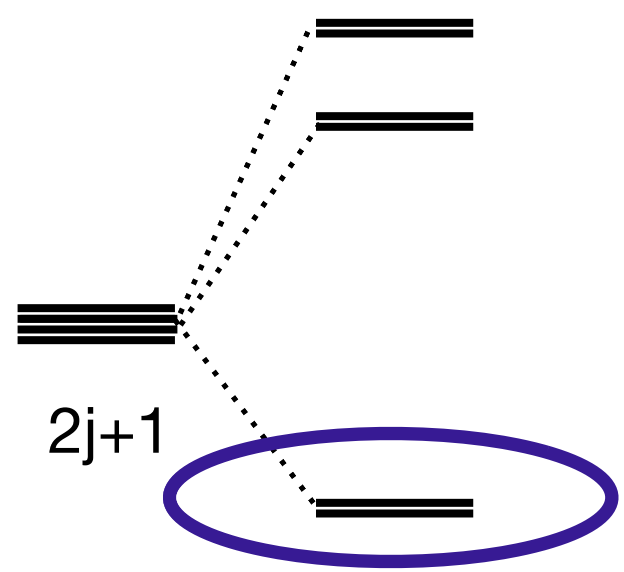

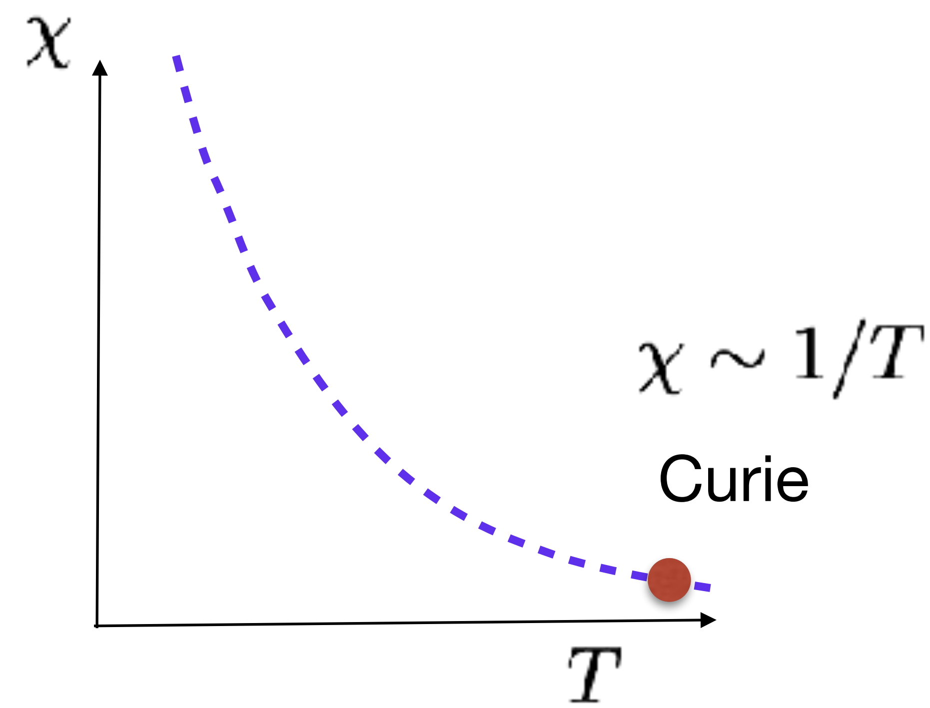

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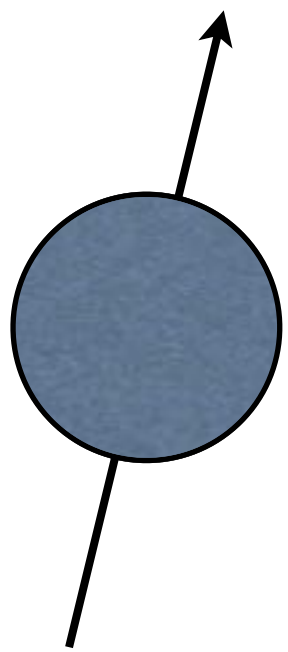

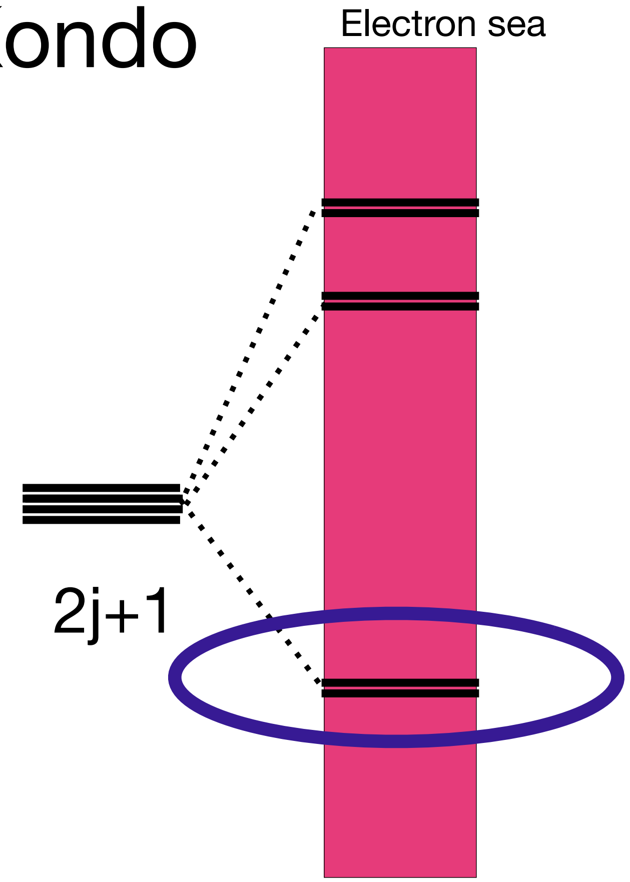

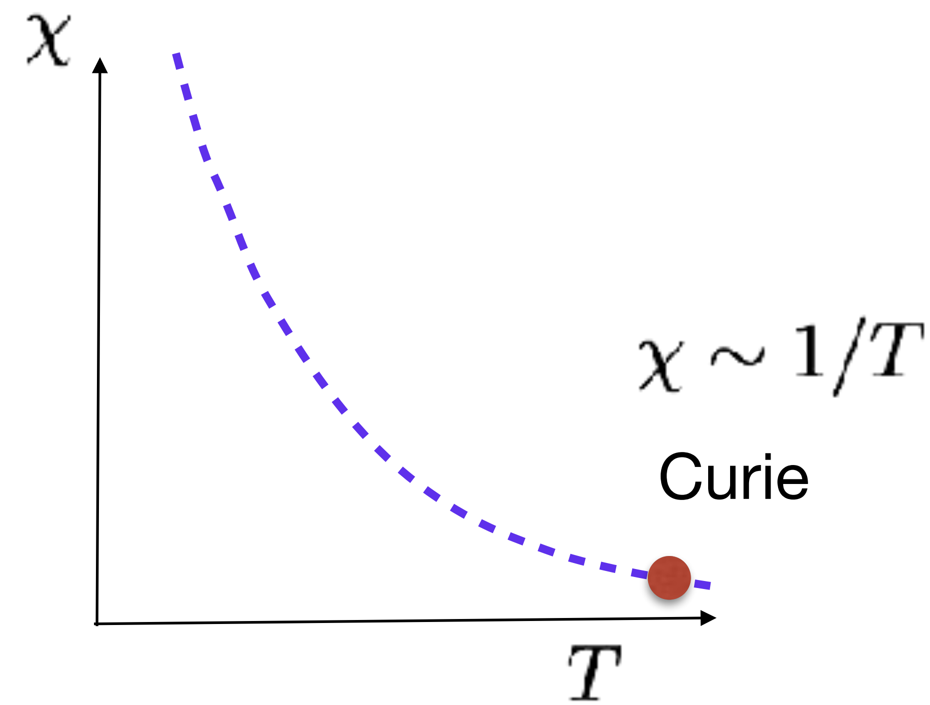

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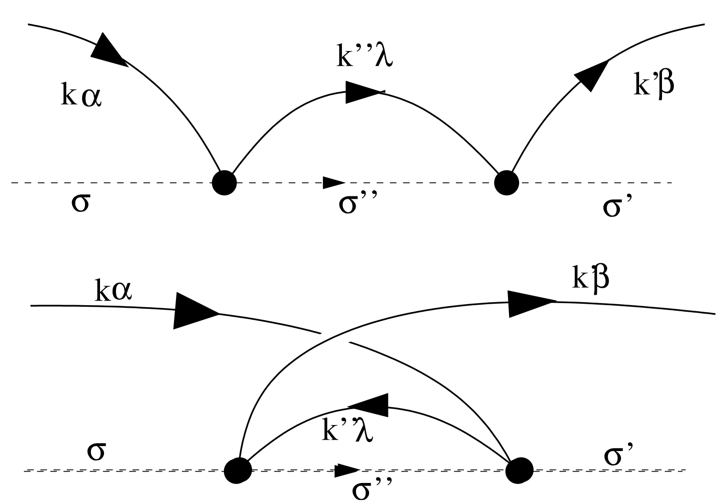


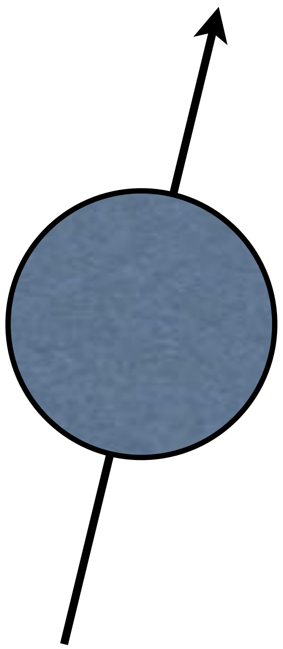
$$\approx -\frac{J^2\rho\delta D}{D}(\sigma^b\sigma^a)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

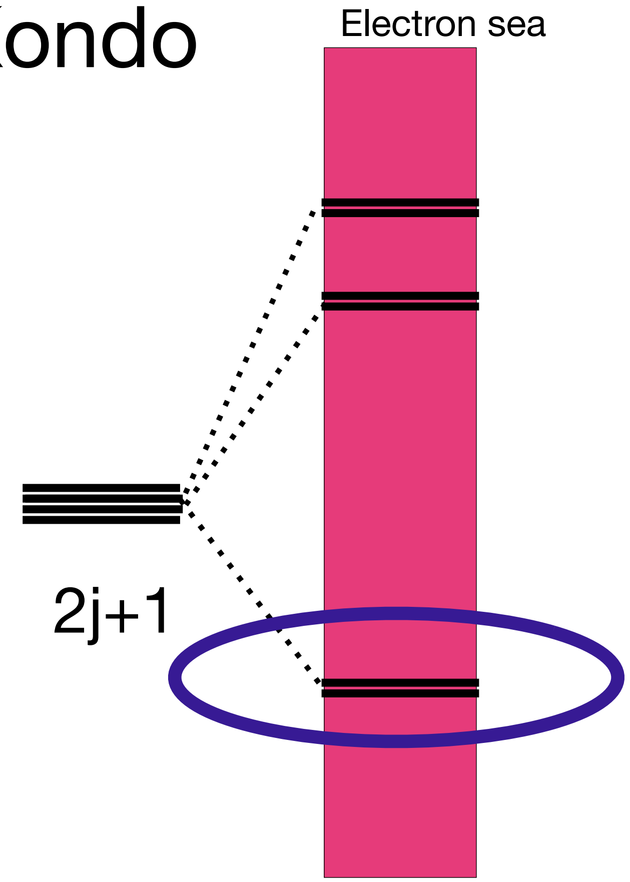
$$\approx \frac{J^2\rho\delta D}{D}(\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$

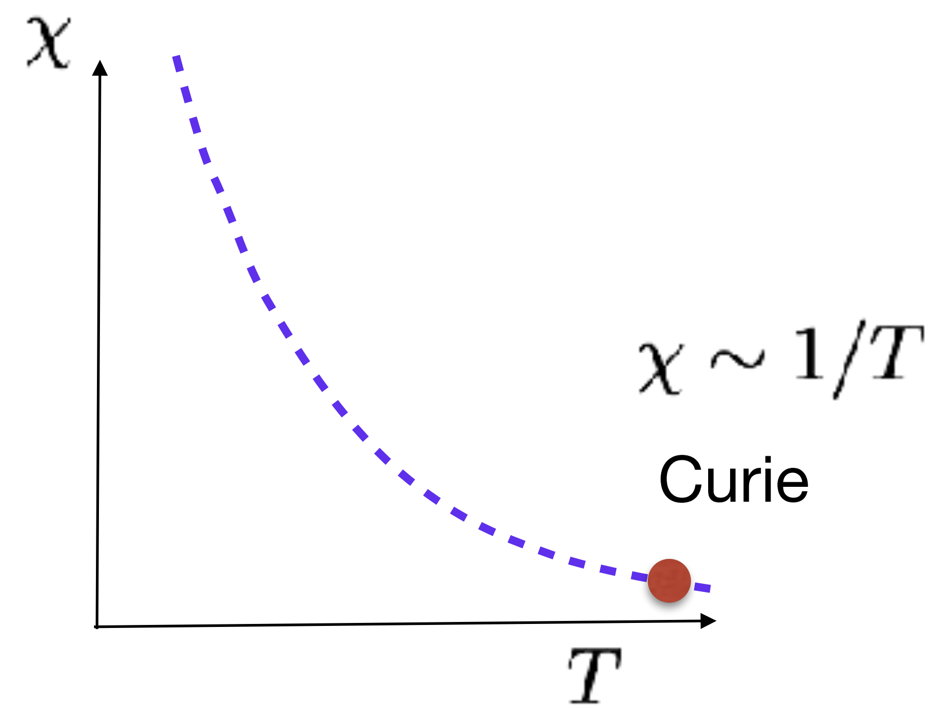
$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \quad \vec{S}\cdot\vec{\sigma}(0)$$

J. Kondo, 1962

Heavy Fermions + Kondo







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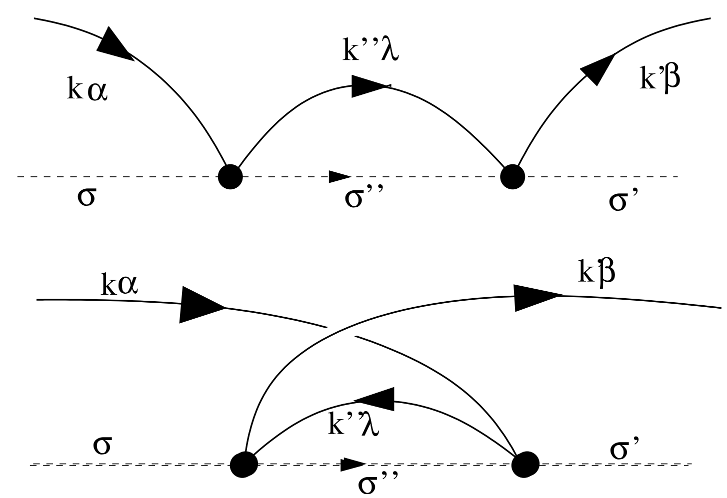
$$J \to J(T) = J + 2J^2\rho \ln \frac{D}{T}$$



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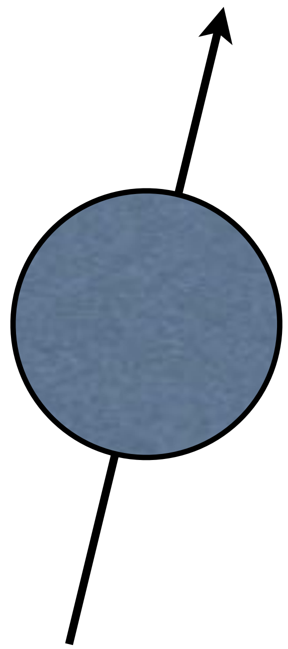

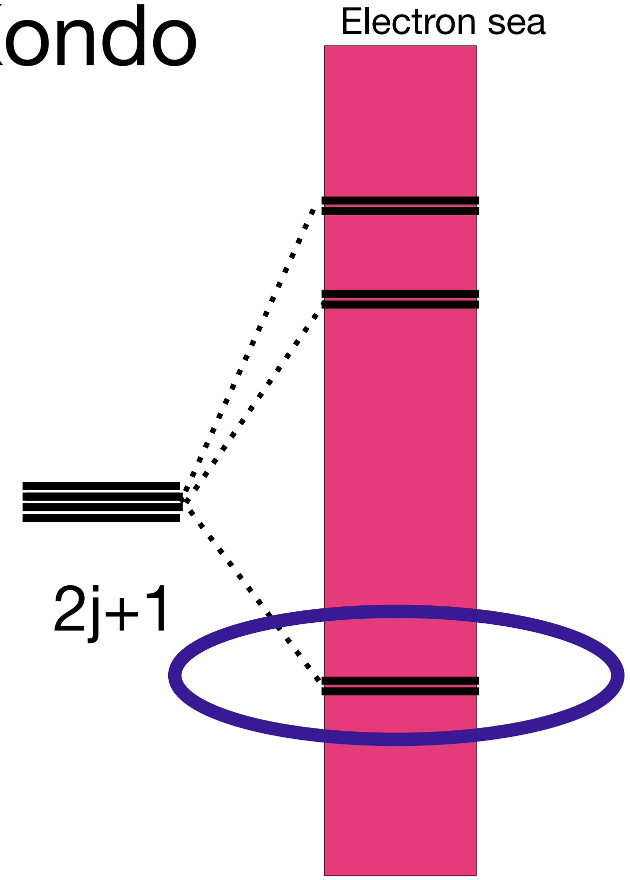

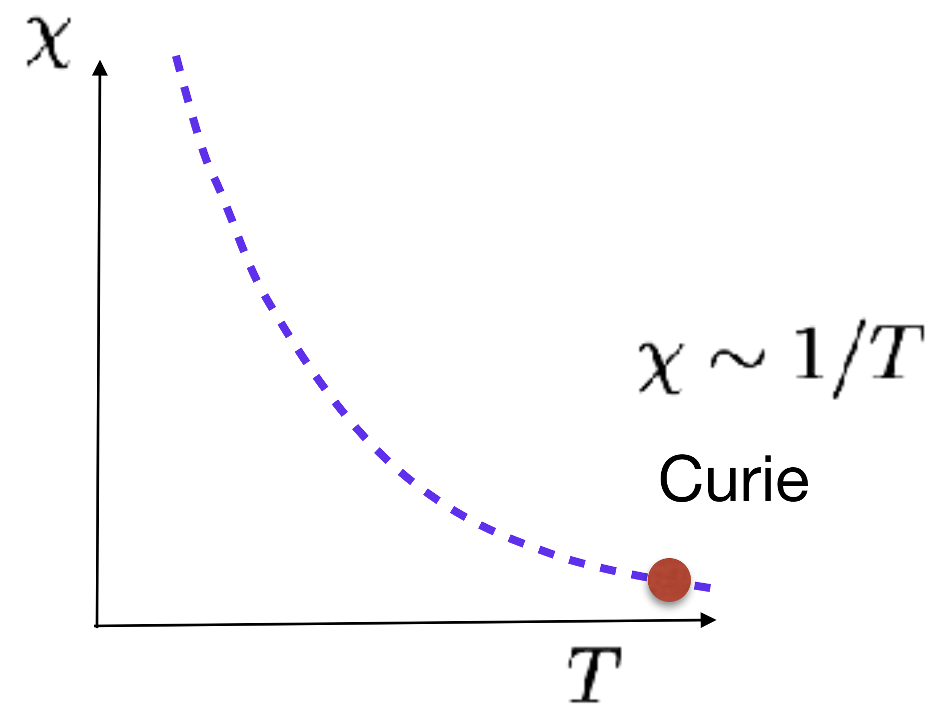

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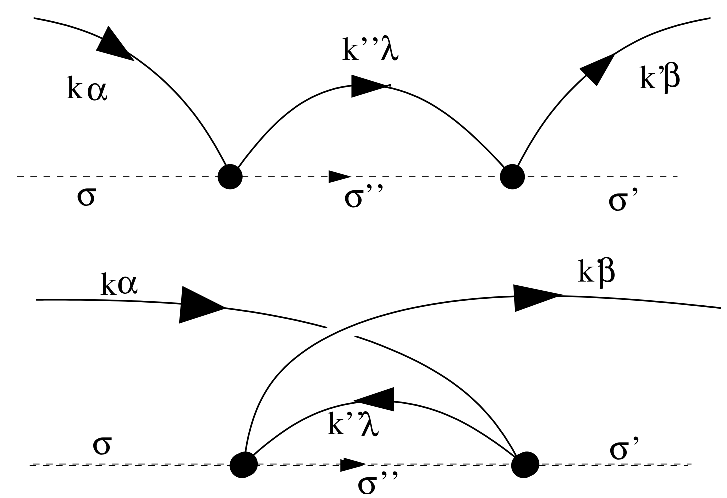

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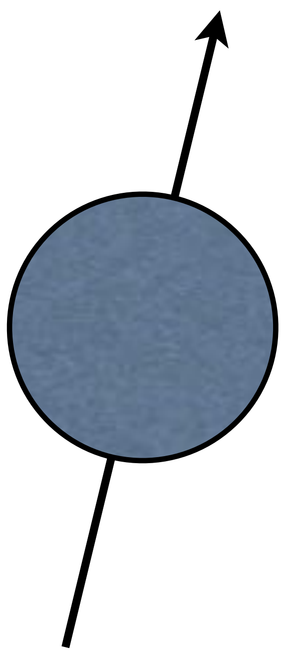

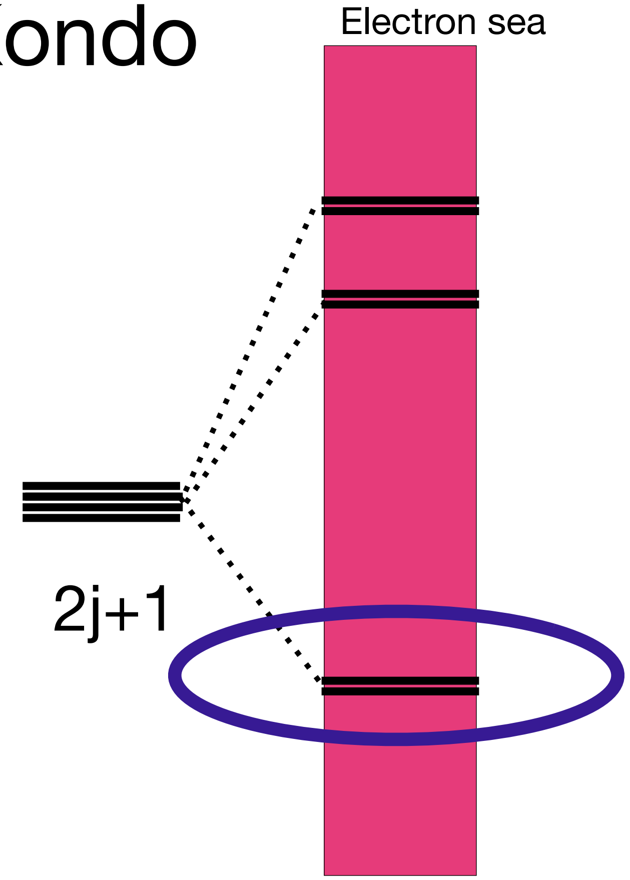

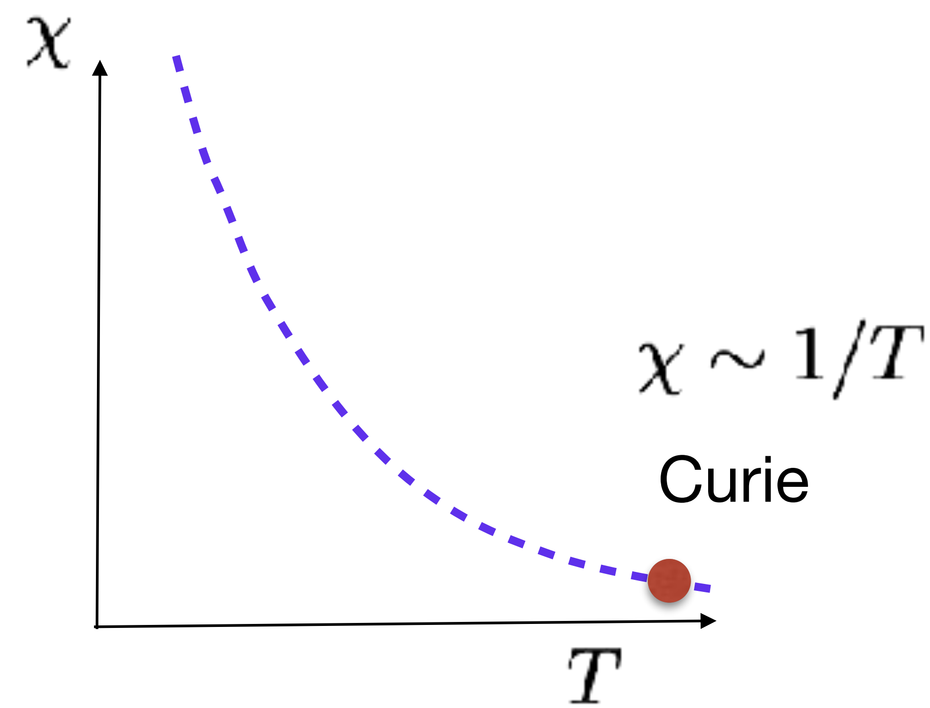

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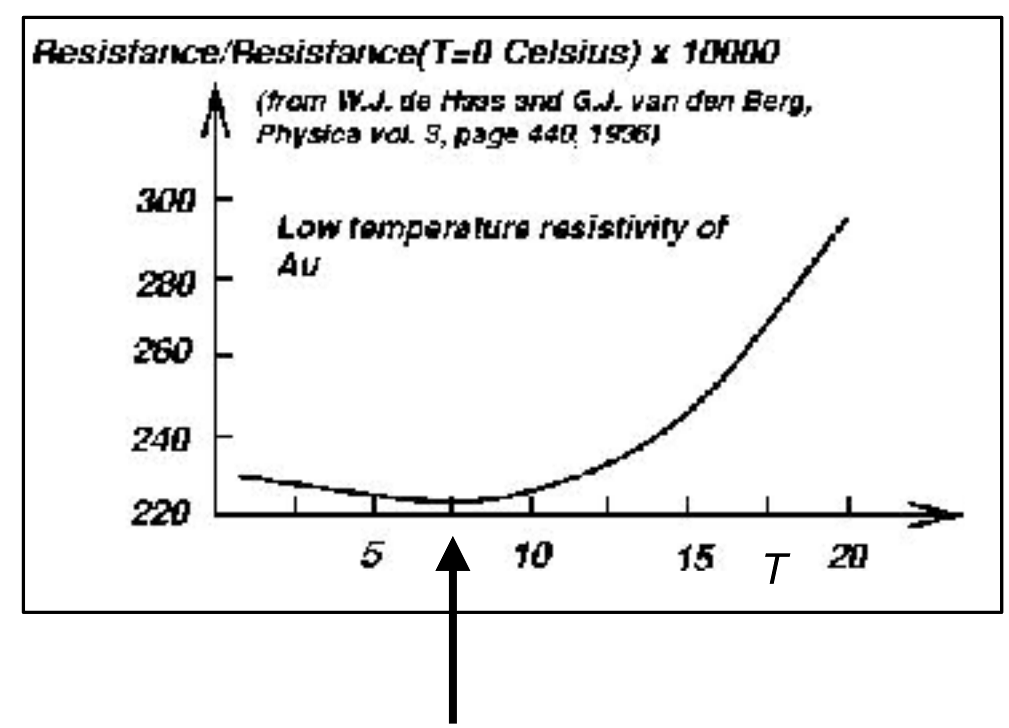
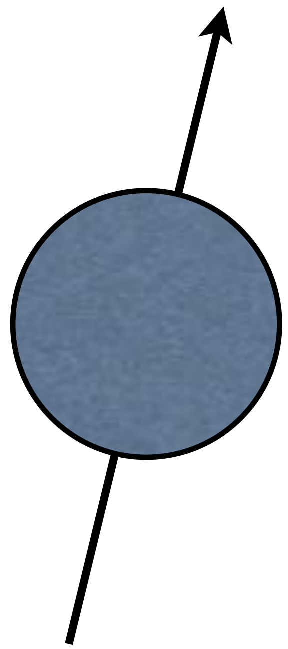
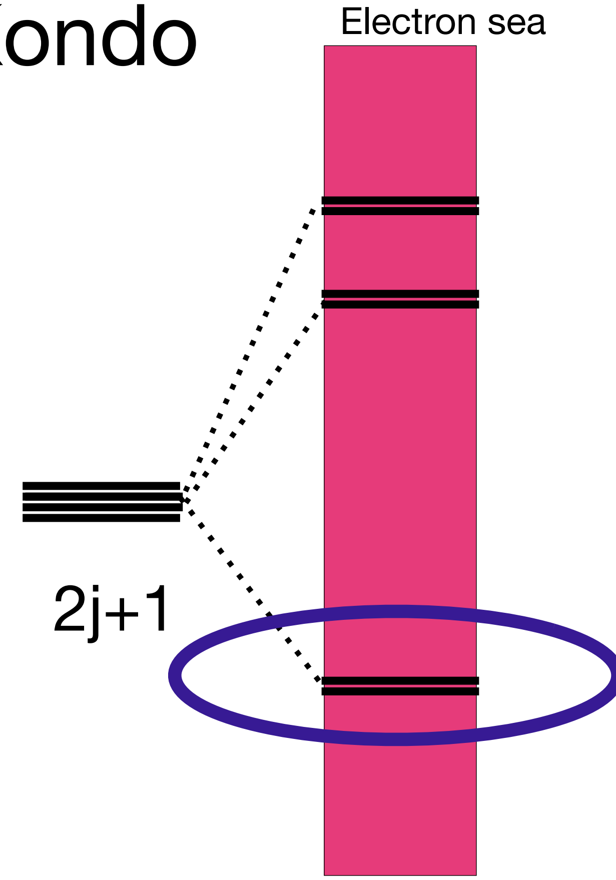
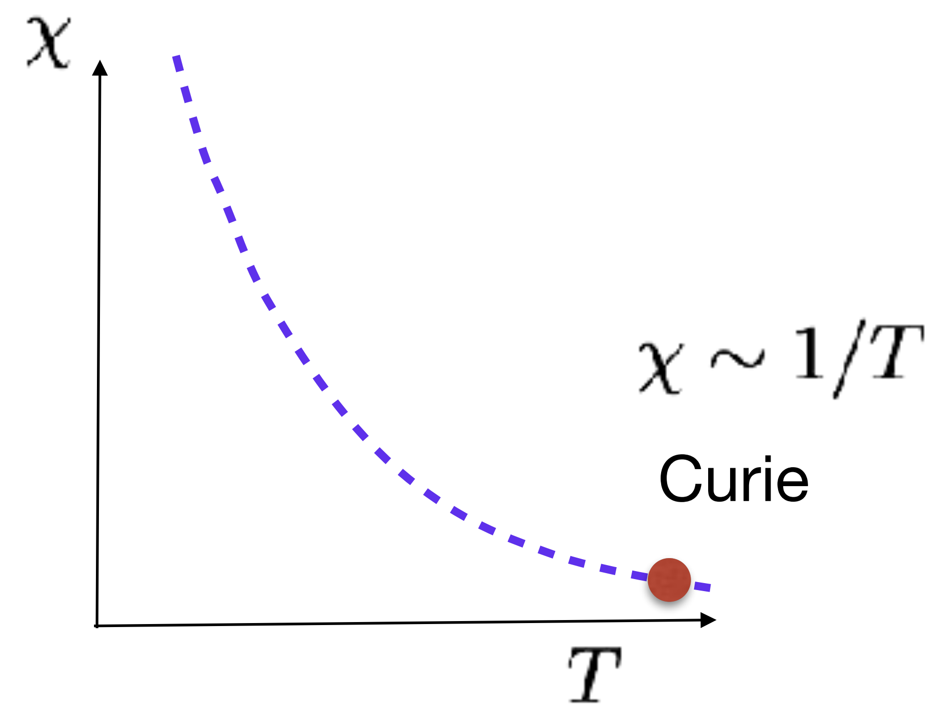


"Kondo Resistance Minimum"

Heavy Fermions + Kondo







Spin (4f,5f):
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"Scales to
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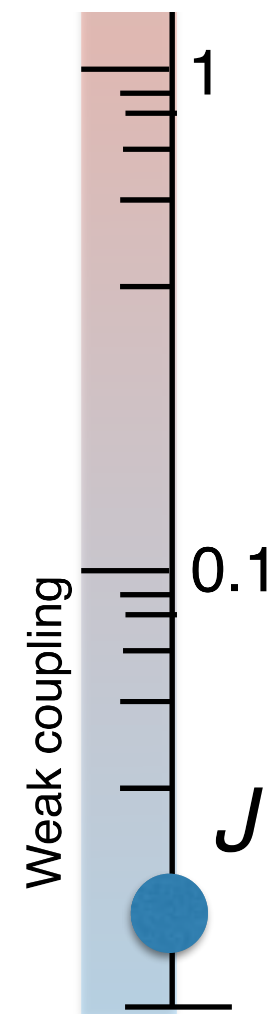

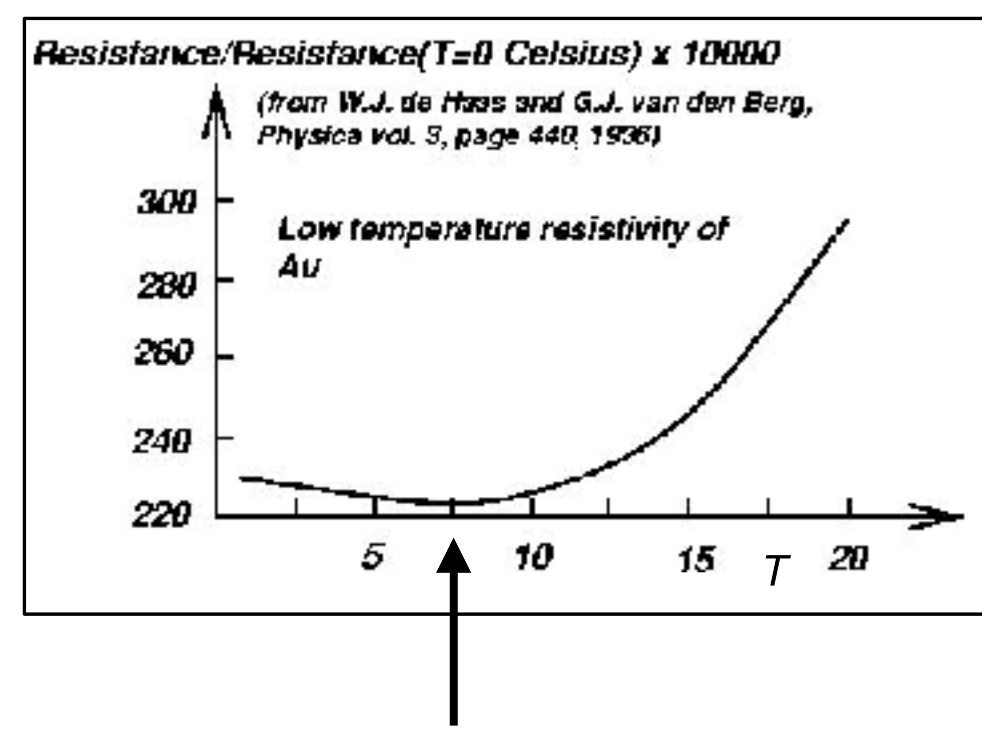

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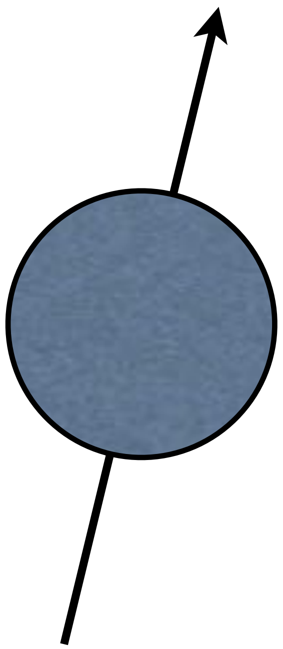

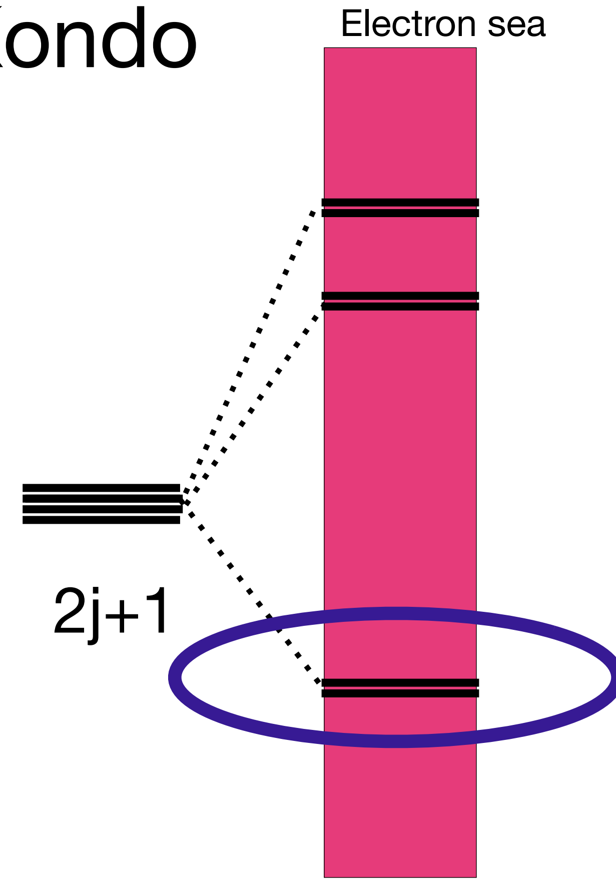

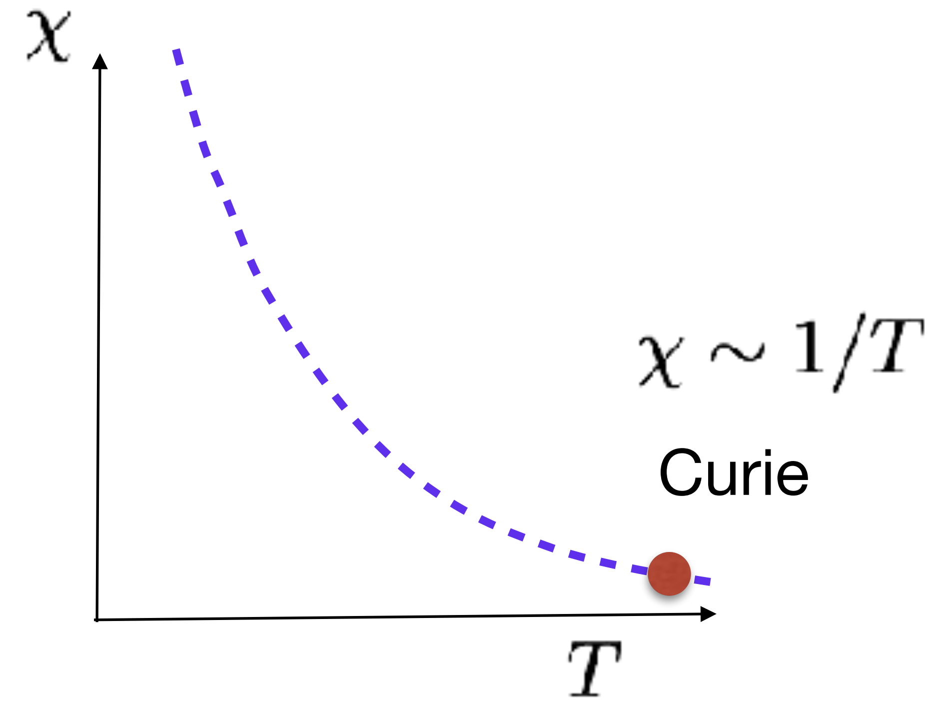

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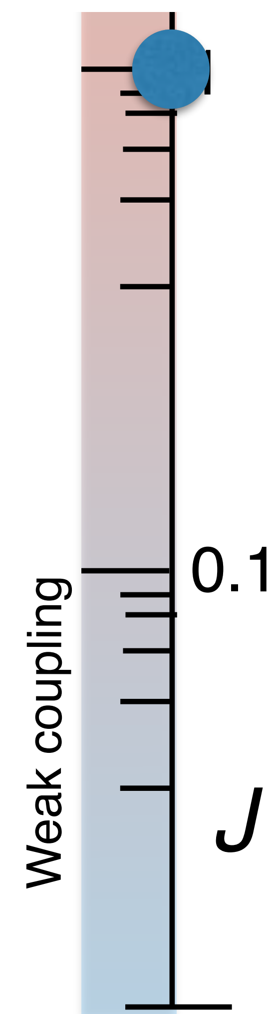

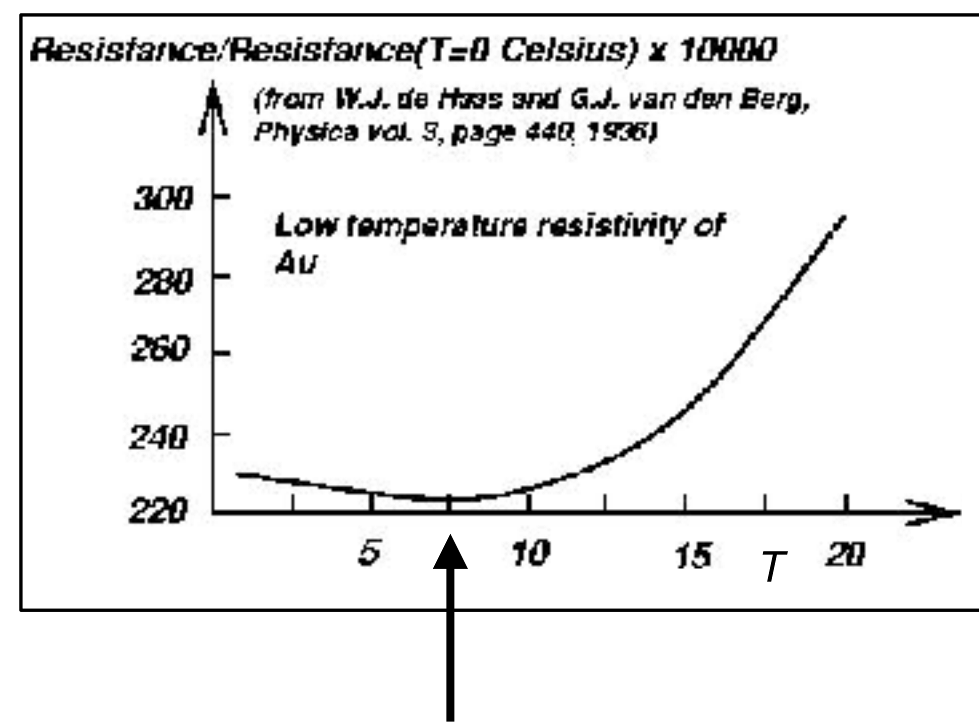

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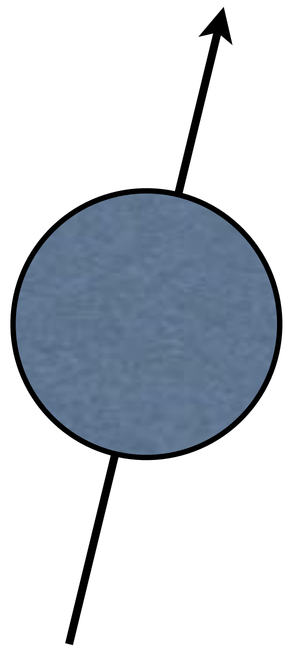
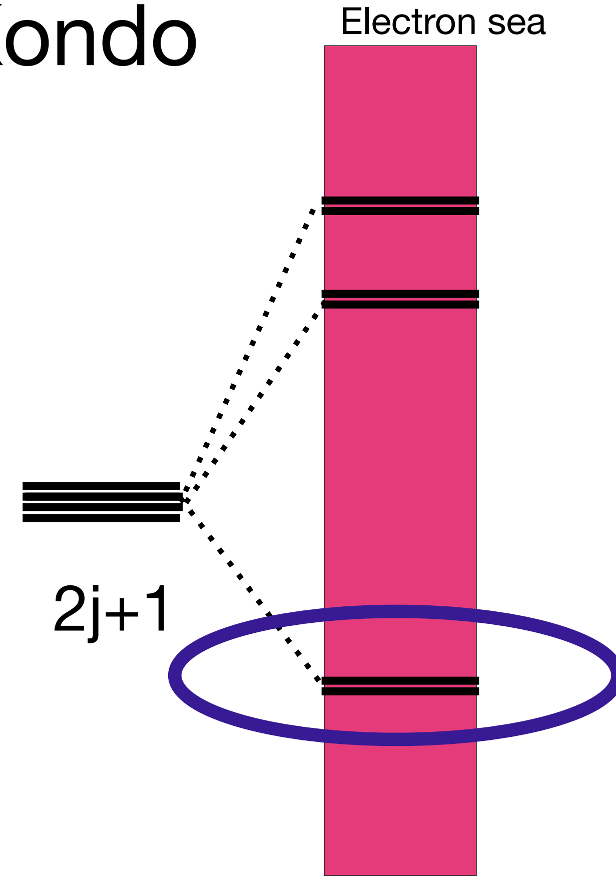
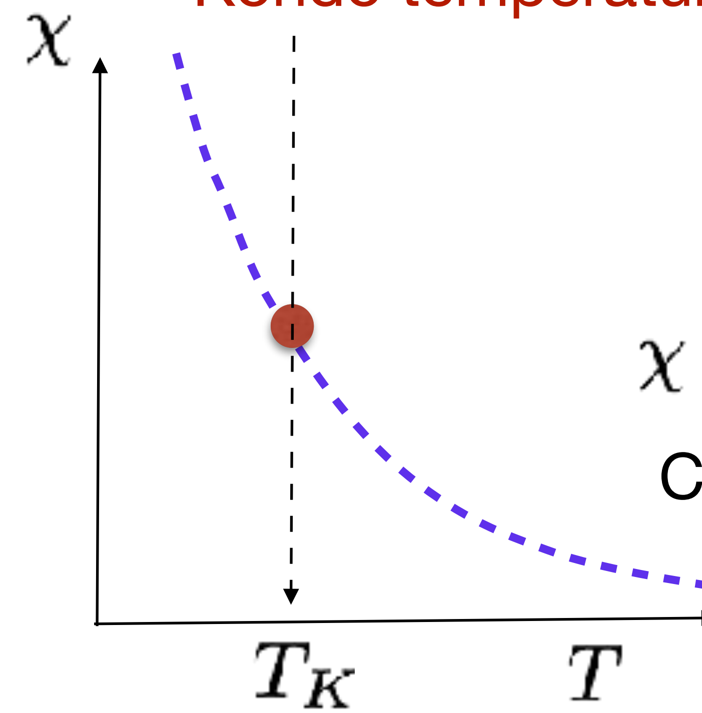


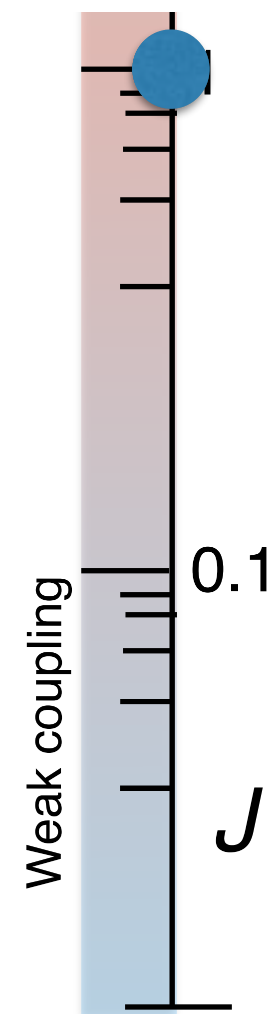


“Kondo temperature”



$\chi \sim 1/T$

Curie



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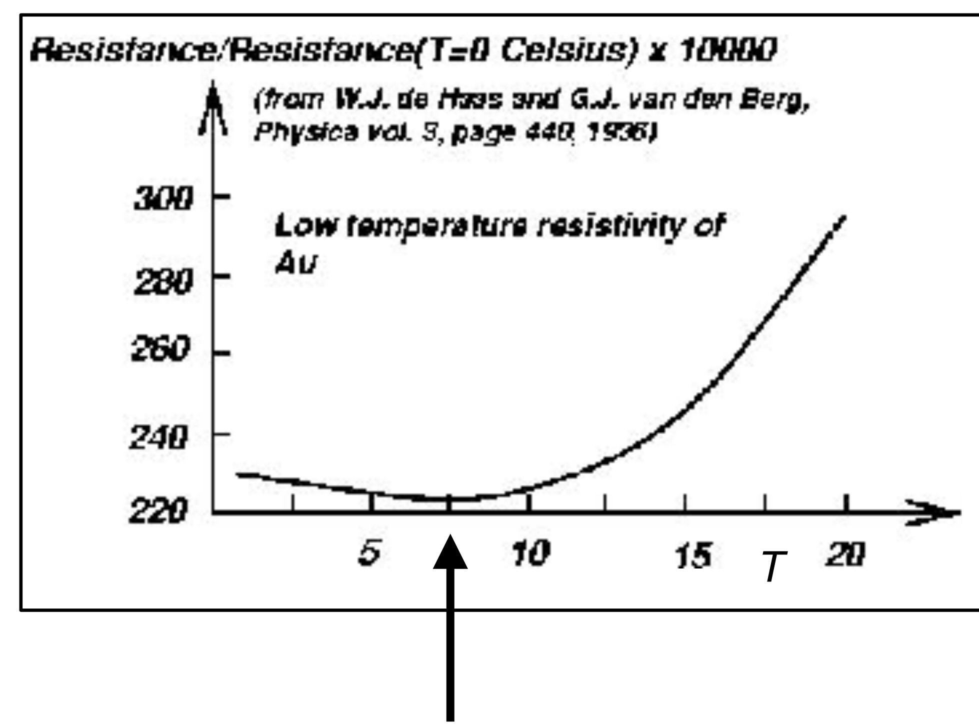

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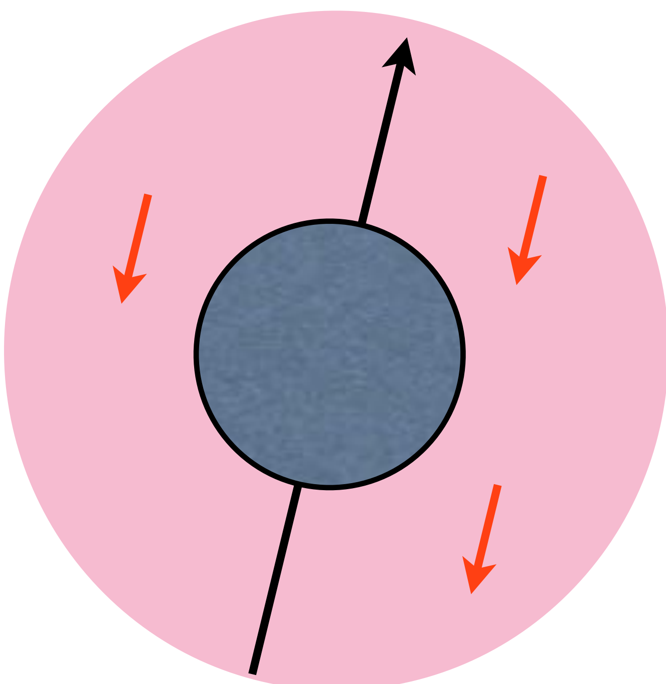

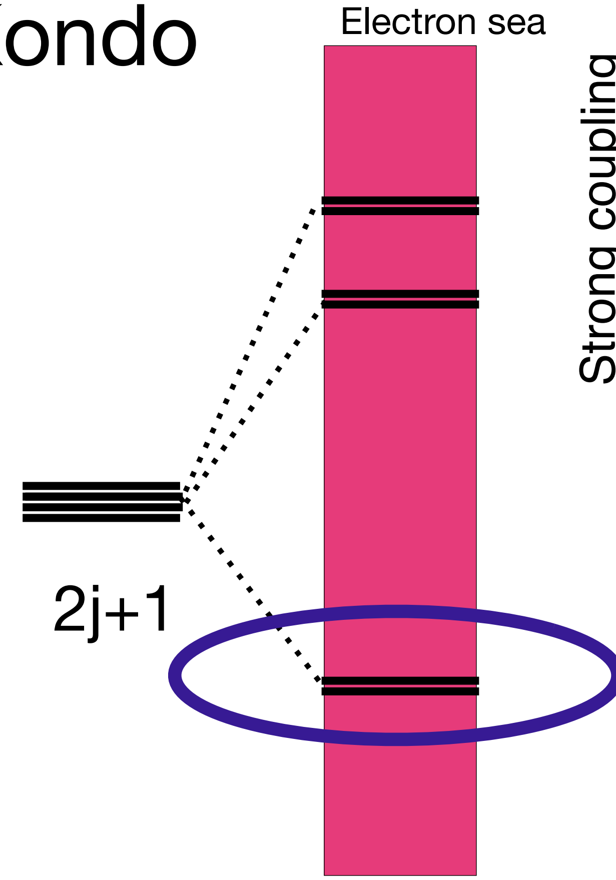

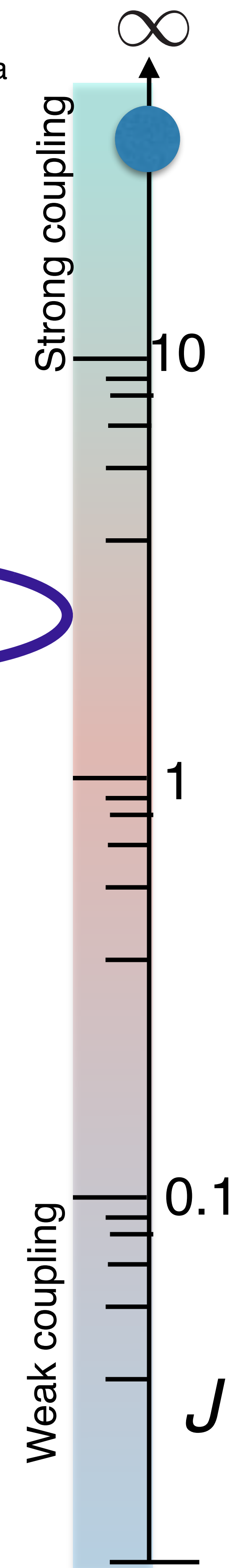

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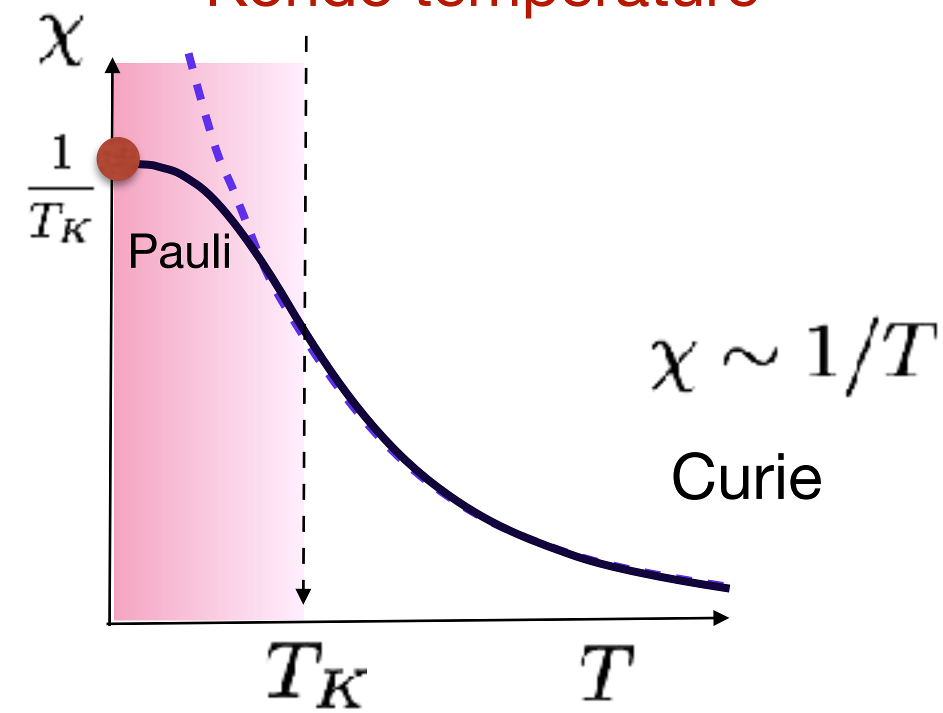

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Spin screened by conduction electrons: <u>entangled</u>

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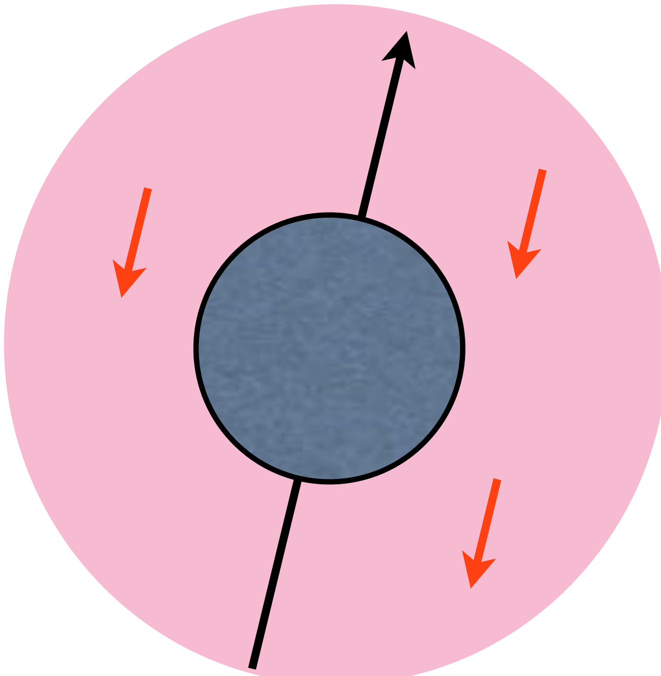

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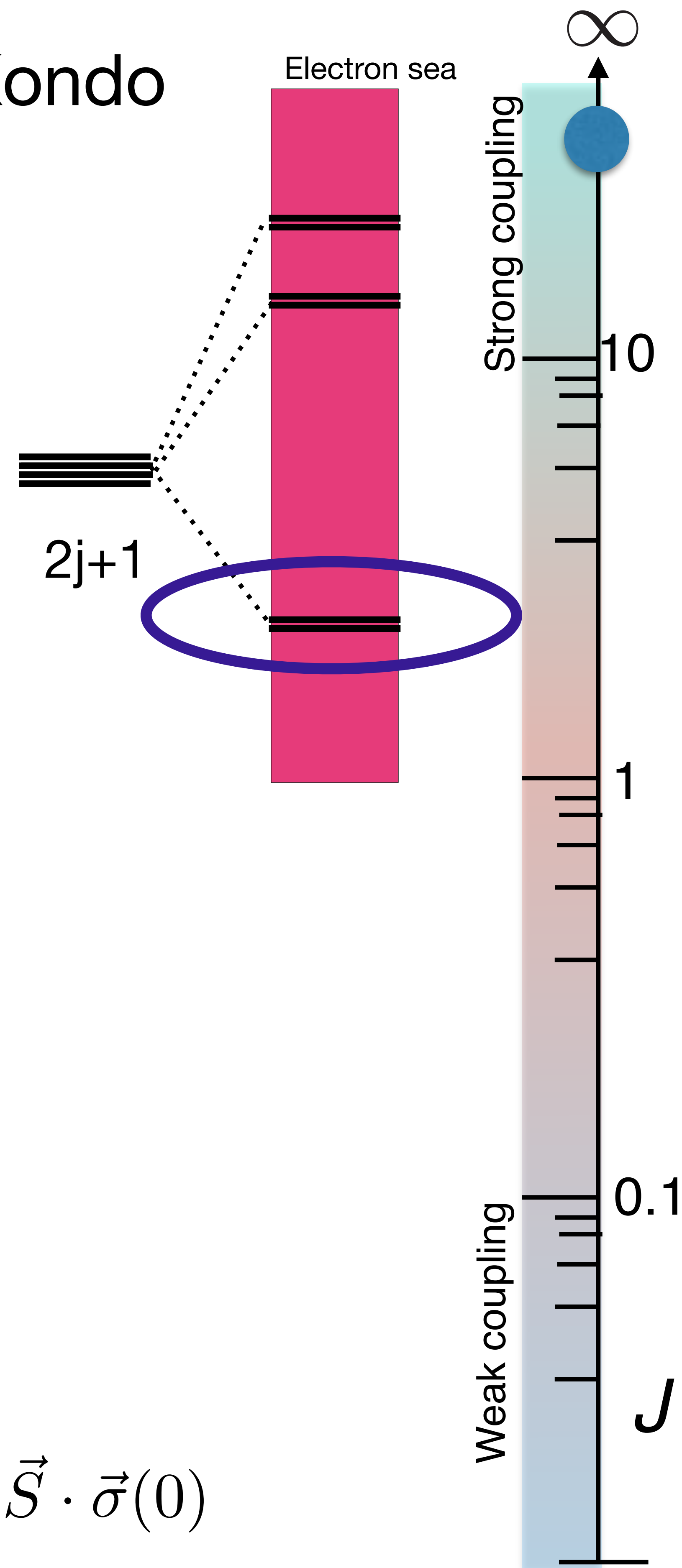

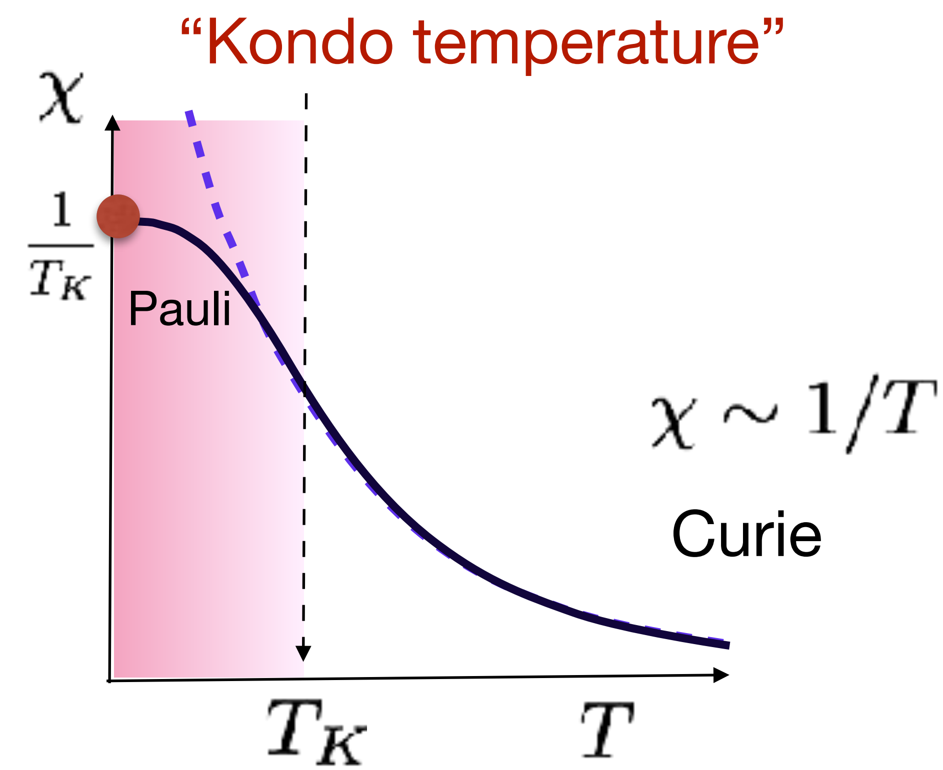

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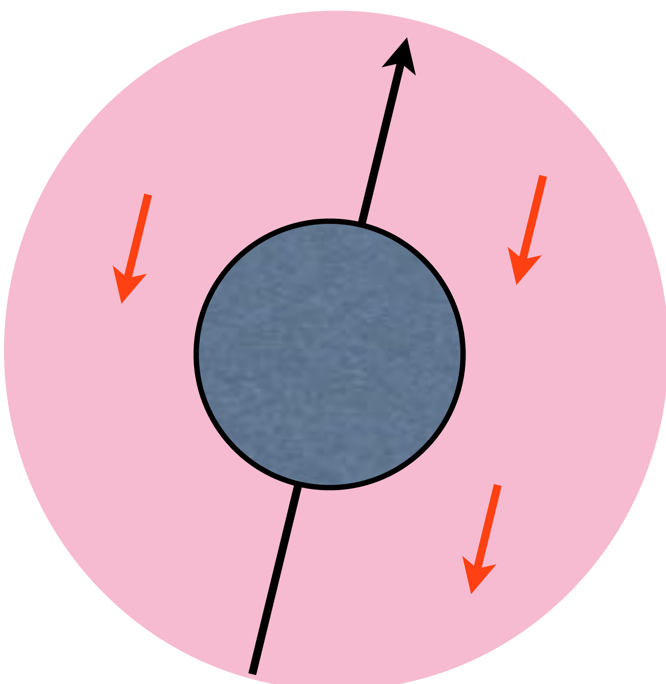

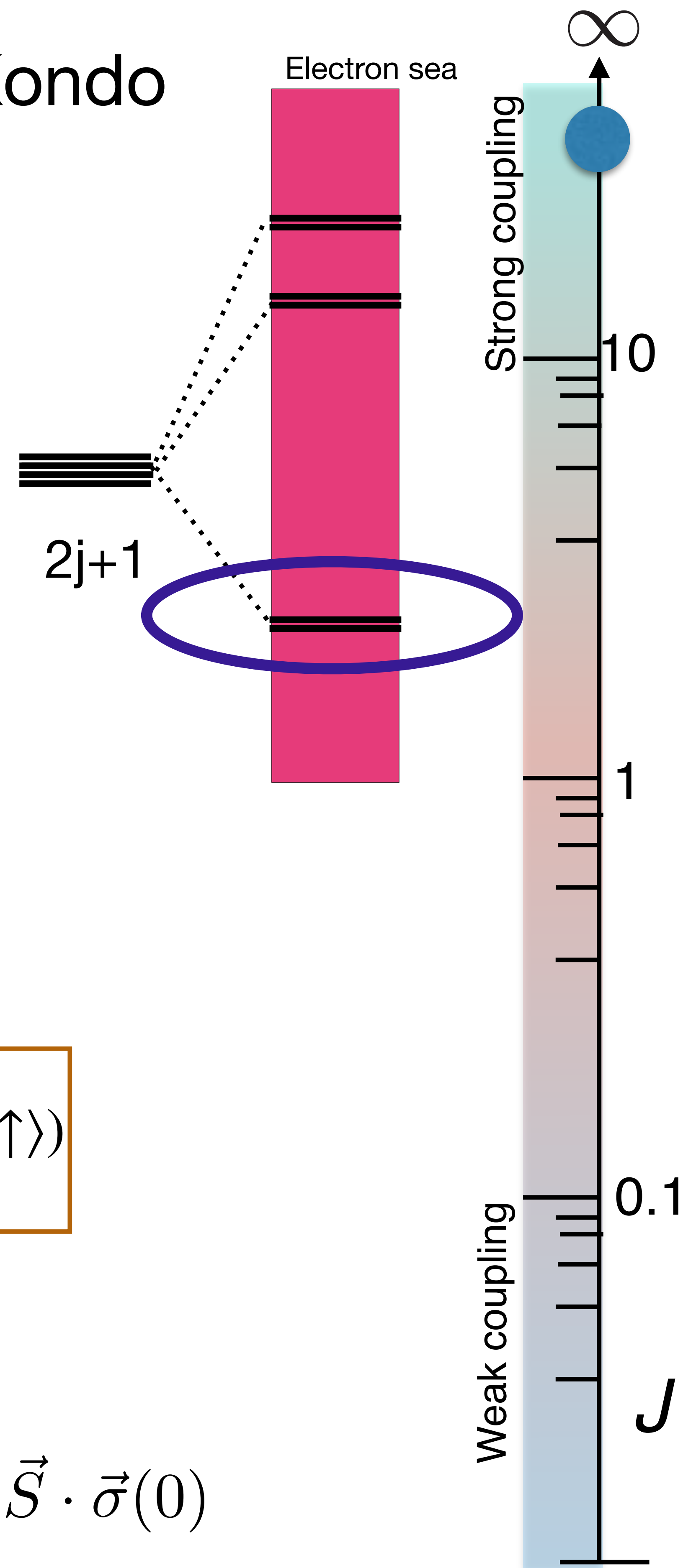

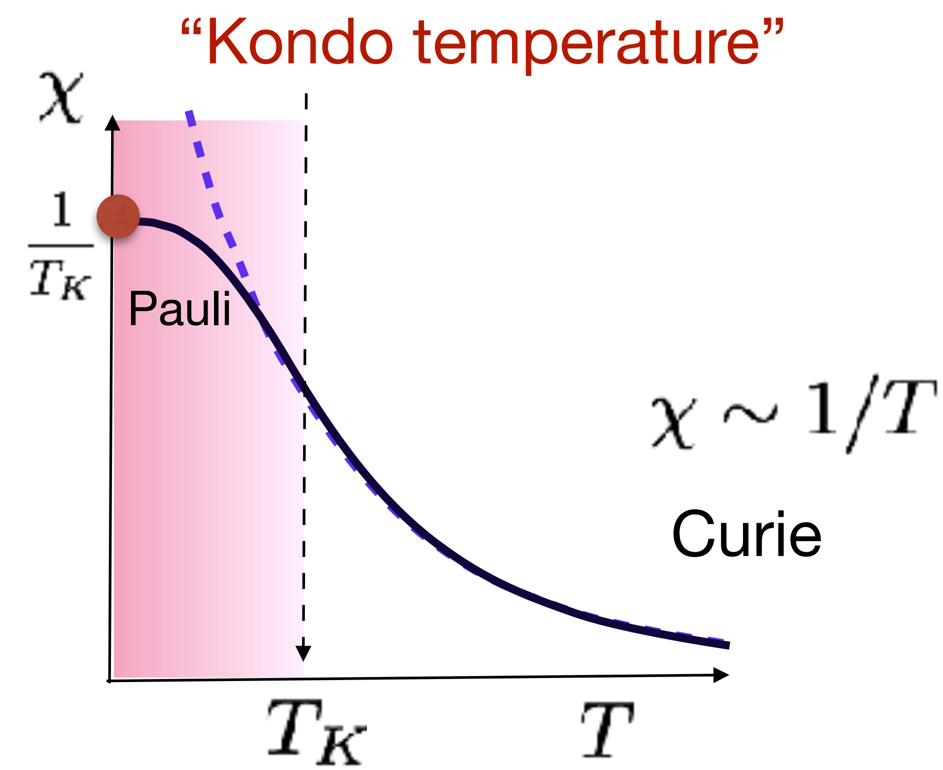

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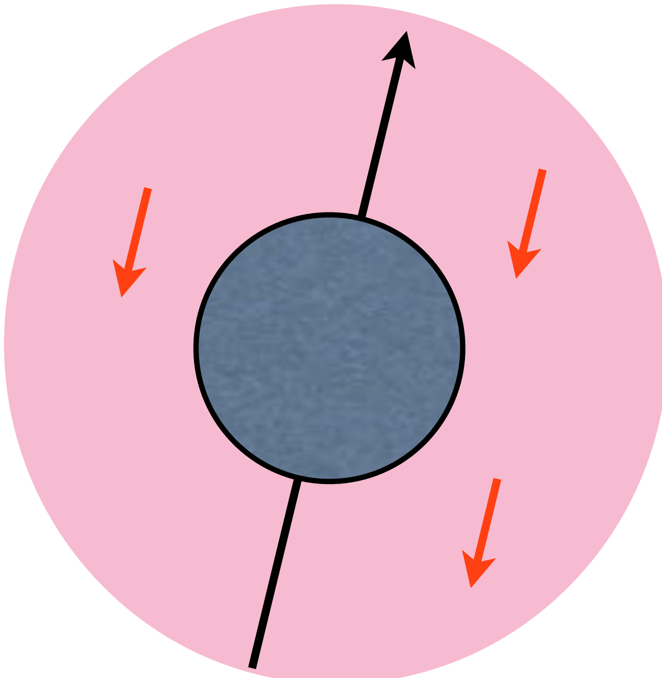

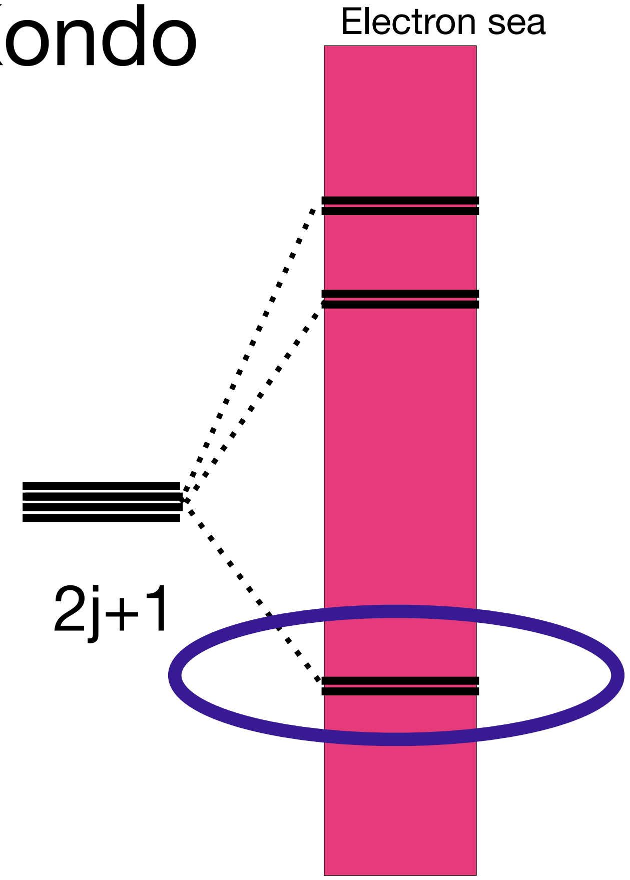

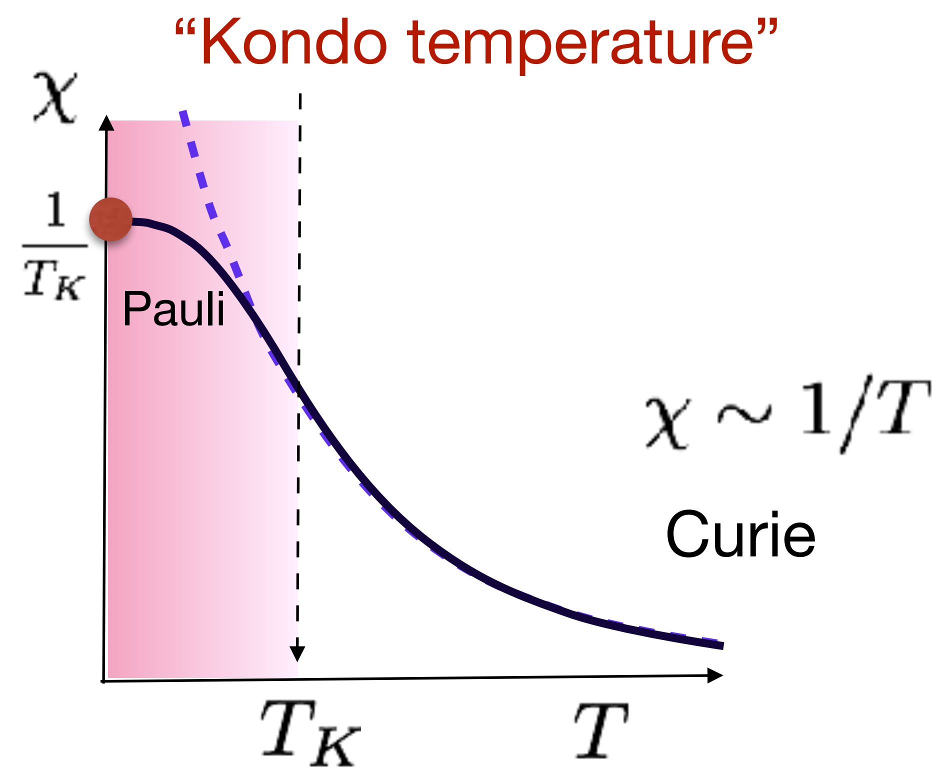

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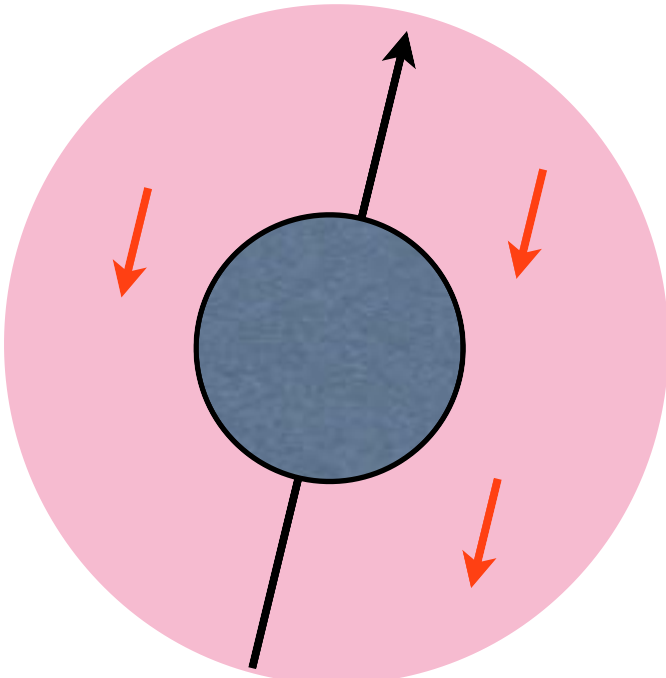

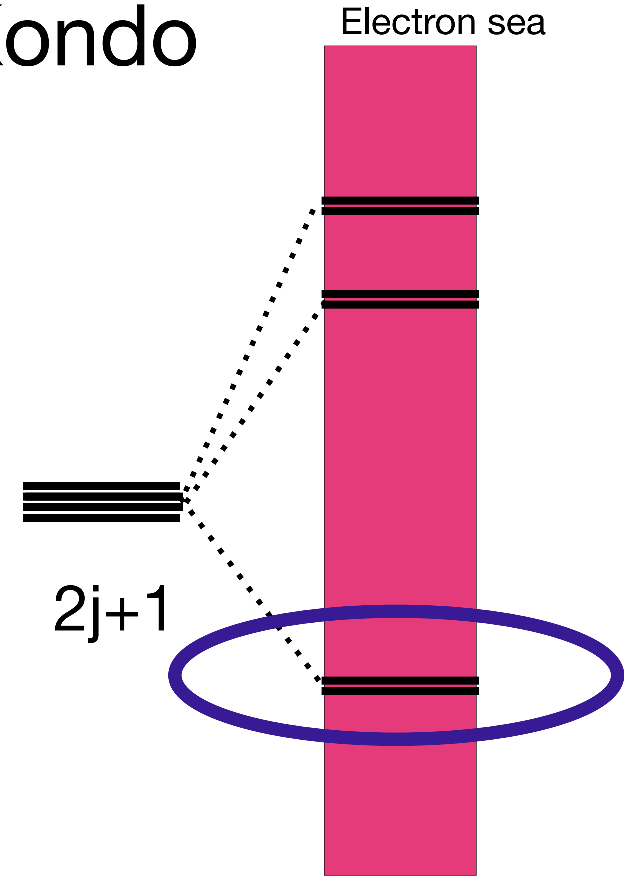

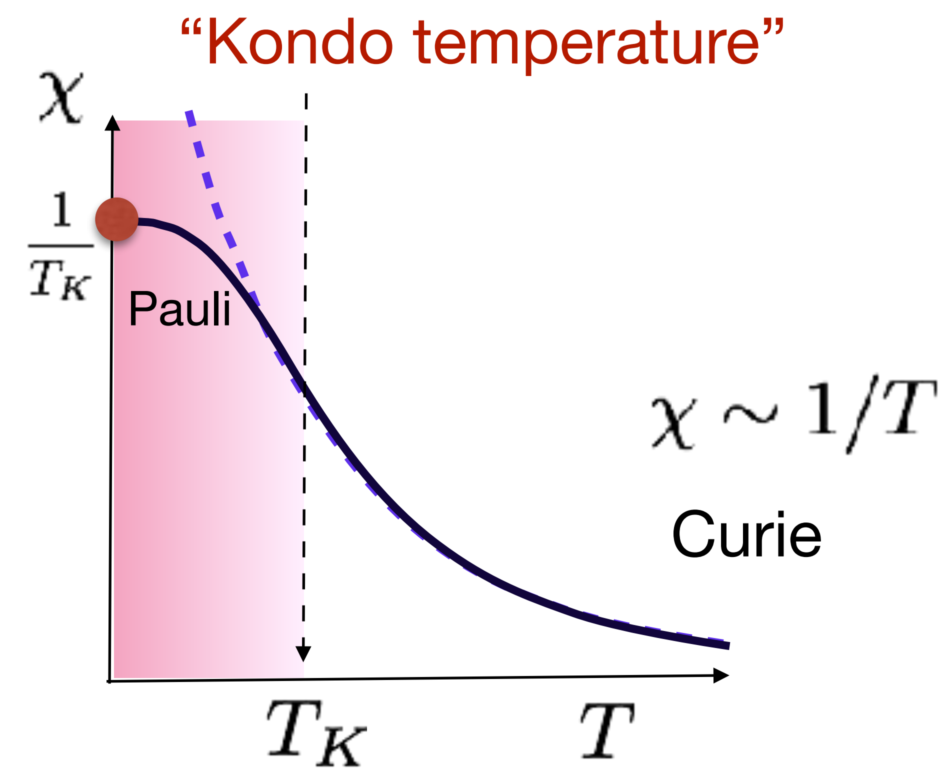

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Spin entanglement entropy

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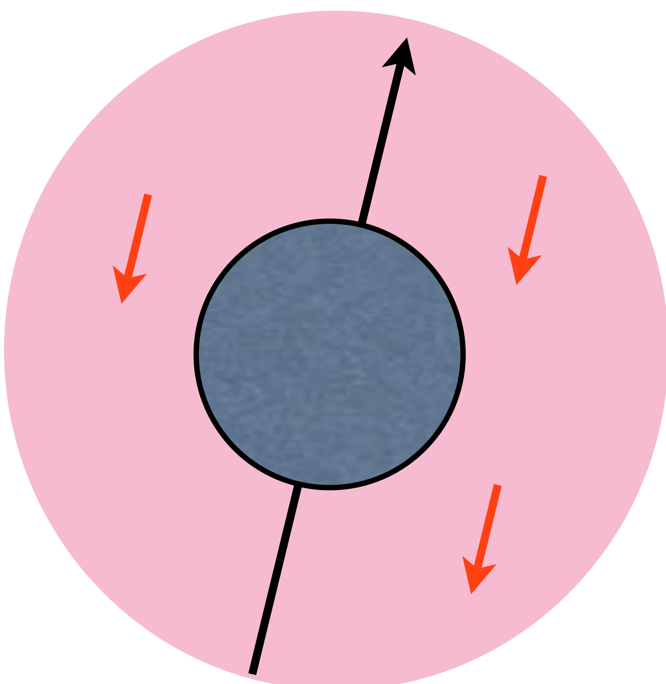

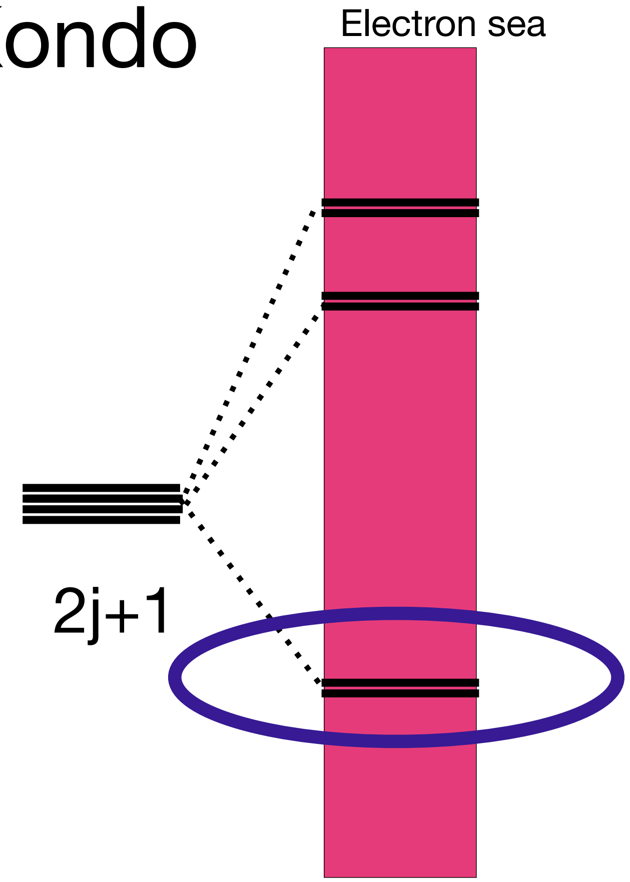

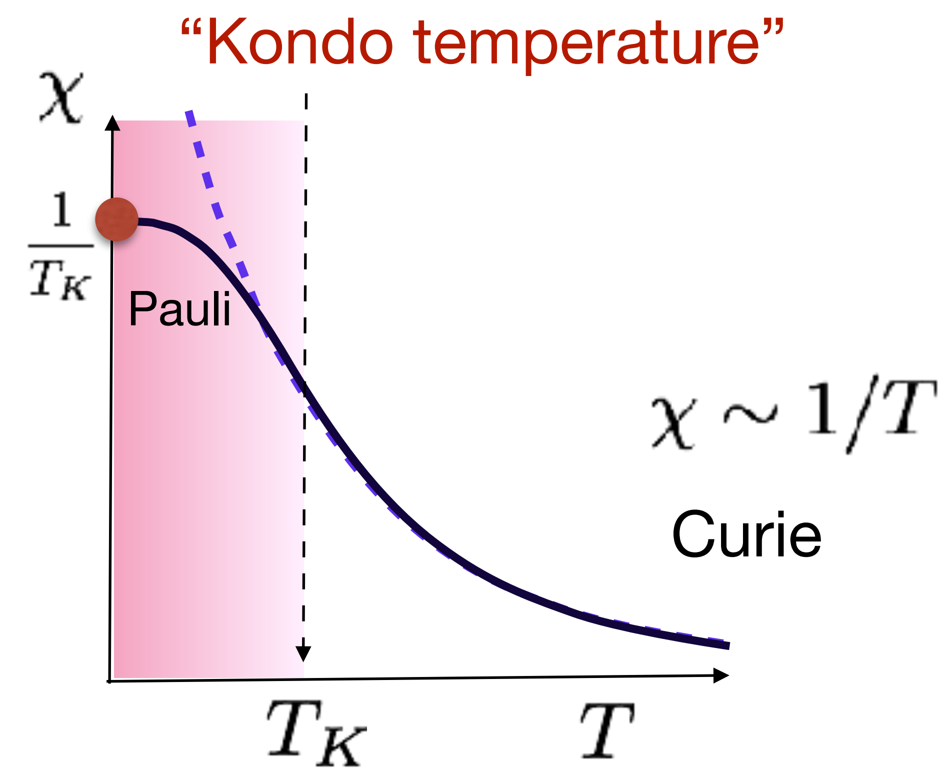

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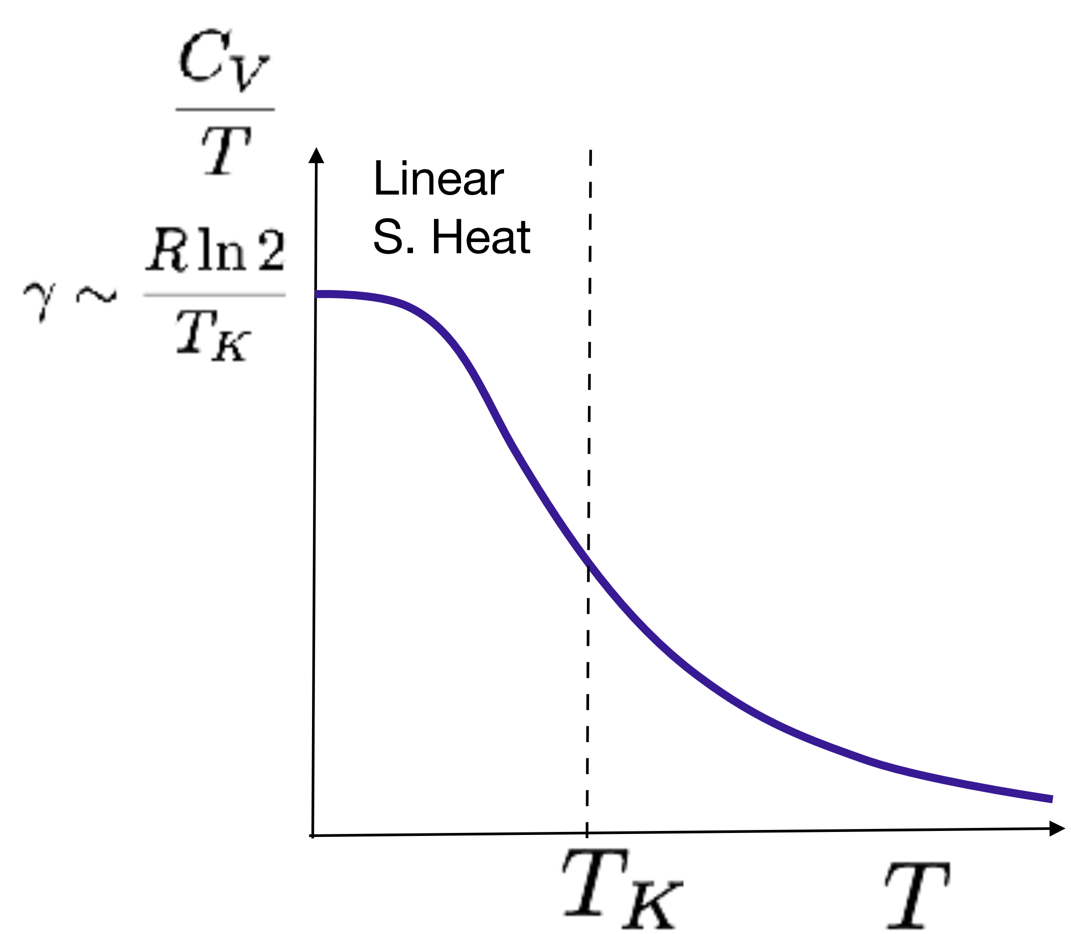

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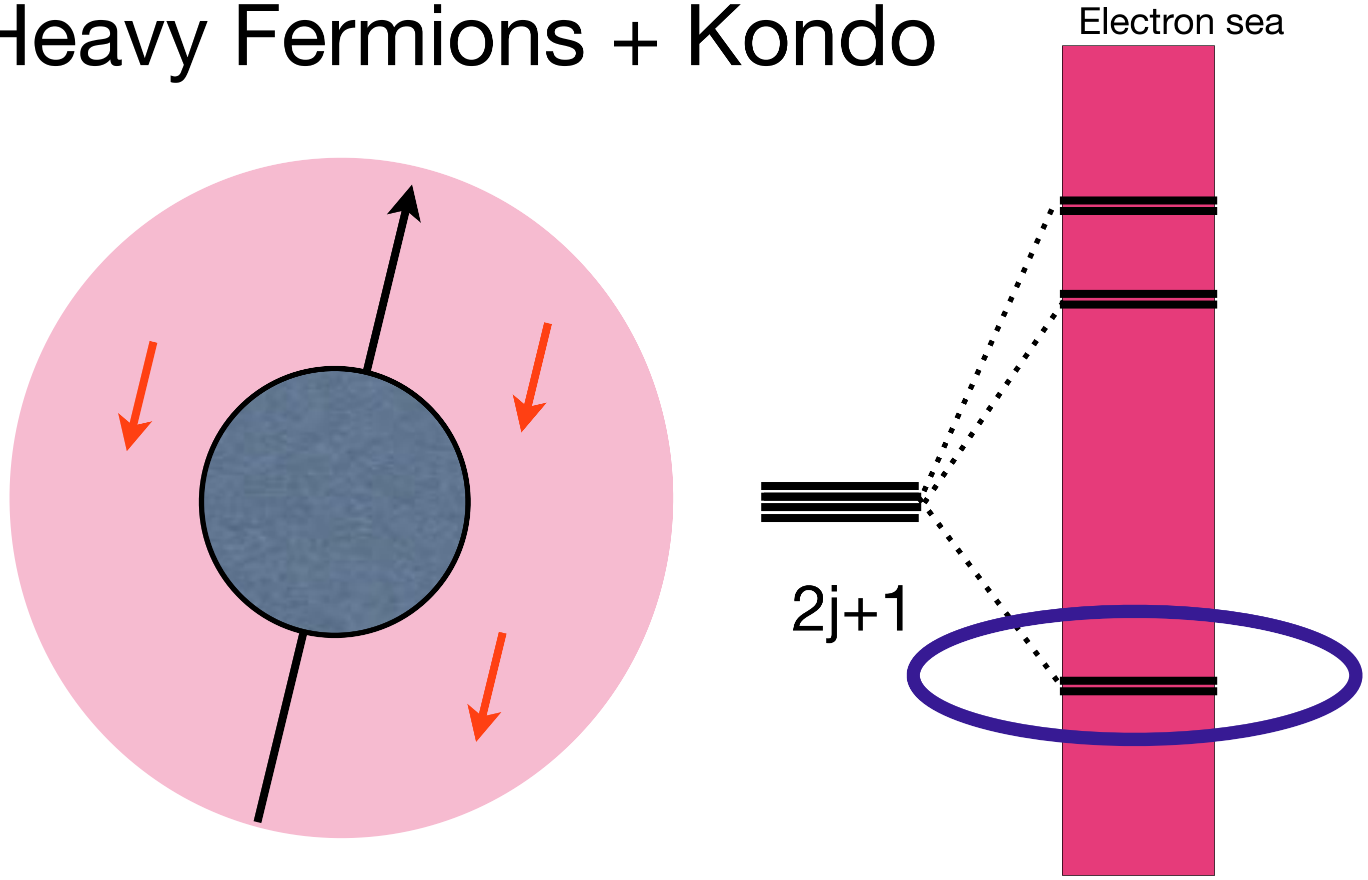

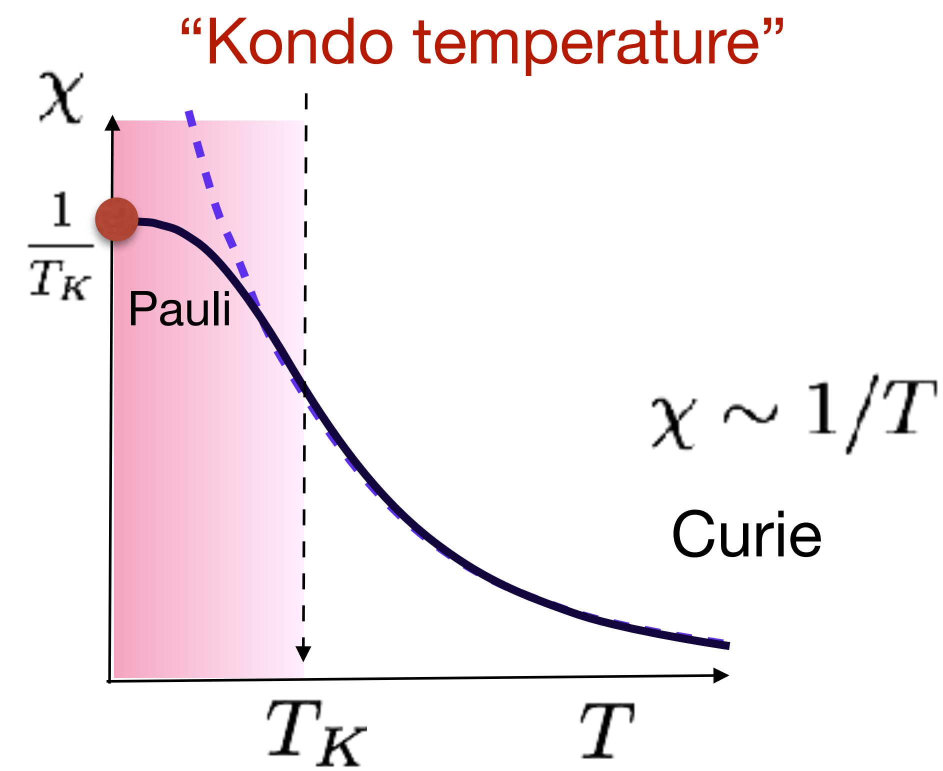

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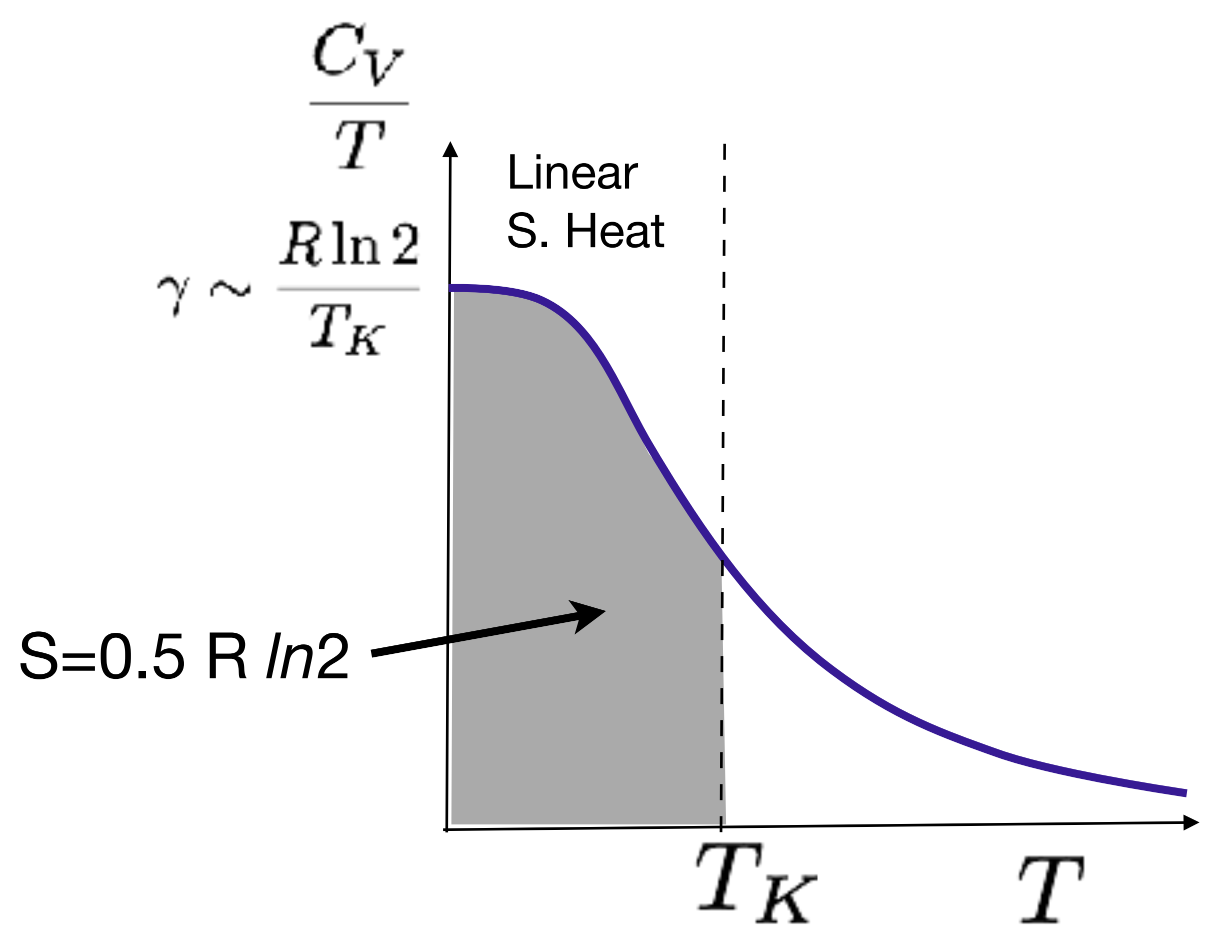

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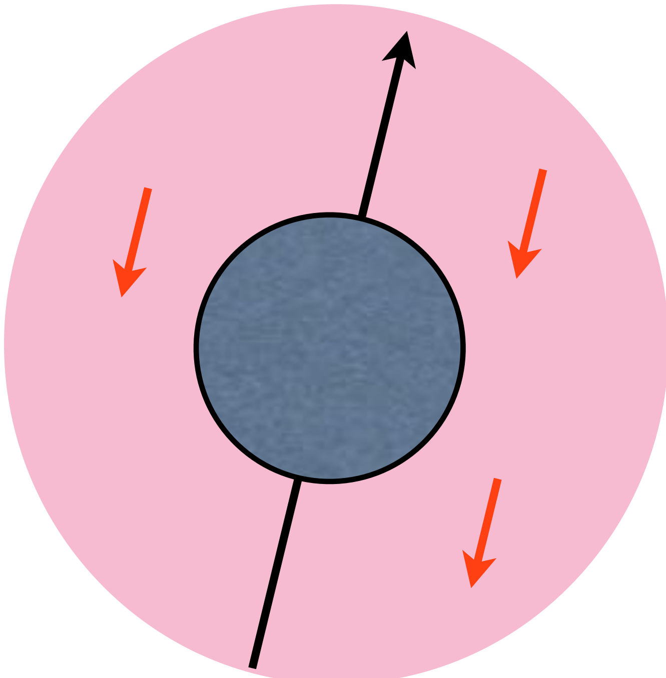

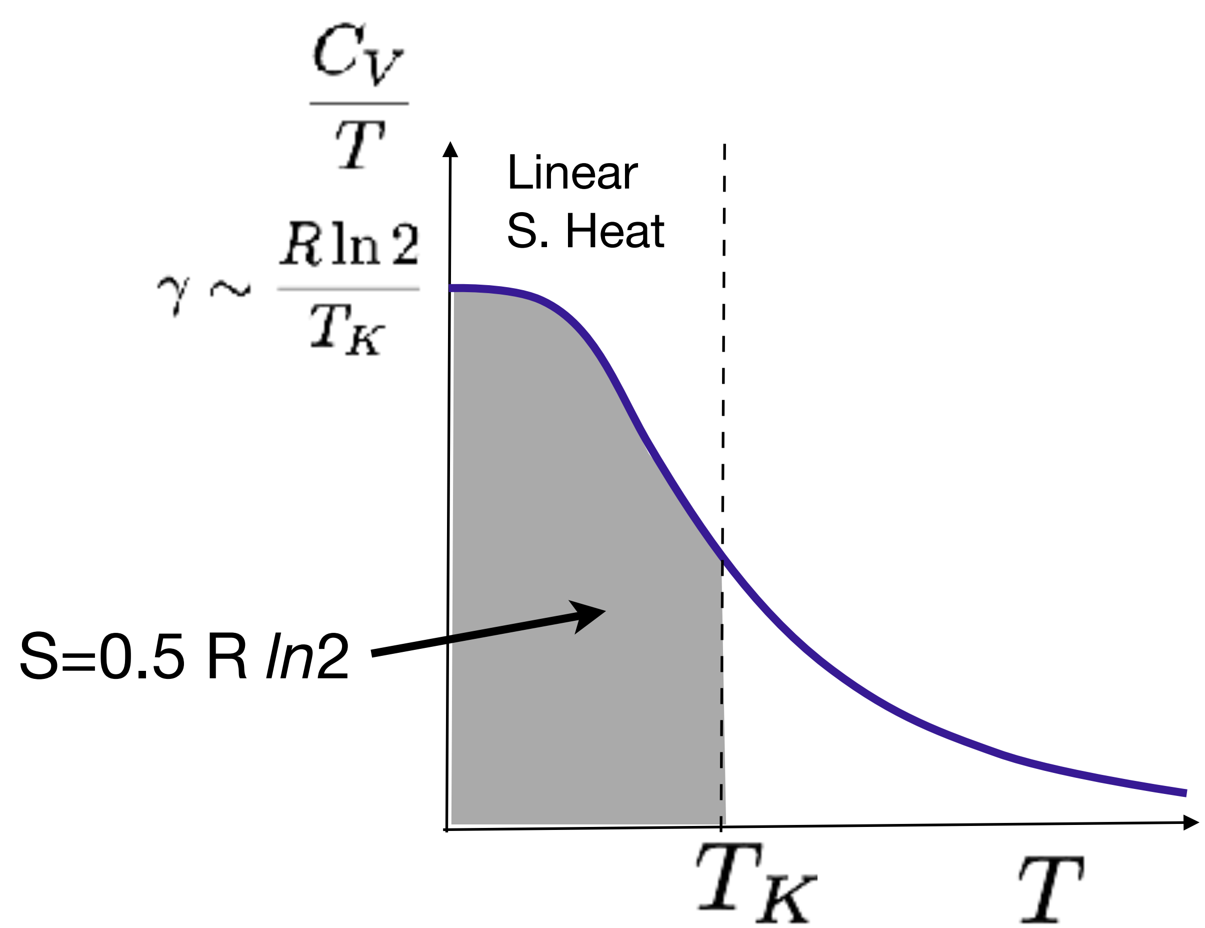

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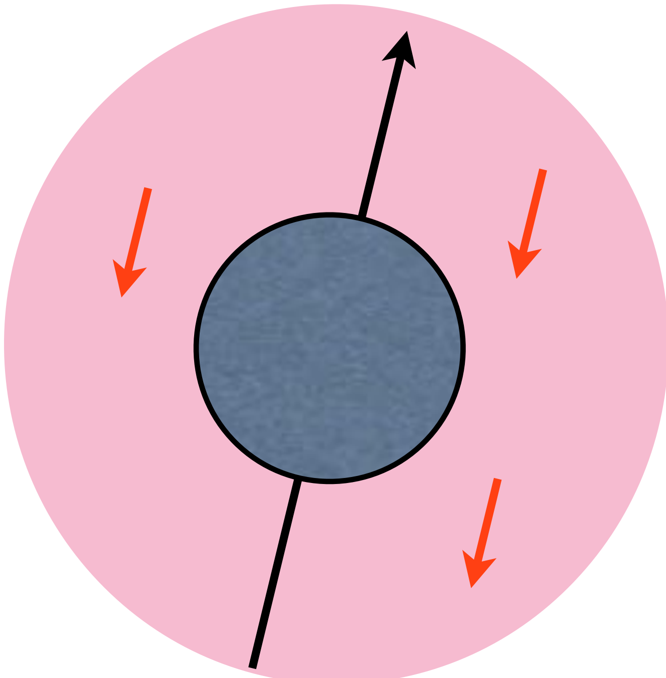

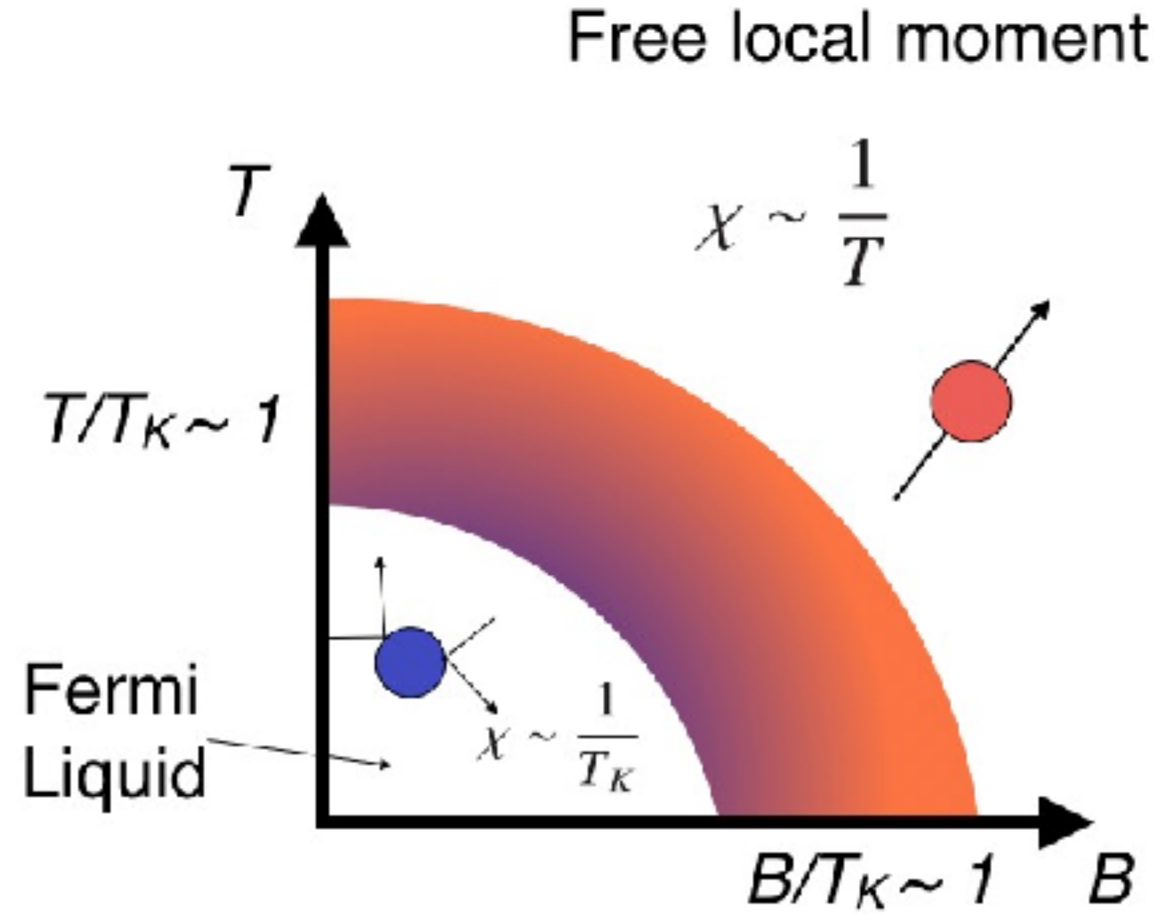

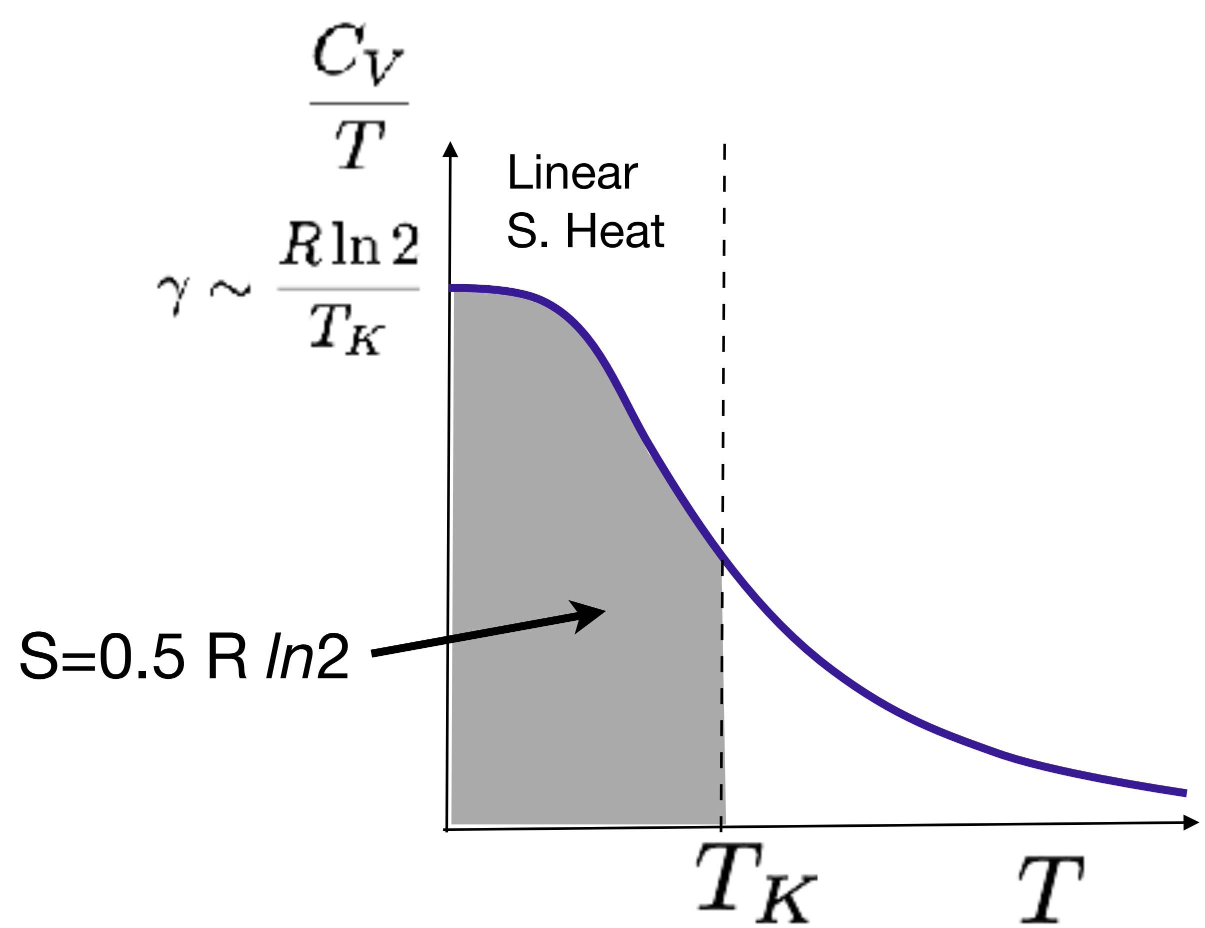

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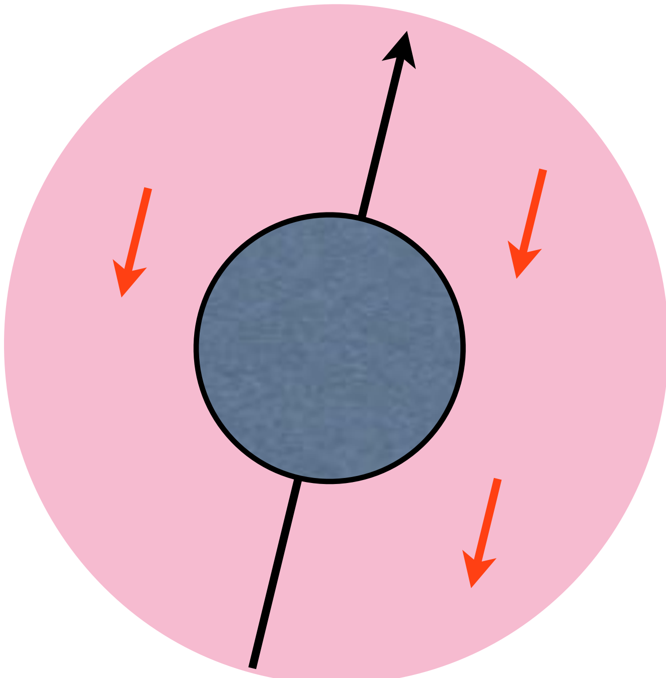

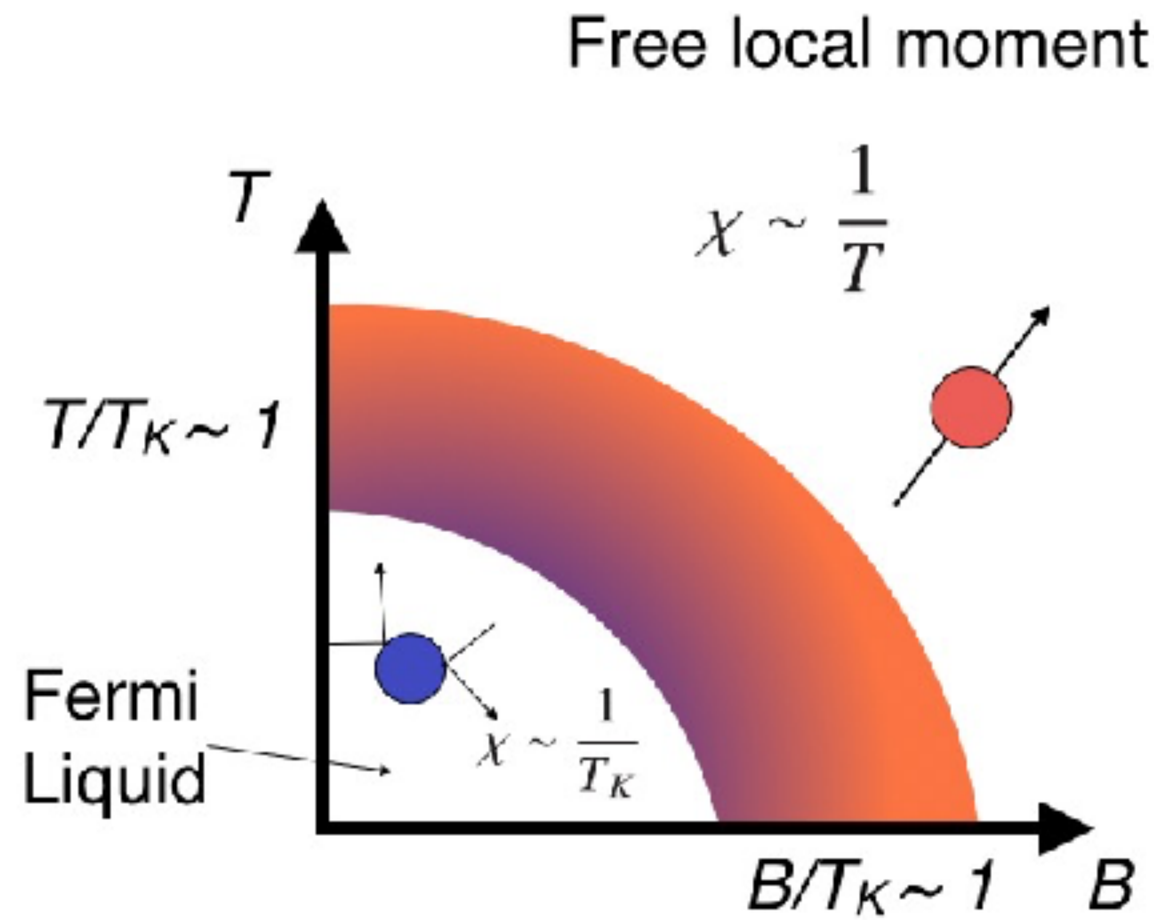

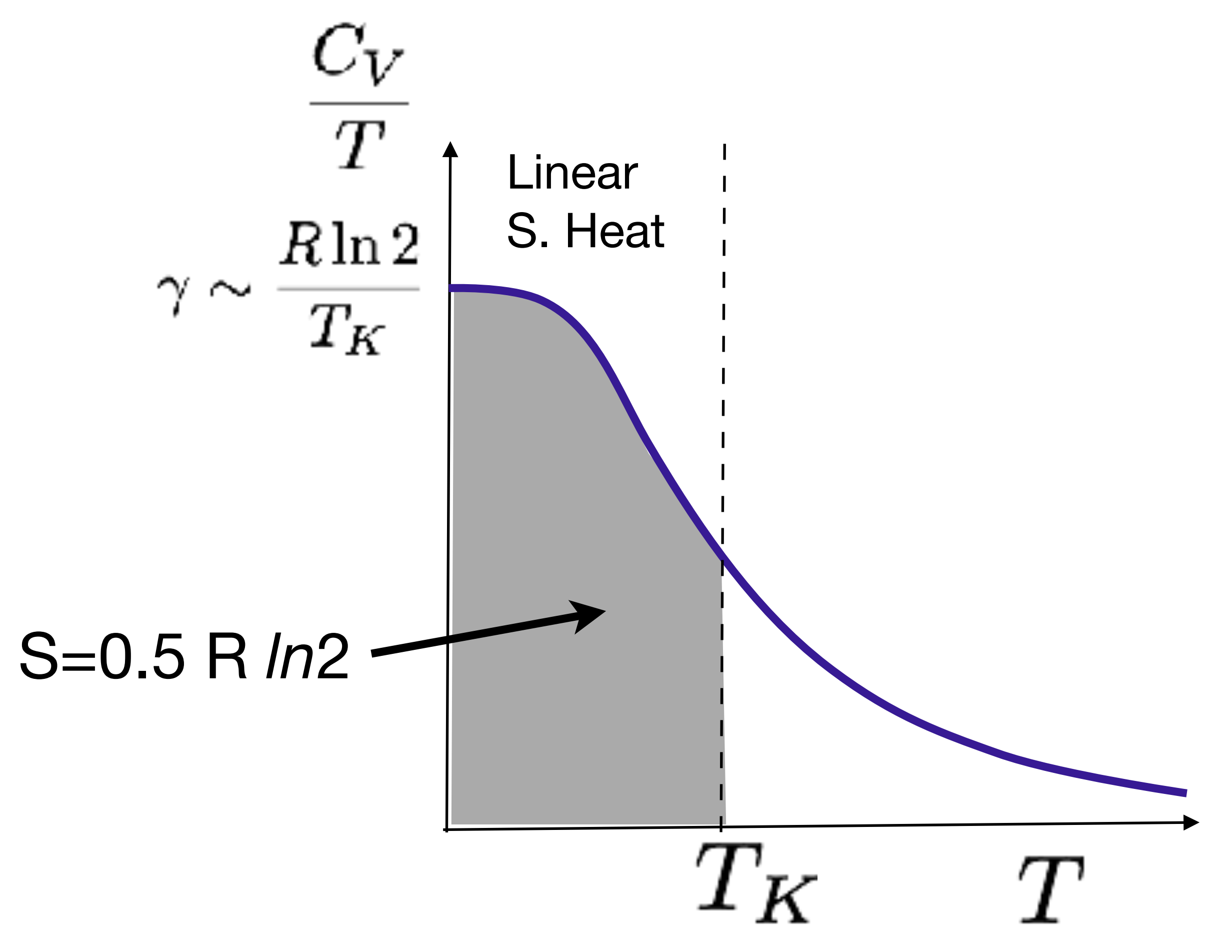

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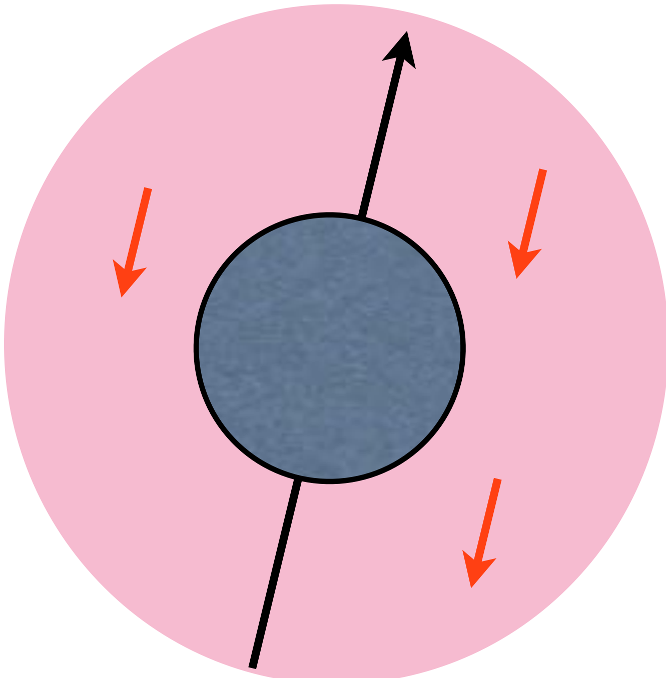
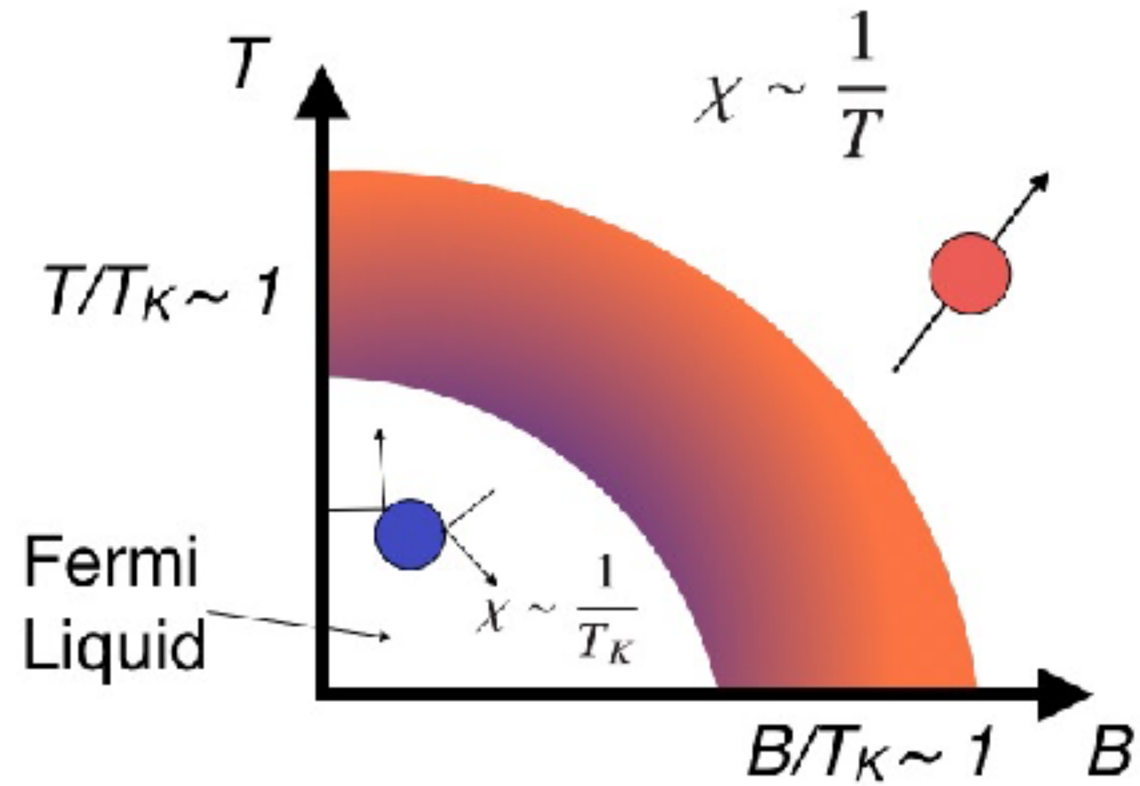
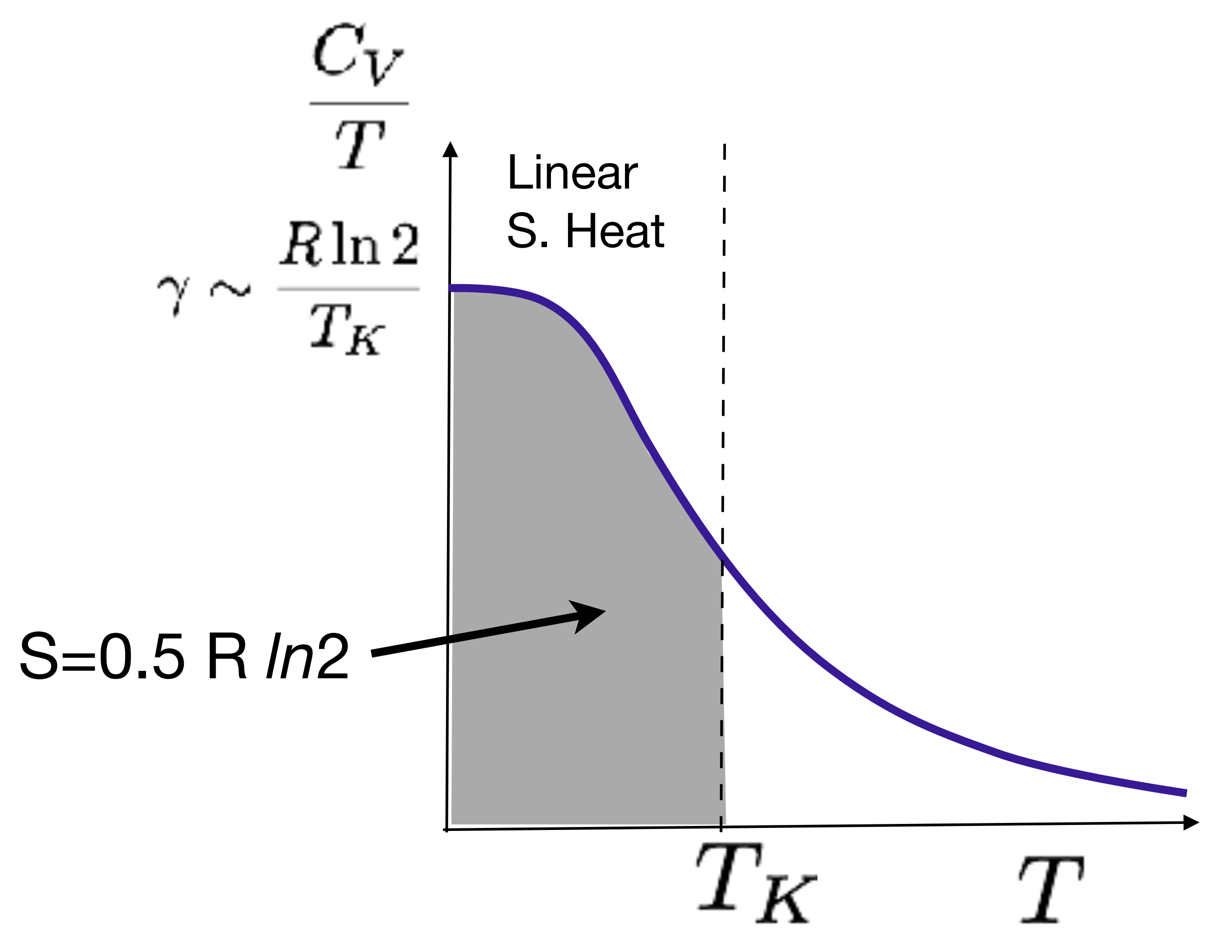




$$\frac{R(T)}{R_U} = n_i \Phi\left(\frac{T}{T_K}\right)$$

Universality



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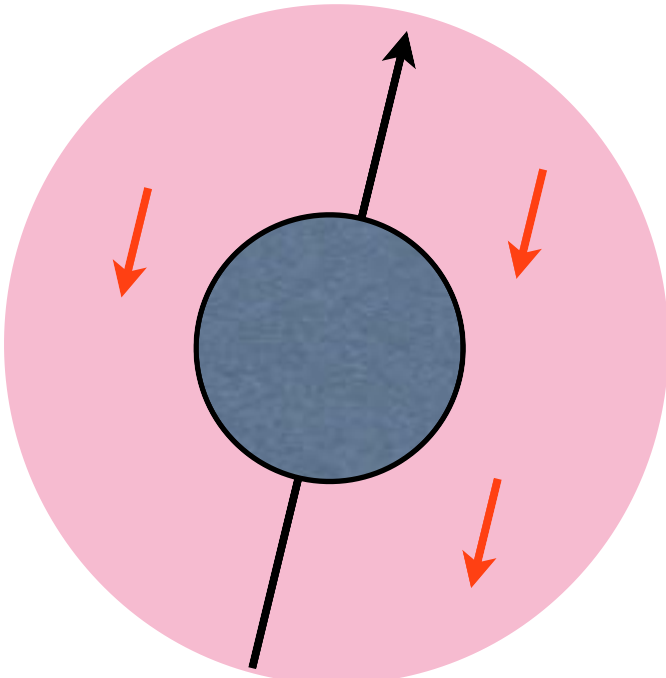

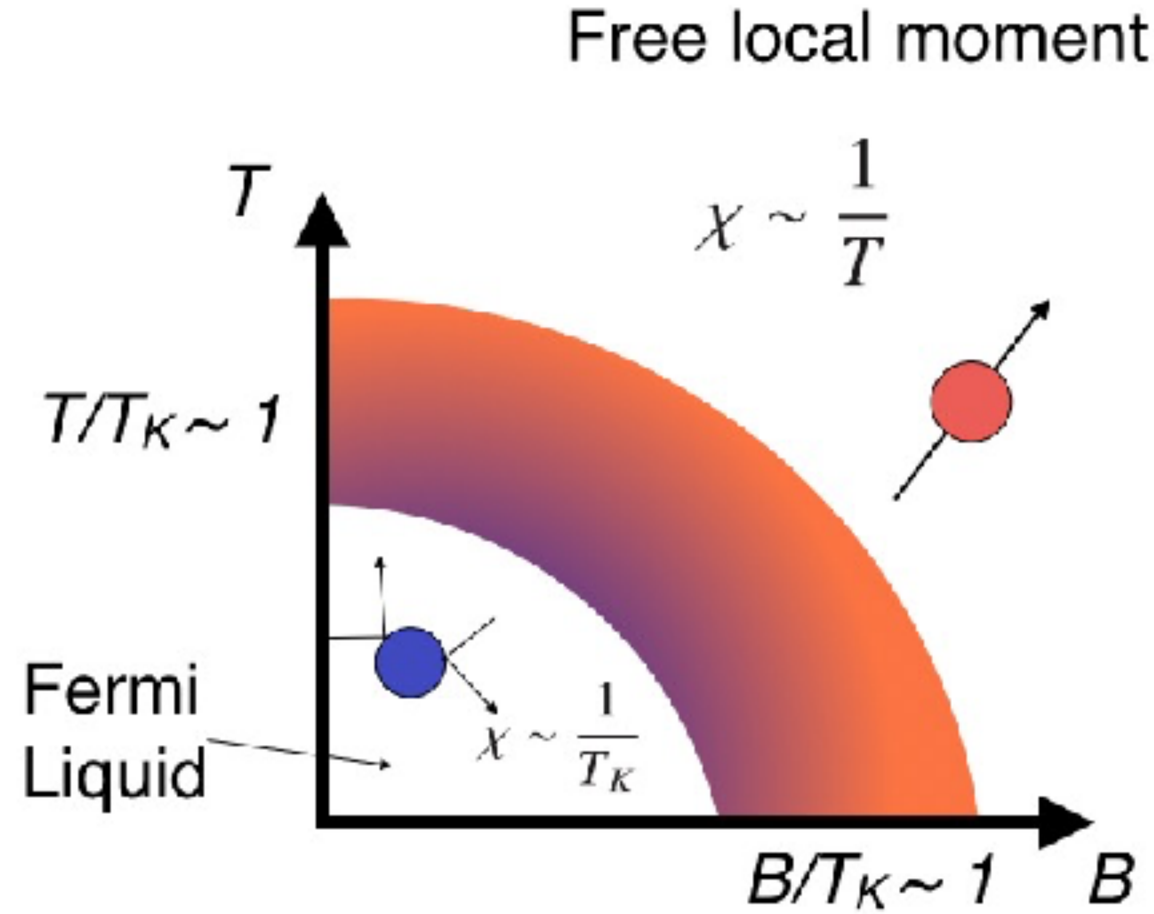

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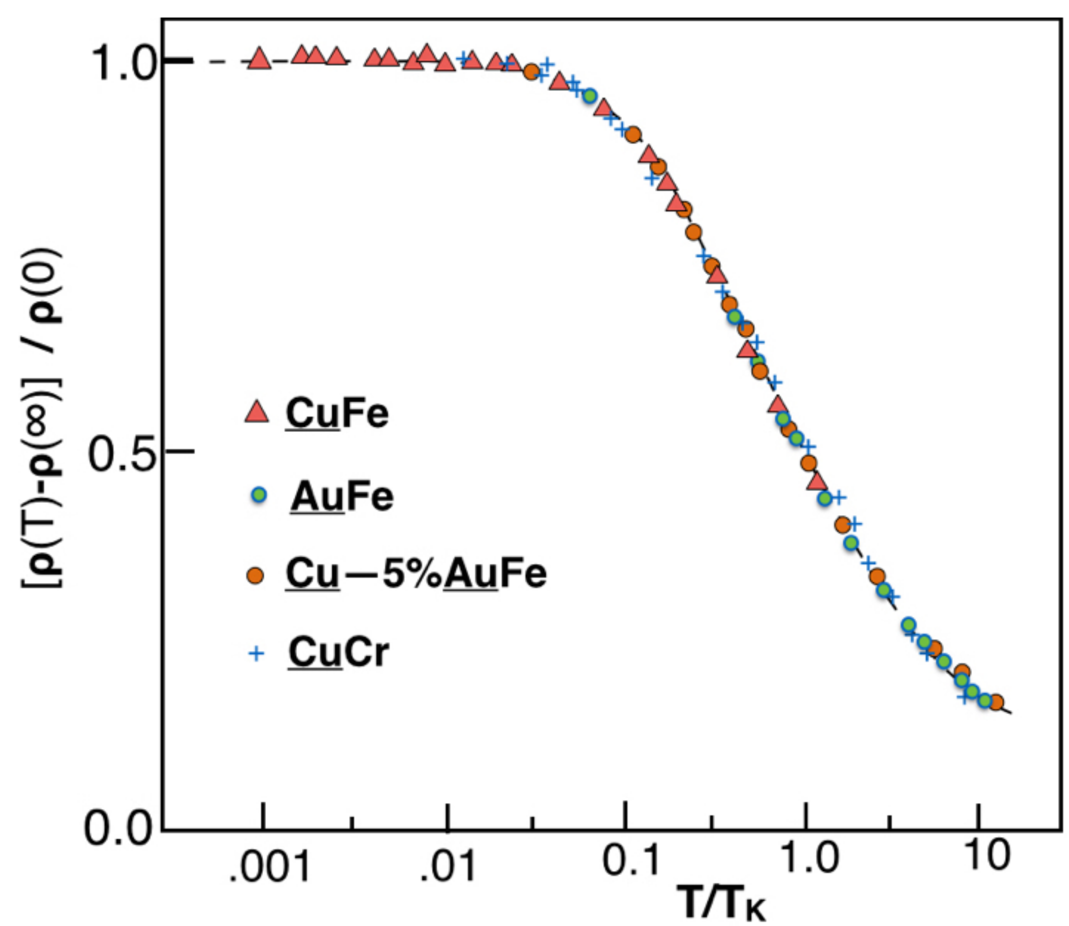

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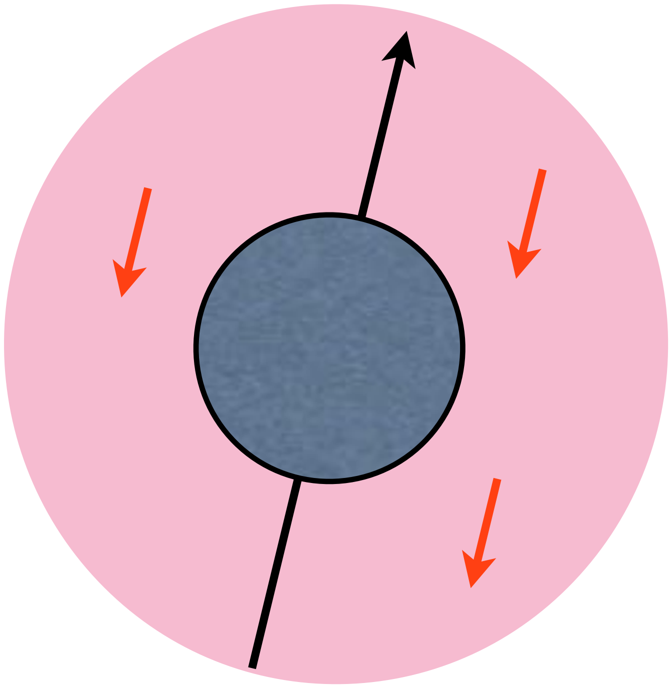

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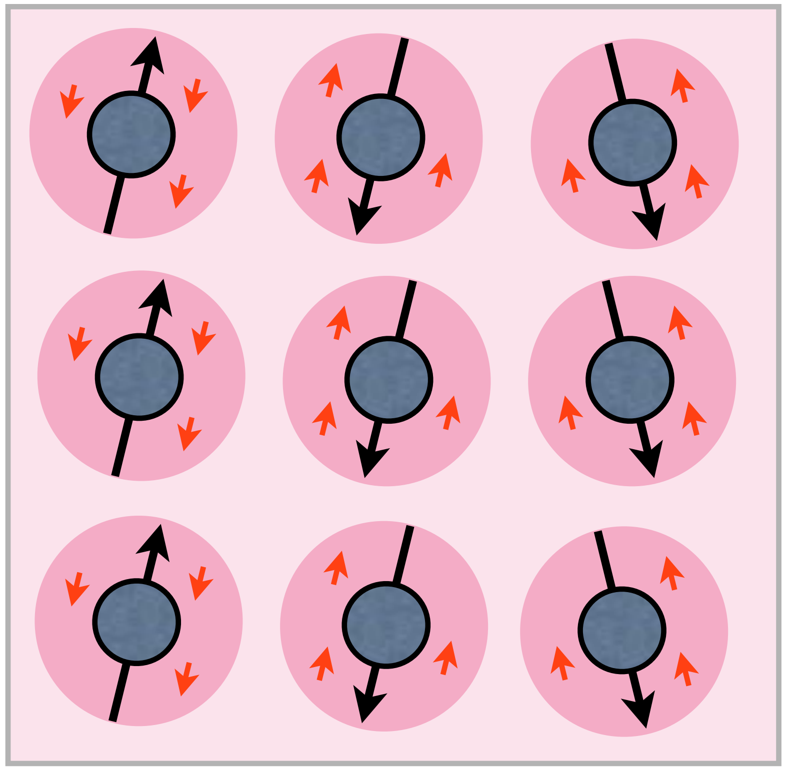

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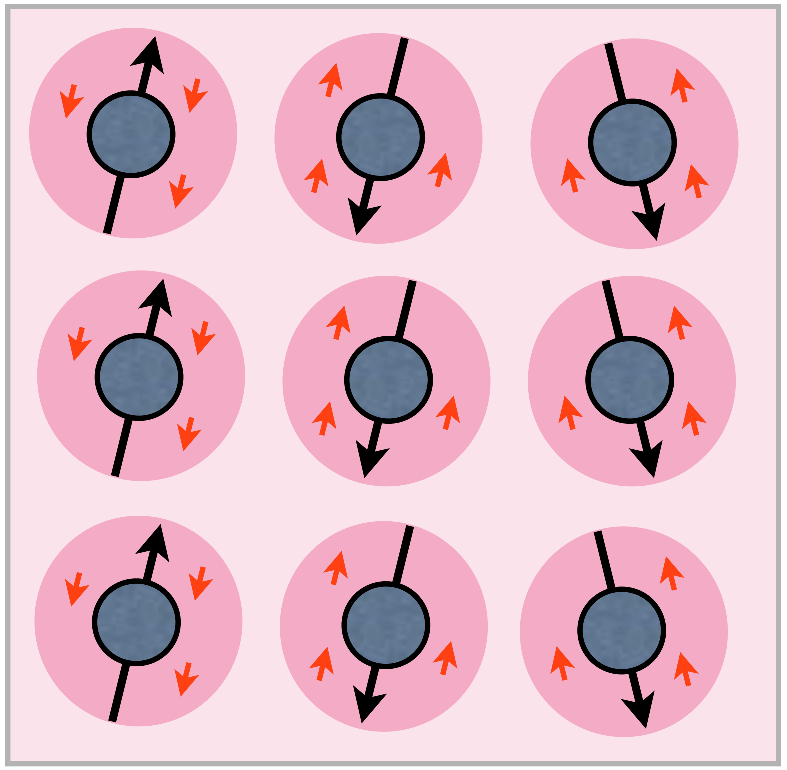

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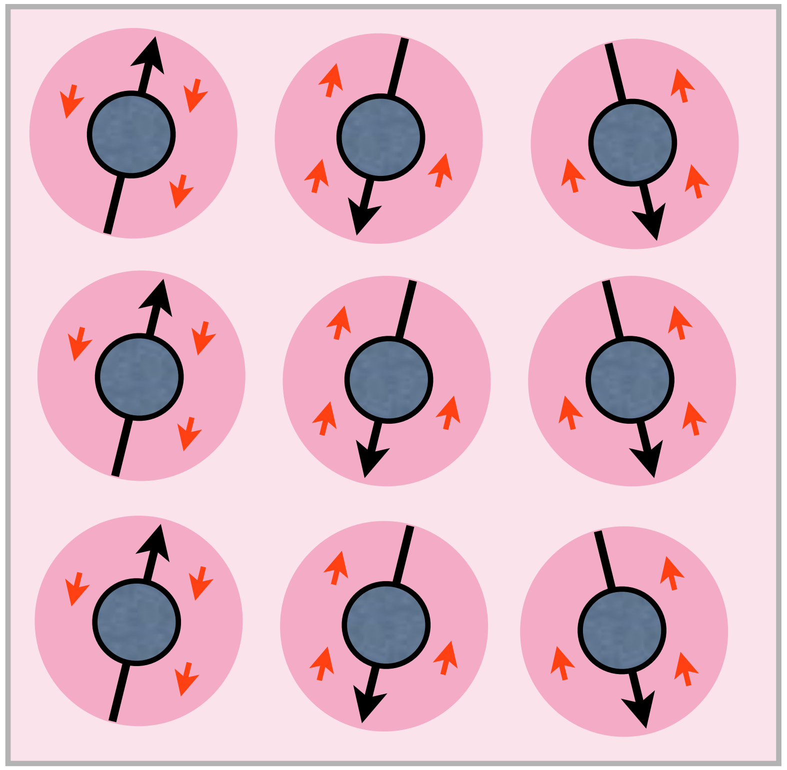

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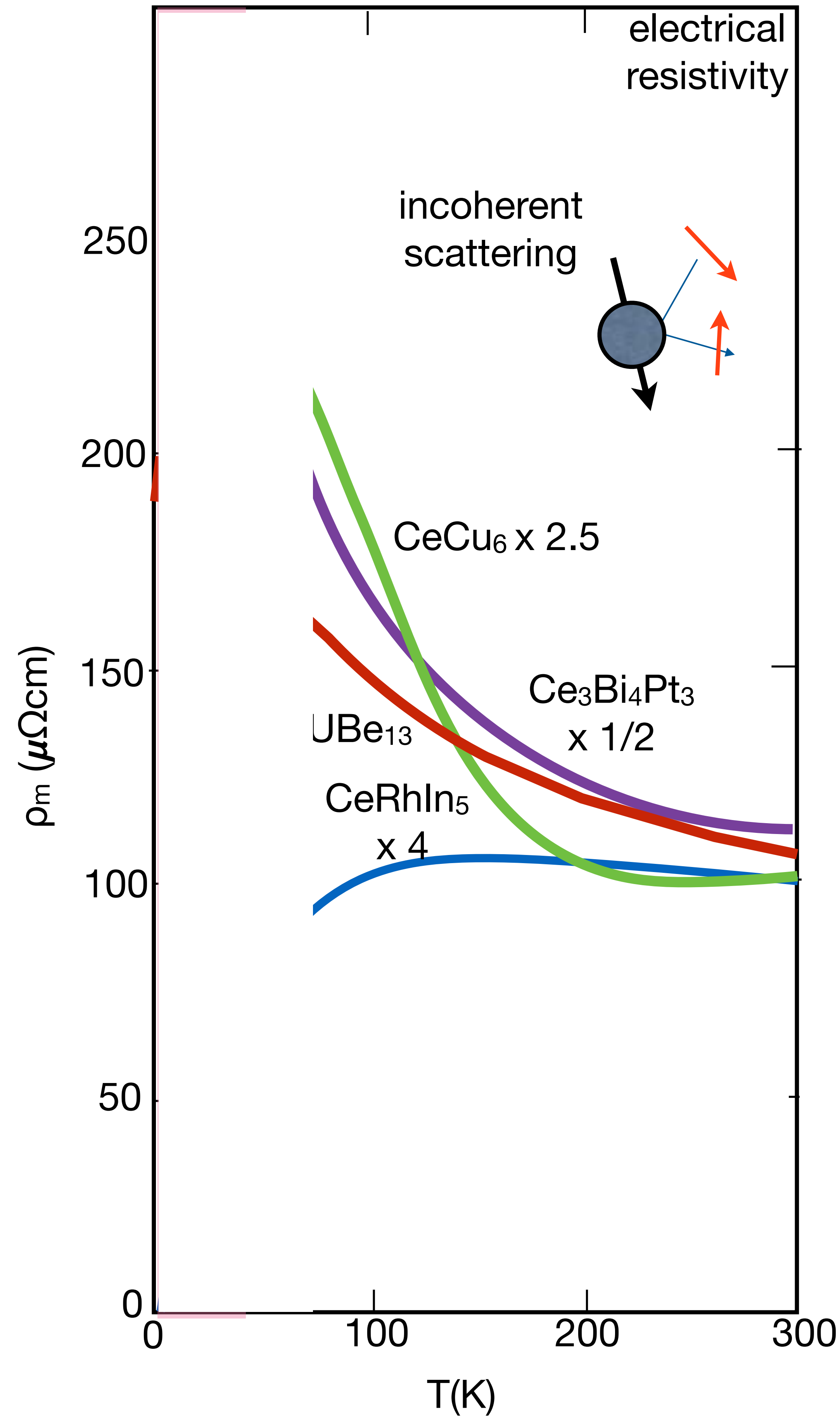

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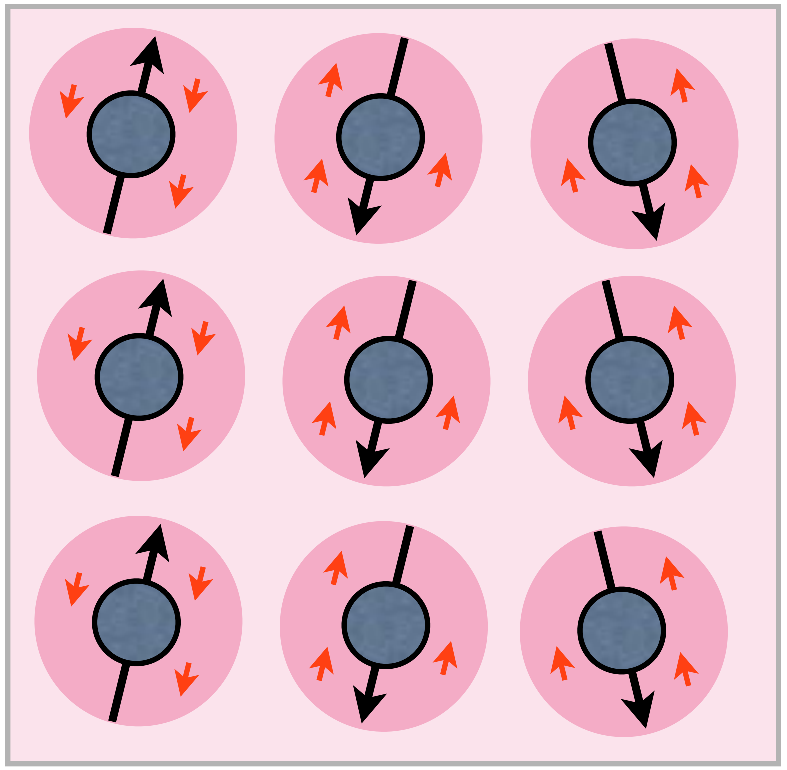

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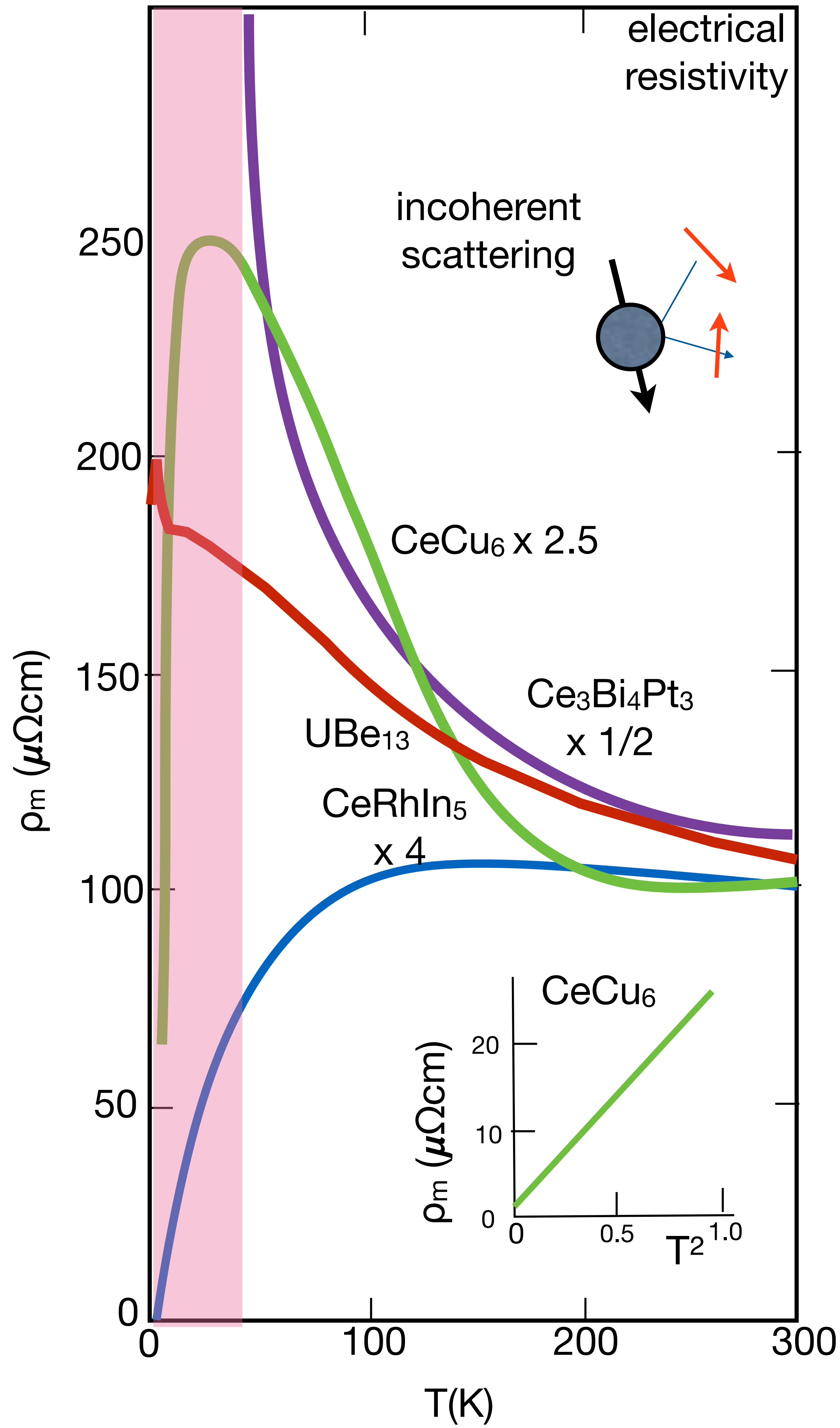

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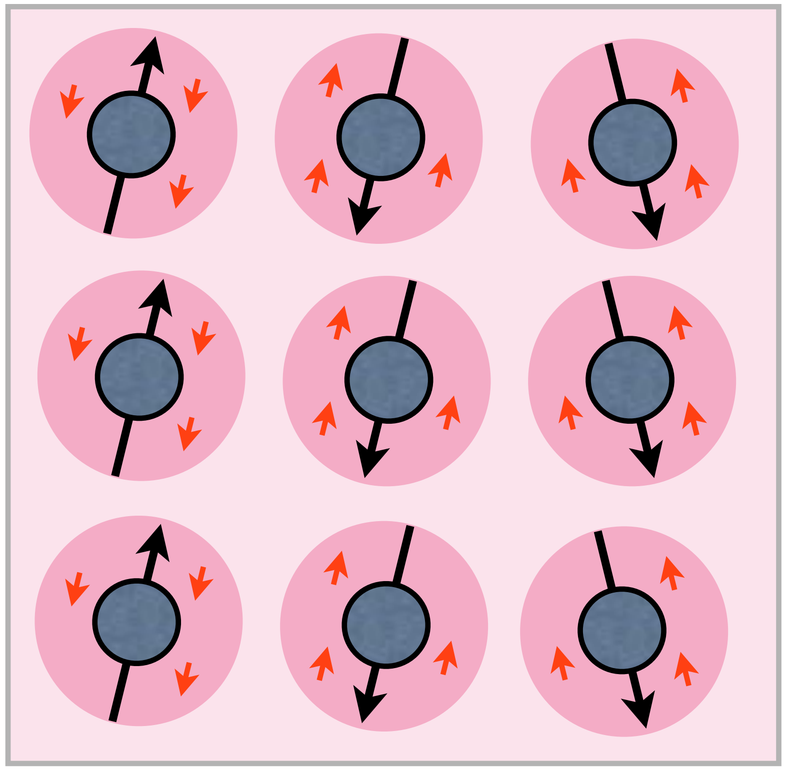
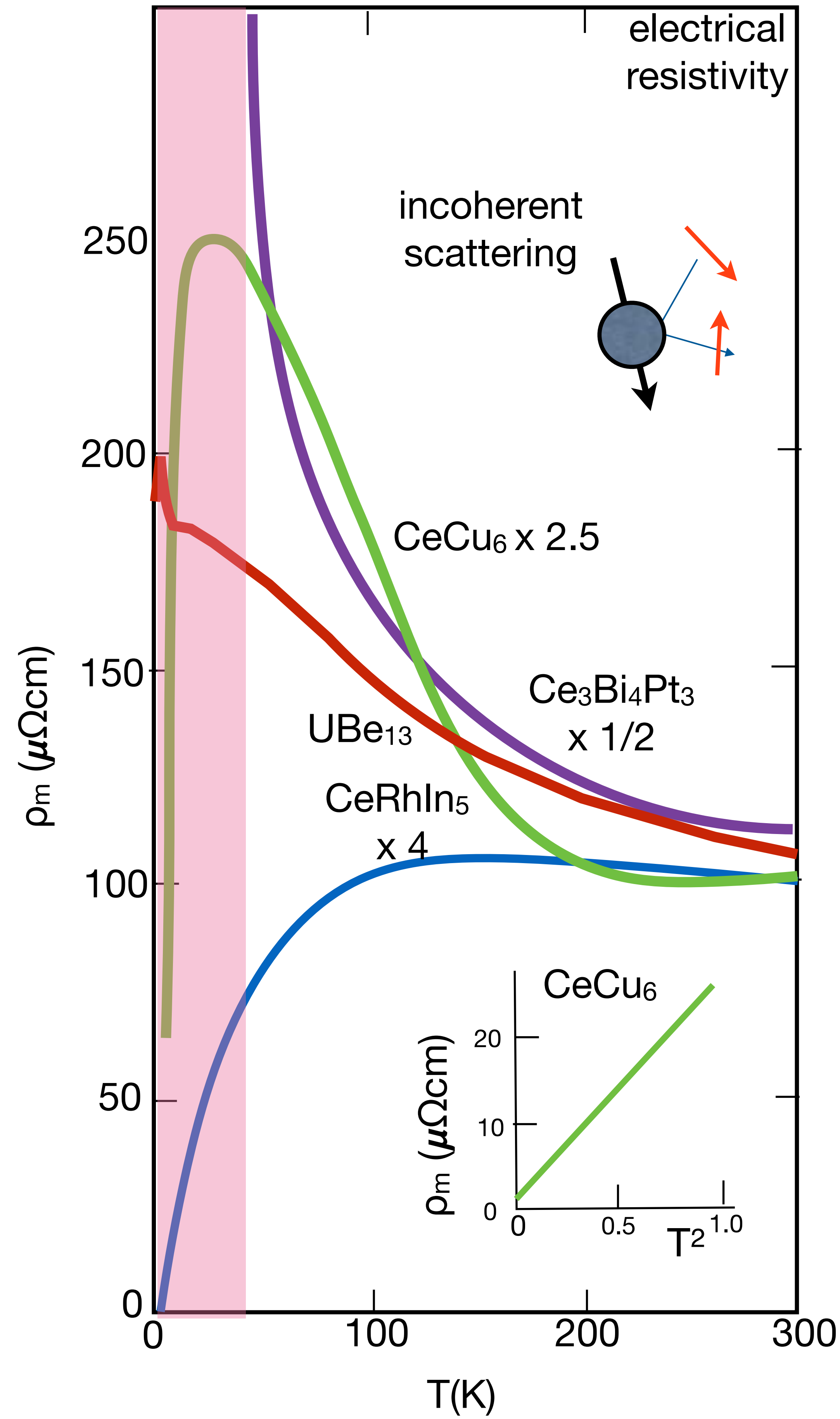


"Kondo Lattice"

Entangled spins and electrons

→ **Heavy Fermion Metals**



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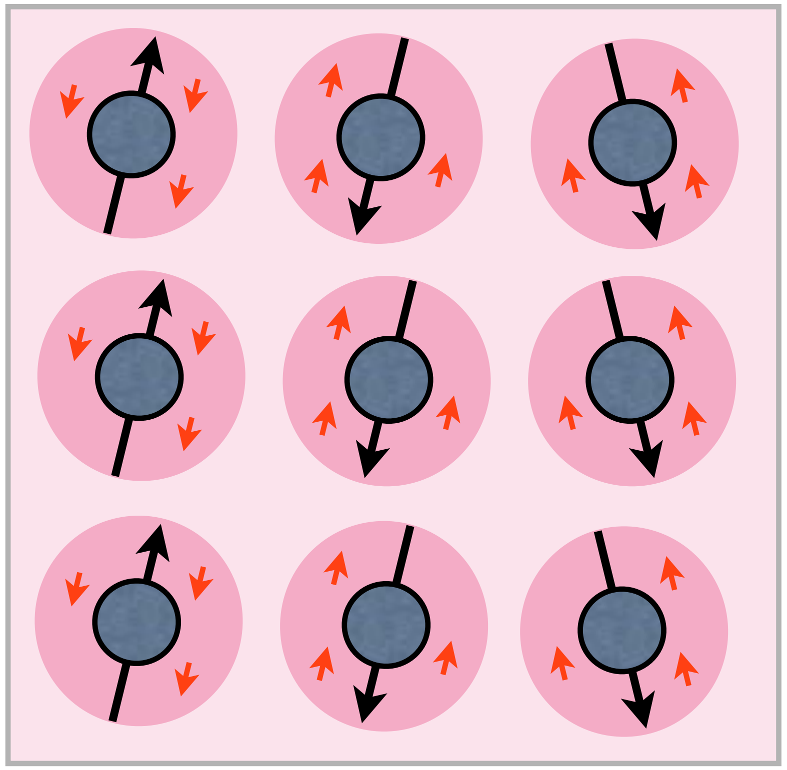

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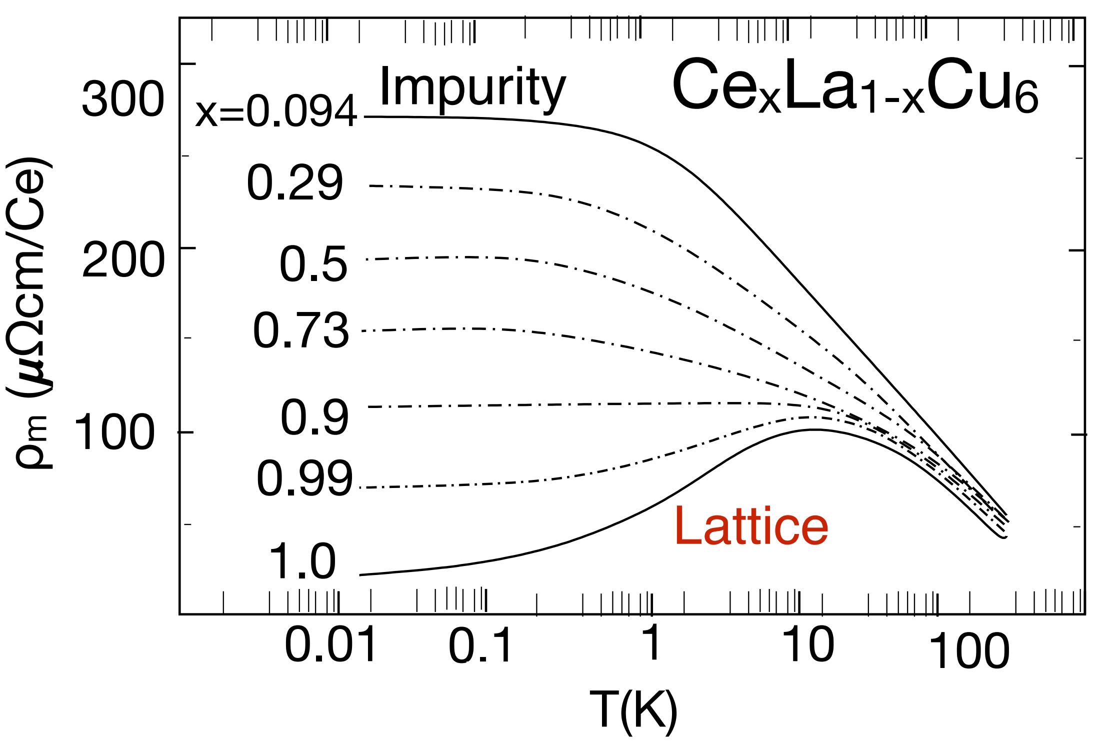

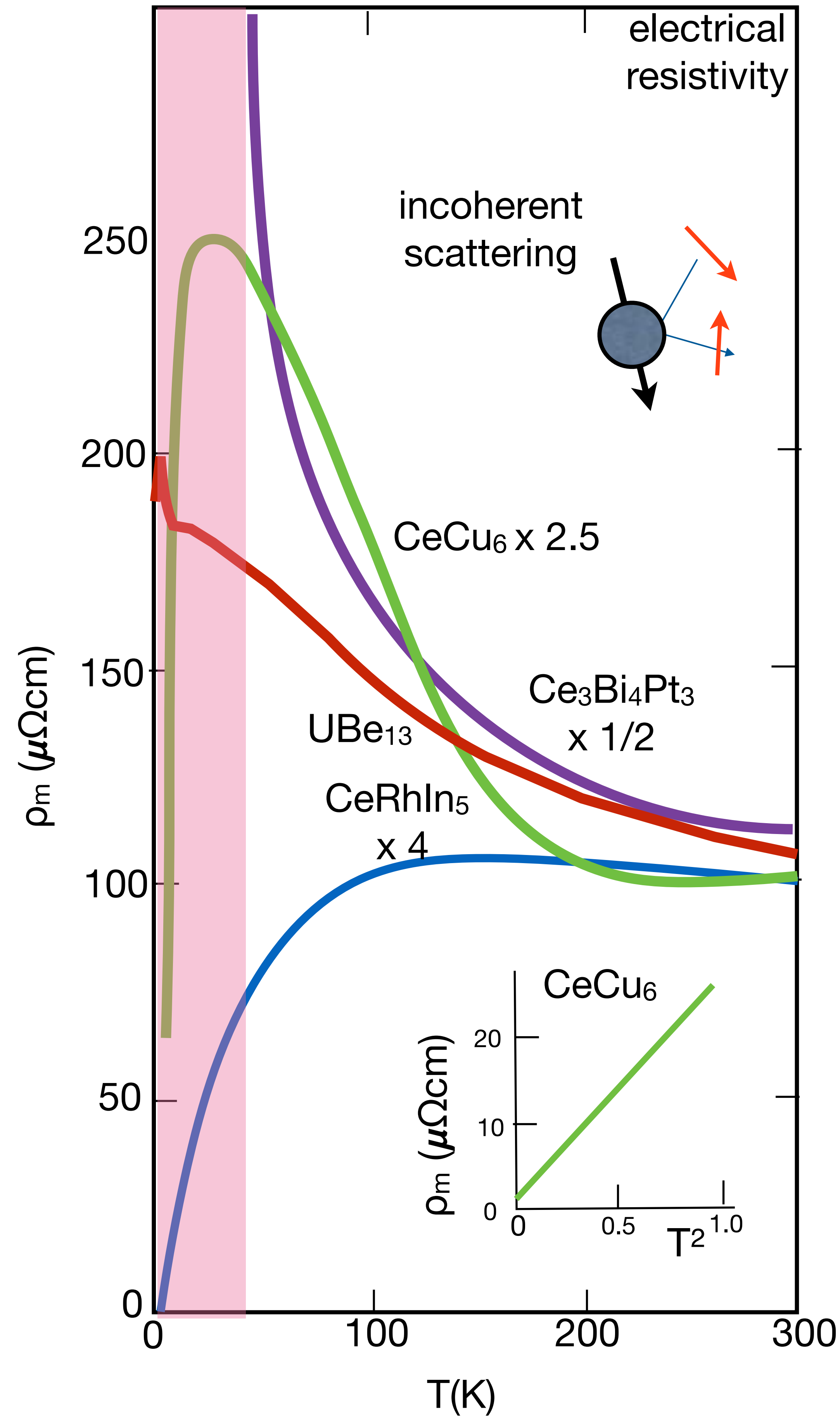

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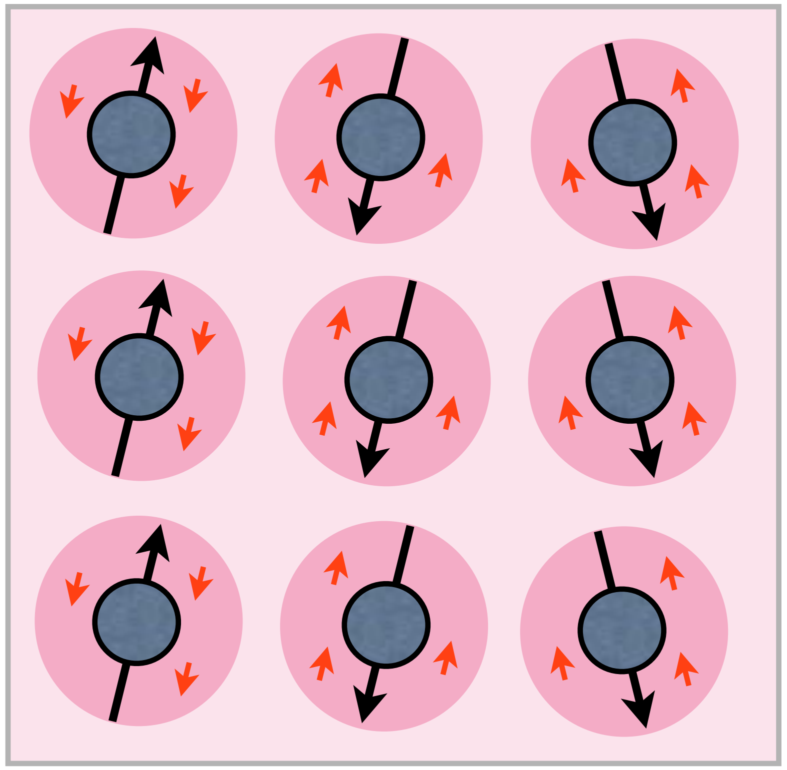

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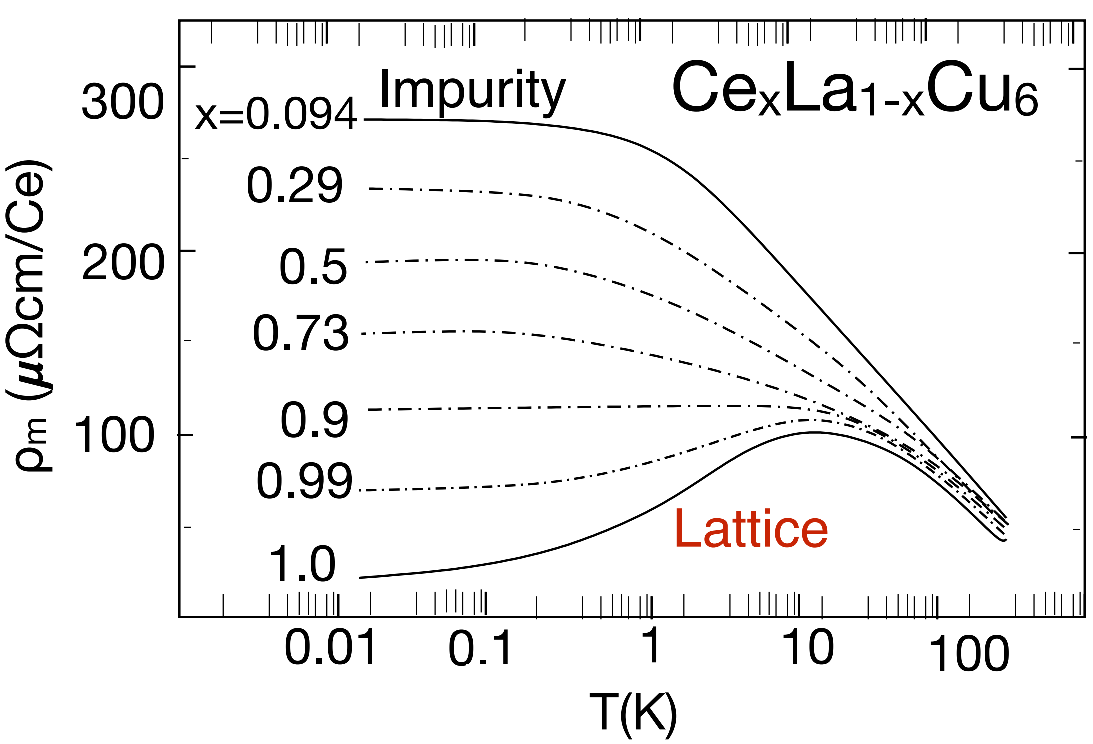

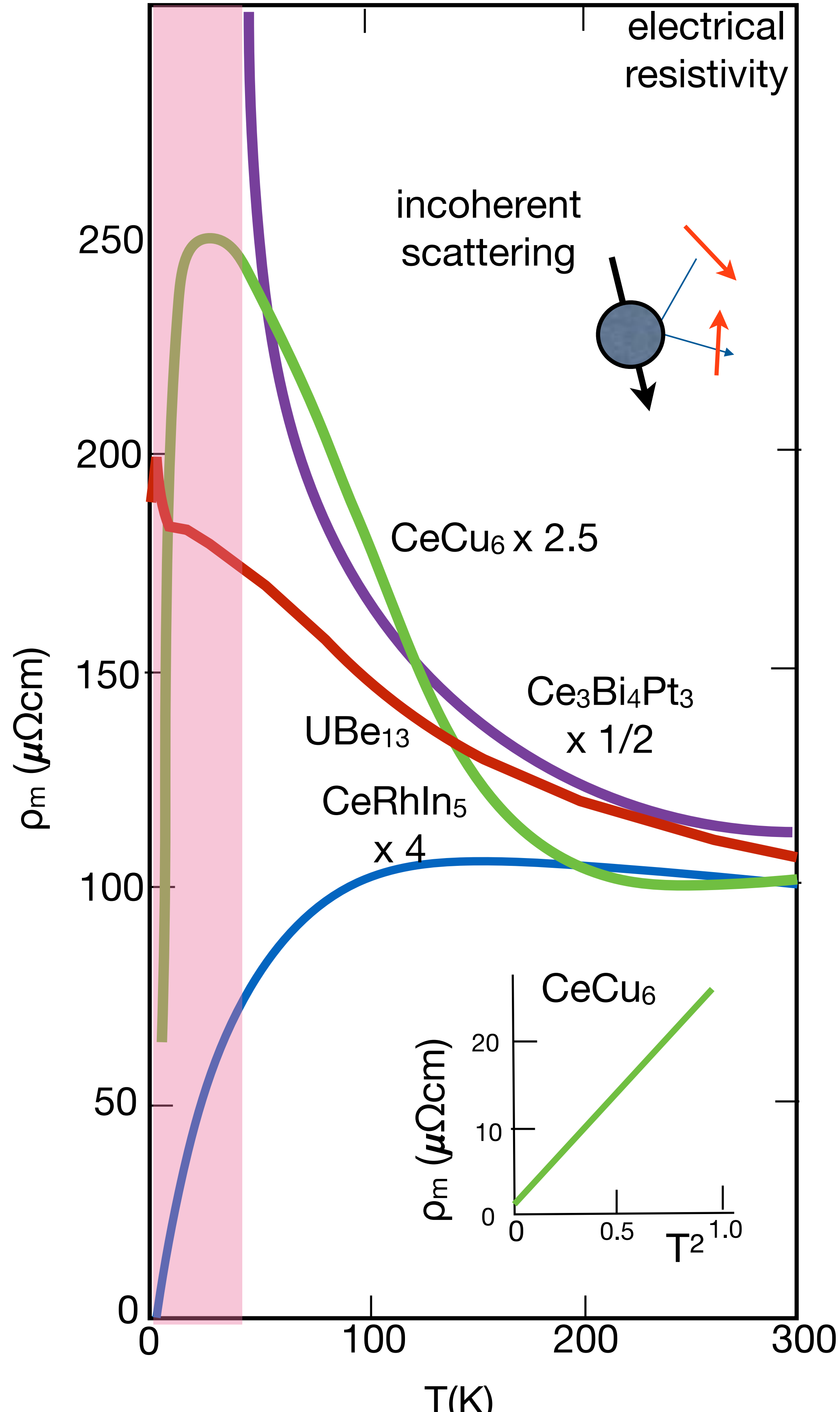

$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

DONIACH'S

Hypothesis.

Doniach (1977)

$$H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_j (\psi^{\dagger}_j \vec{\sigma} \psi_j) \cdot \vec{S}_j$$

Kondo Lattice Model
(Kasuya, 1951)

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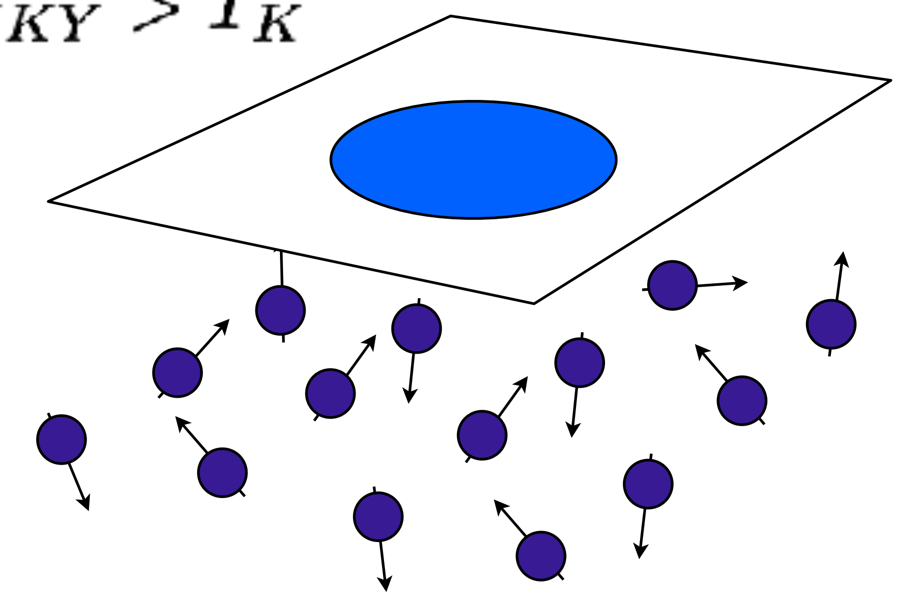
$T_{RKKY} > T_K$



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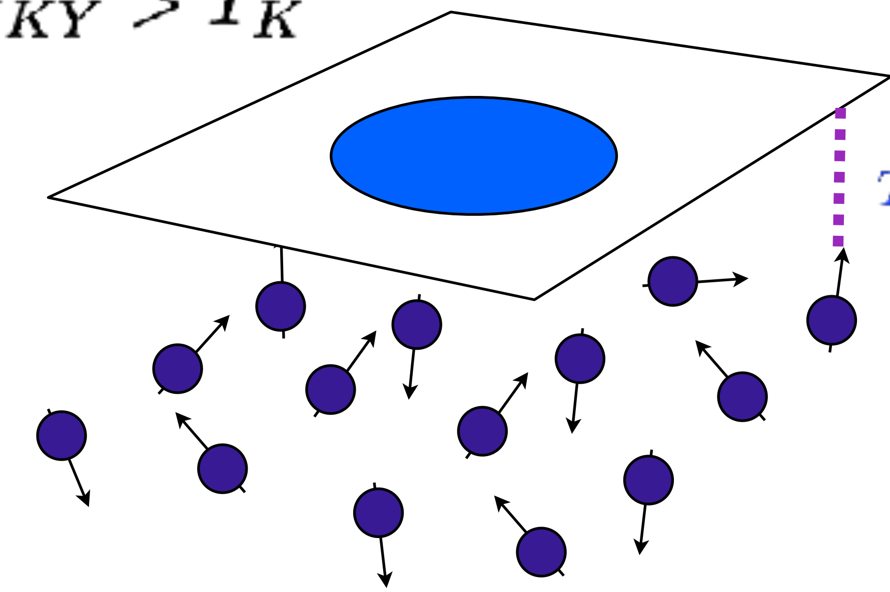

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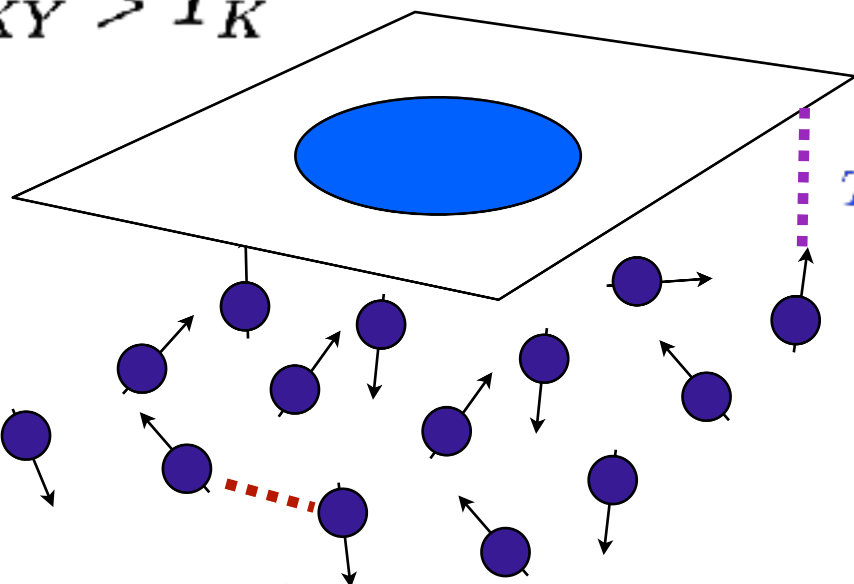

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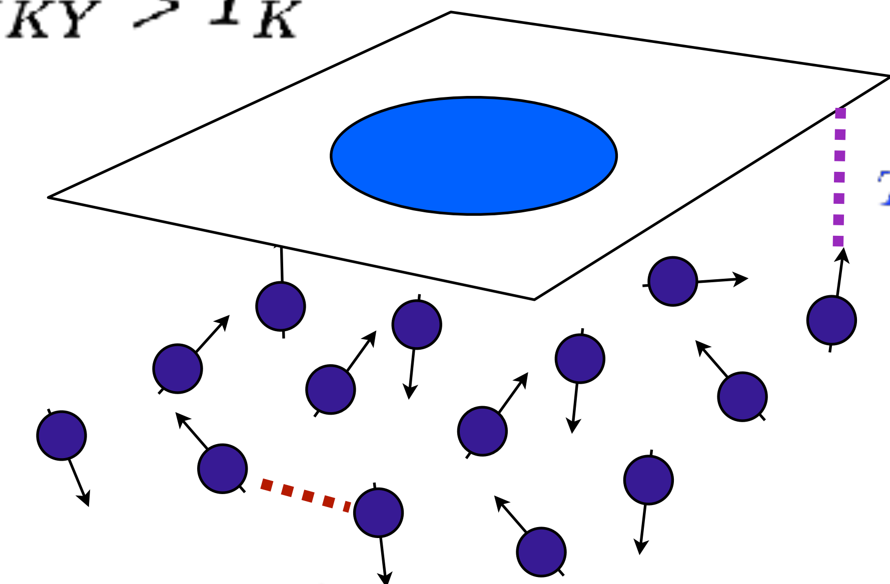

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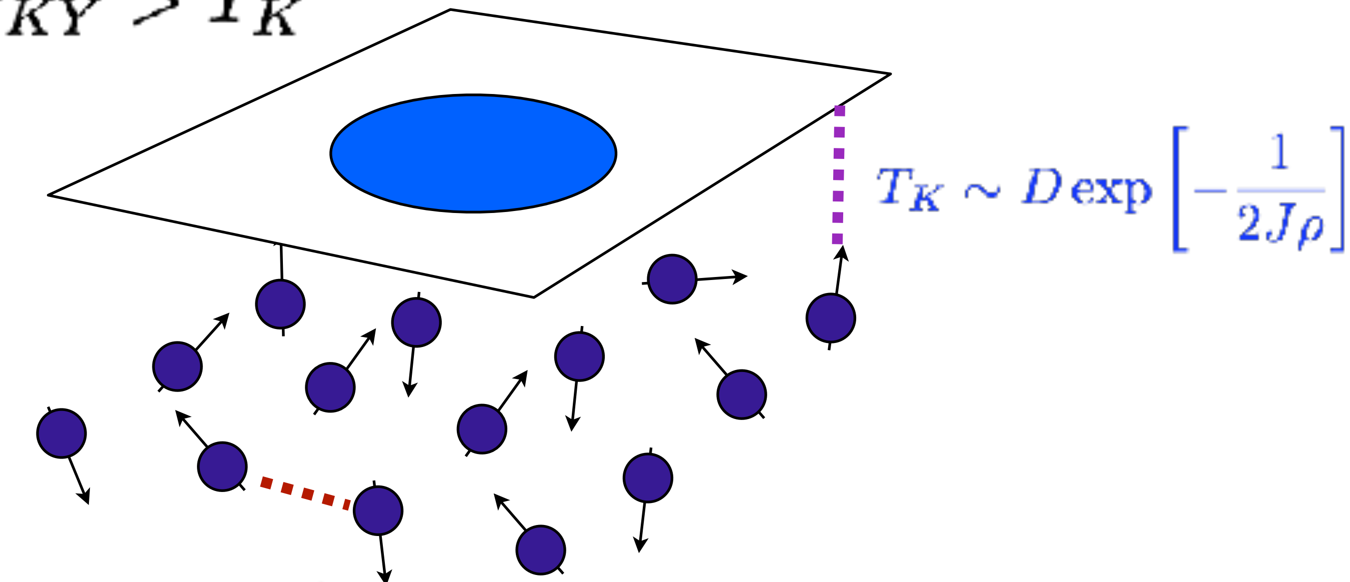

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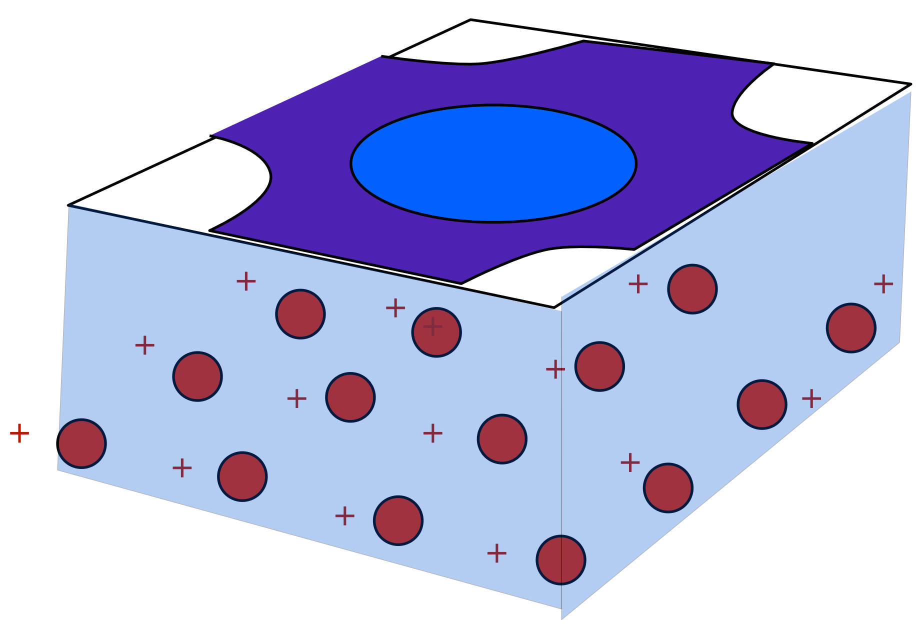
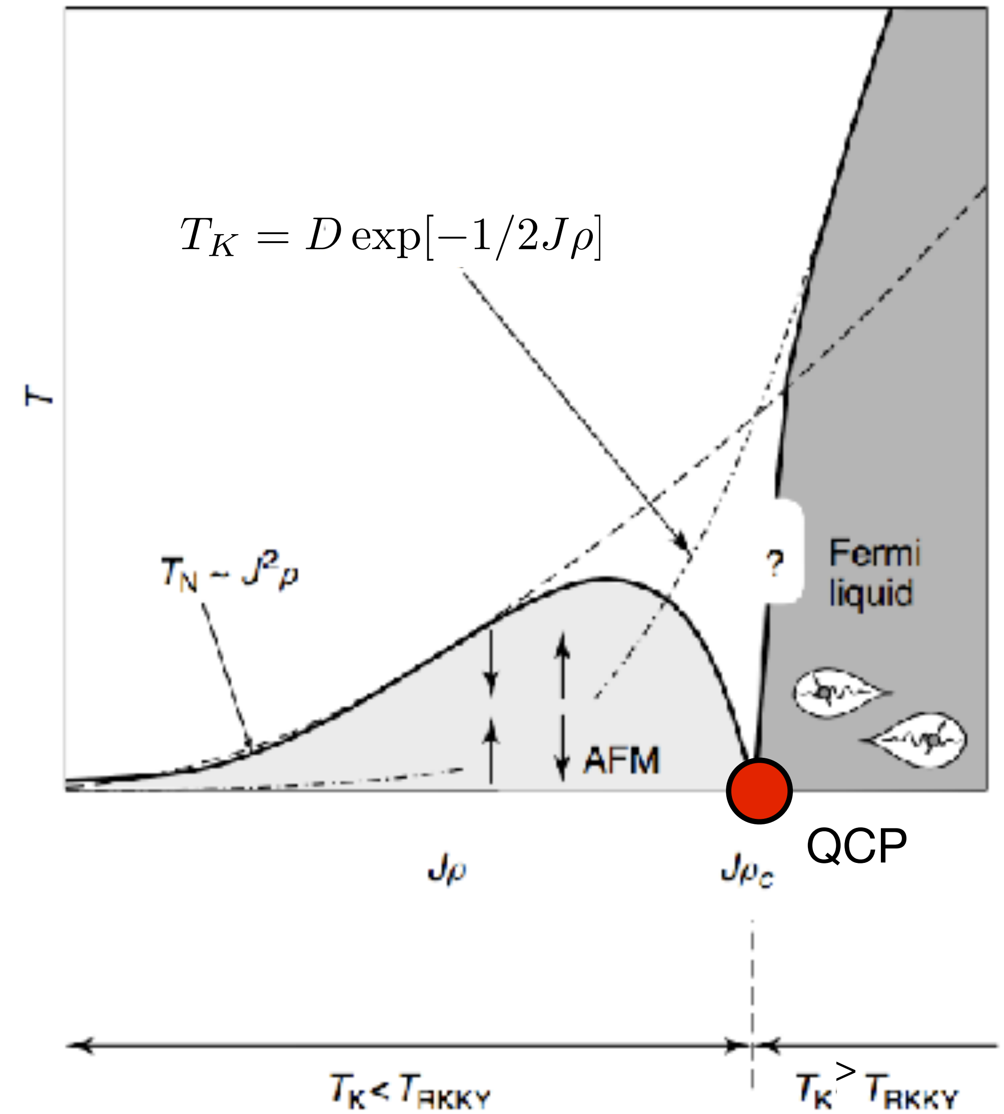
Large Fermi surface of heavy Fermions





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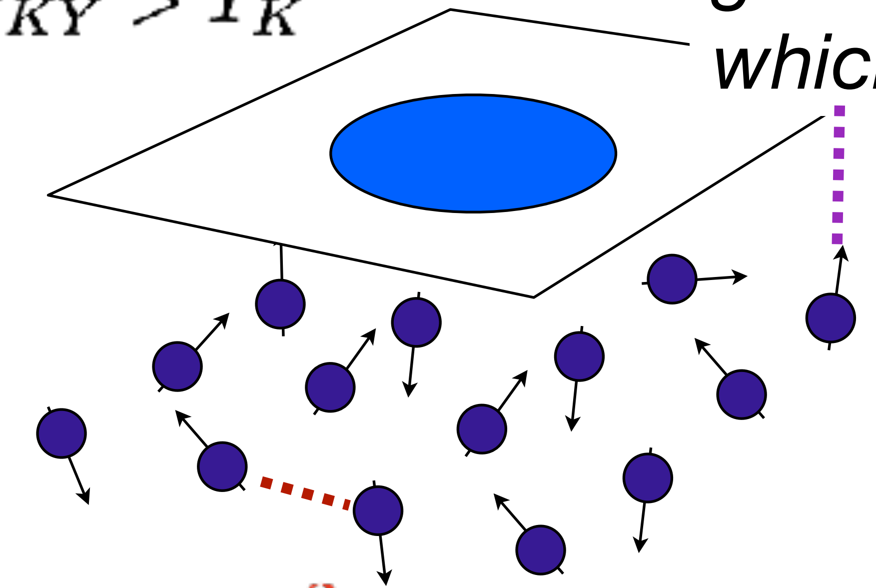

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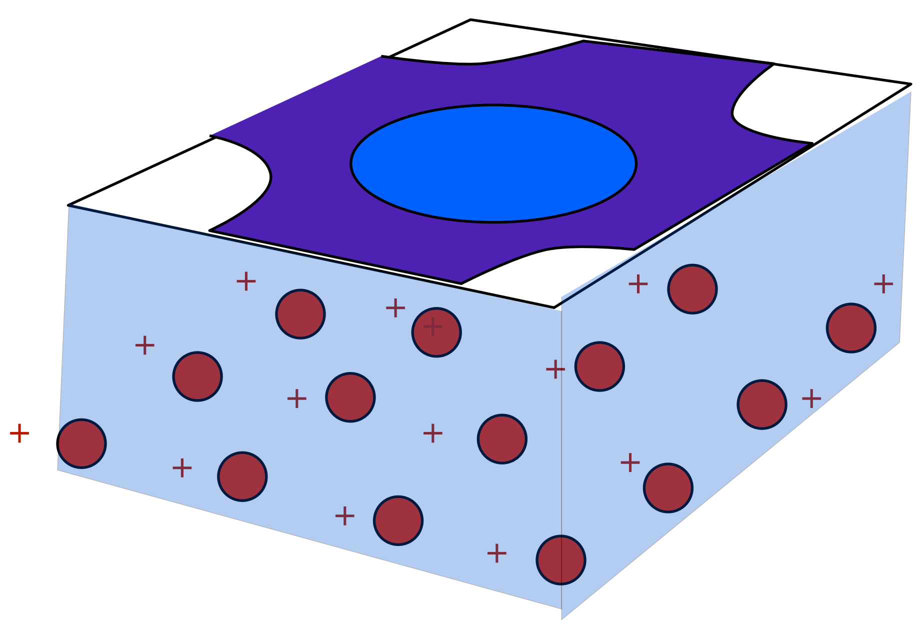

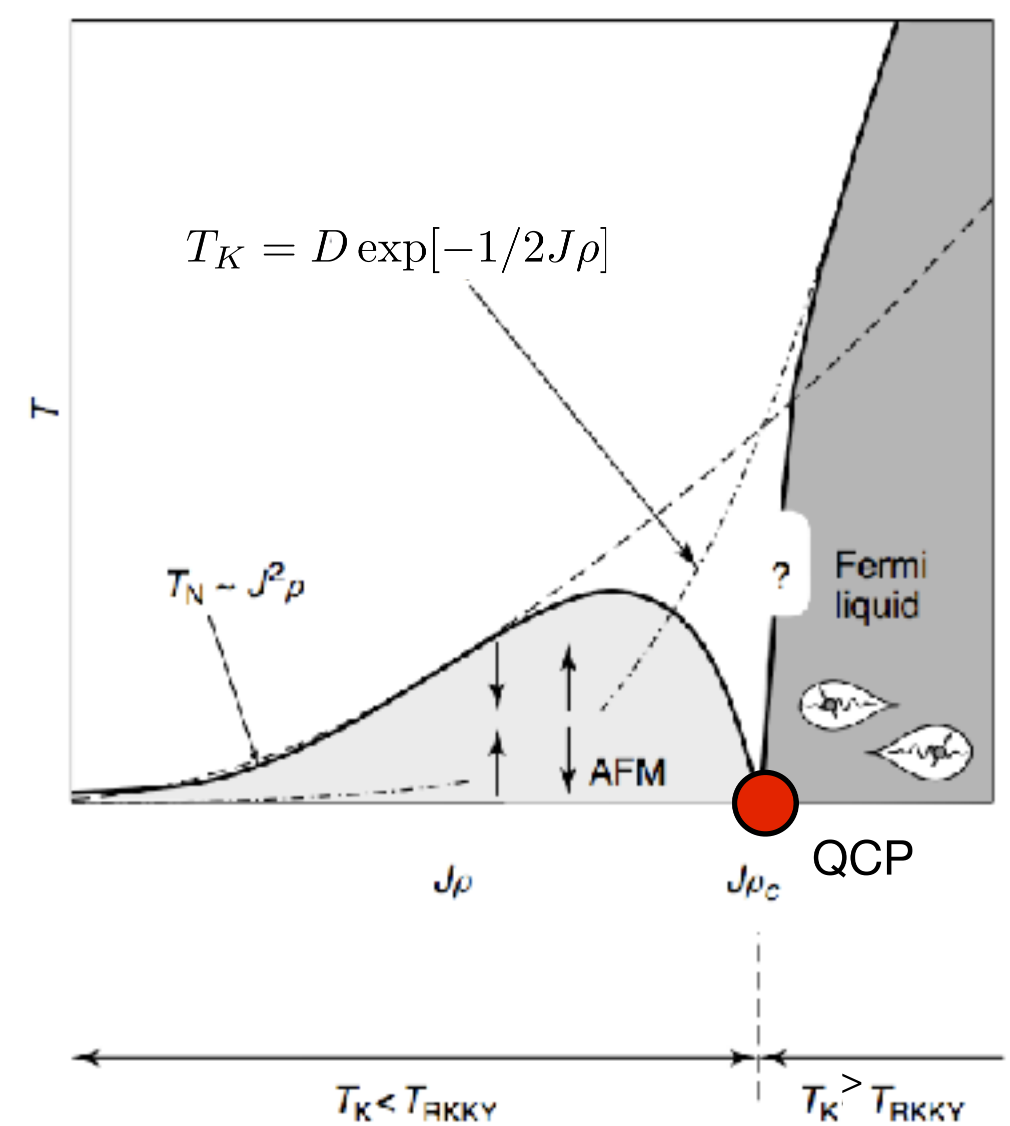

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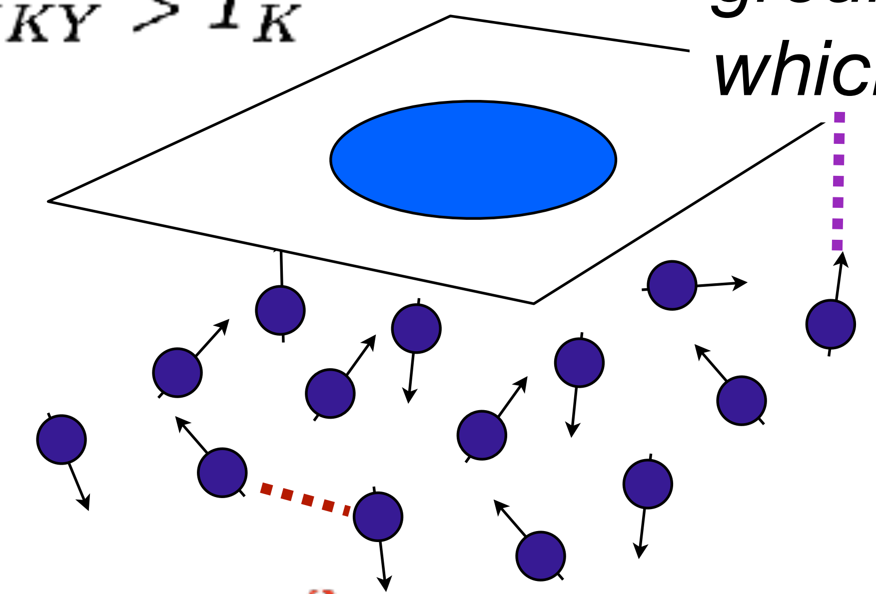

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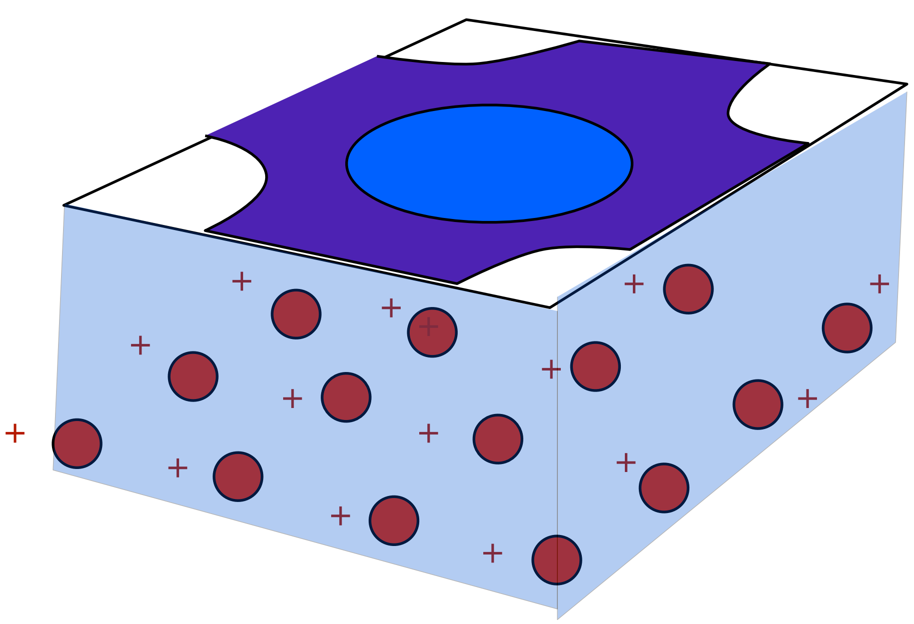

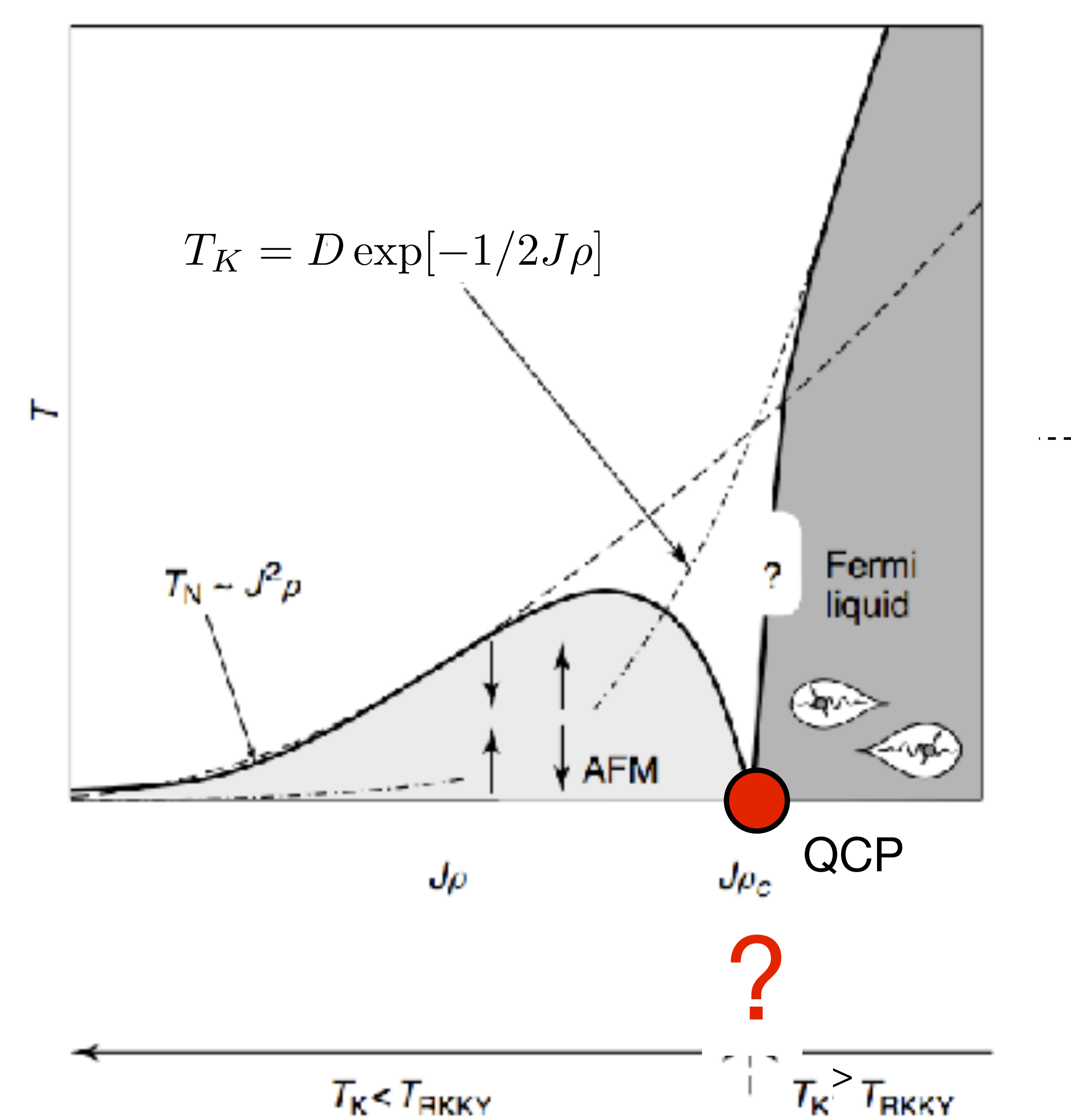

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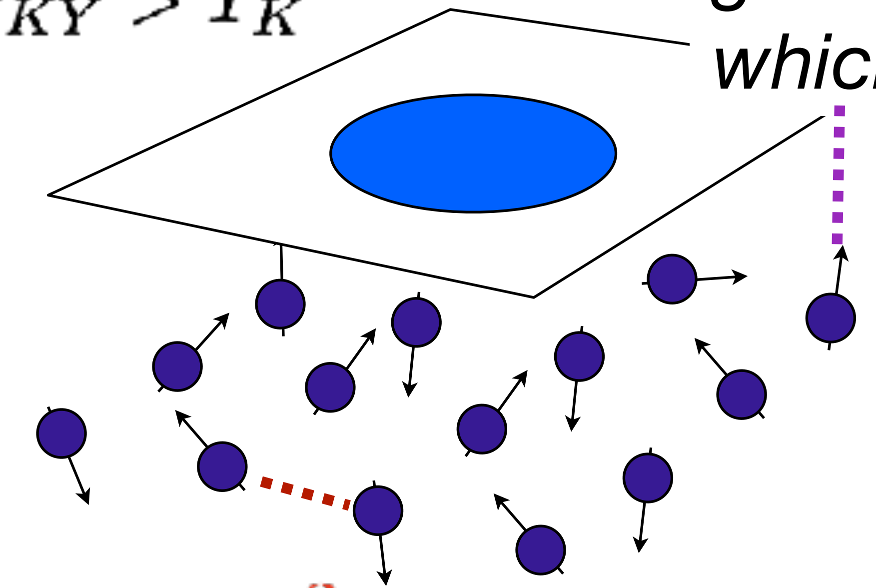

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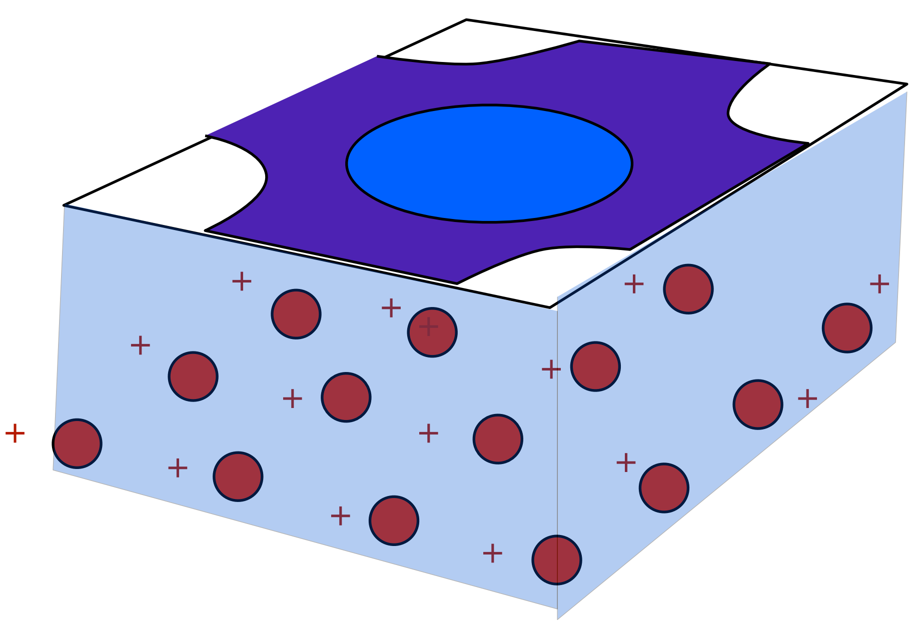
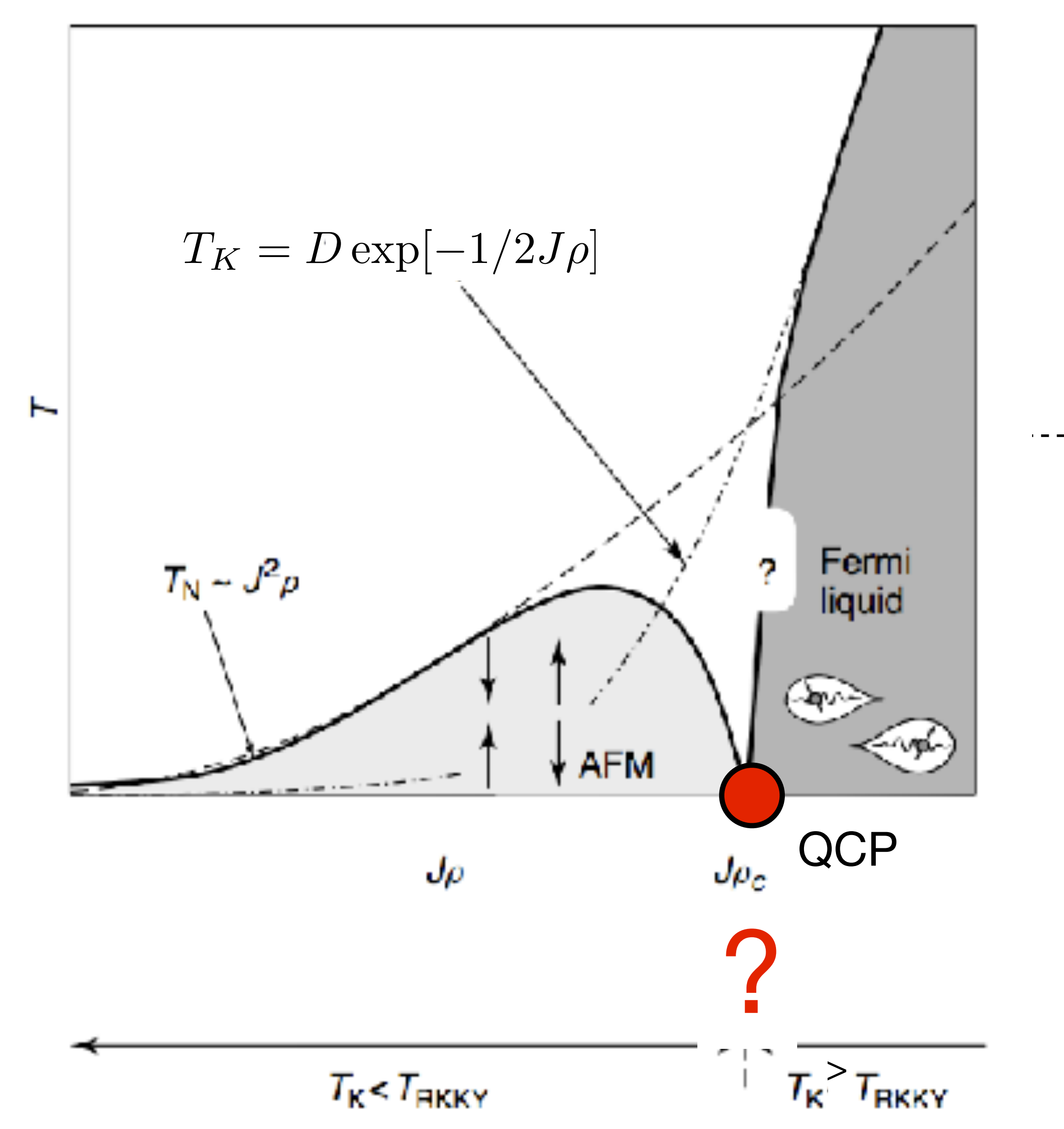
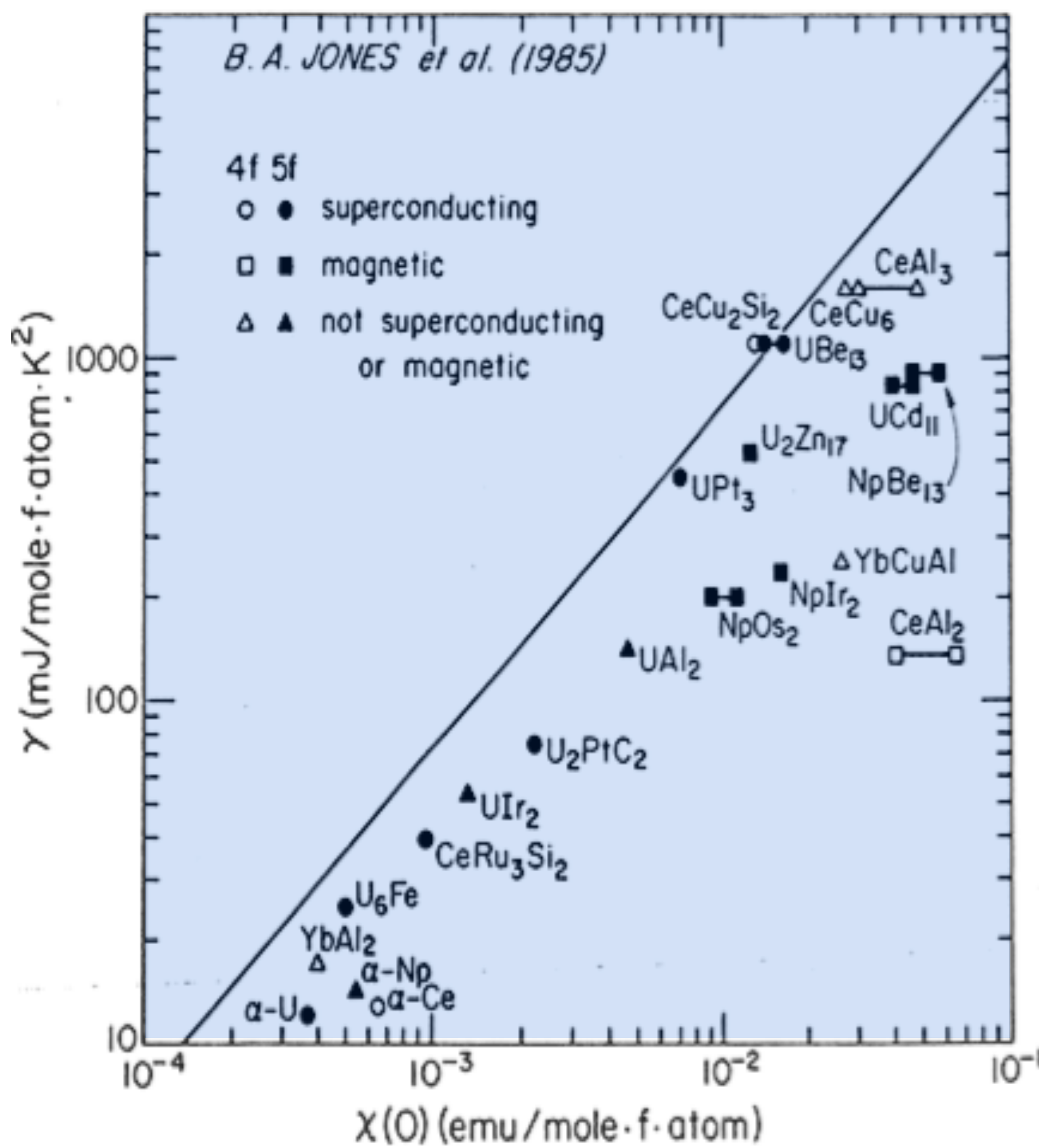




FRACTIONALIZATION?

Heavy Fermions: magnetically polarizable Landau Fermi liquids.

$$E_{\mathbf{p}} = \frac{p^2}{2m^*}, \qquad N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$



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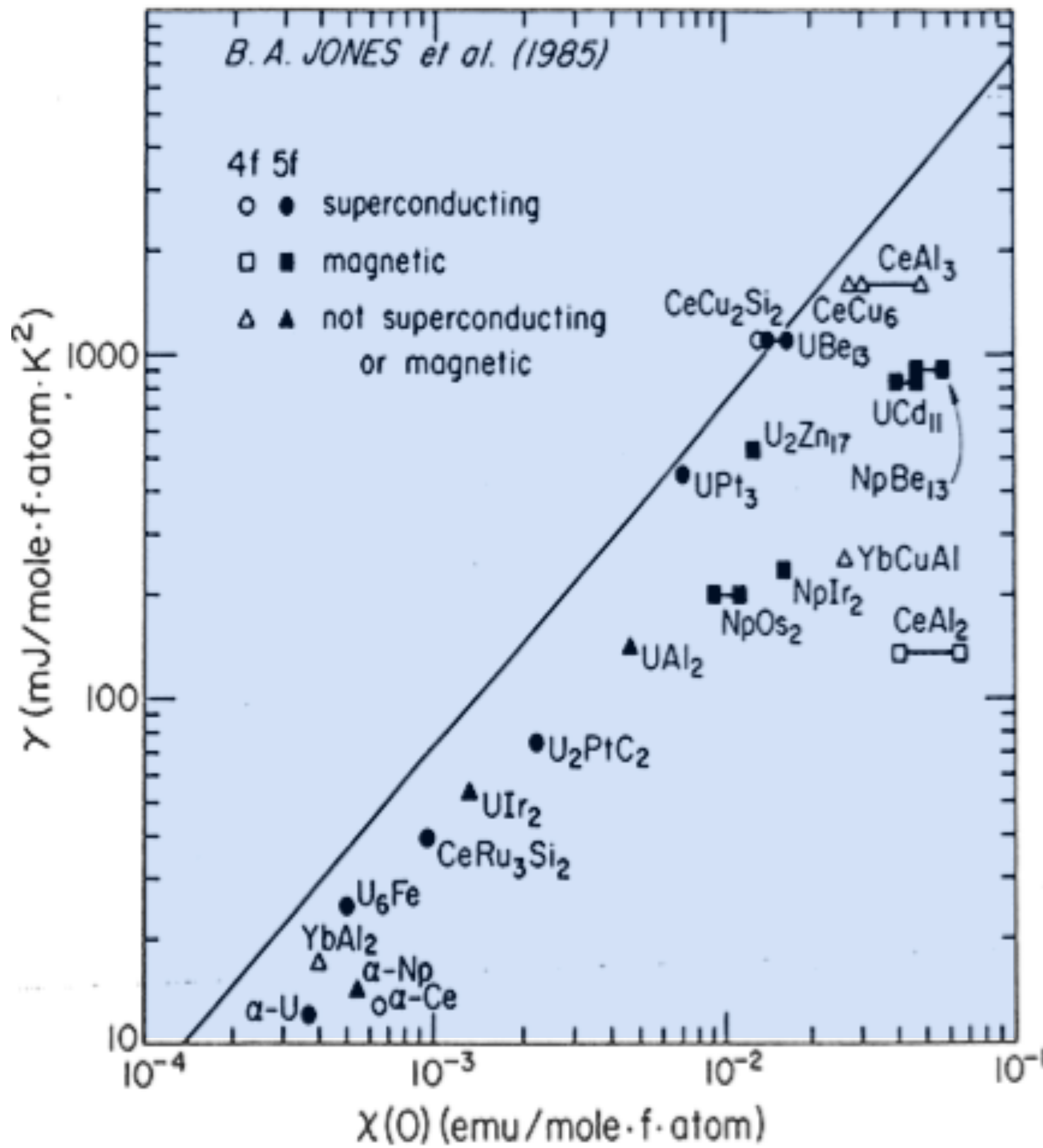

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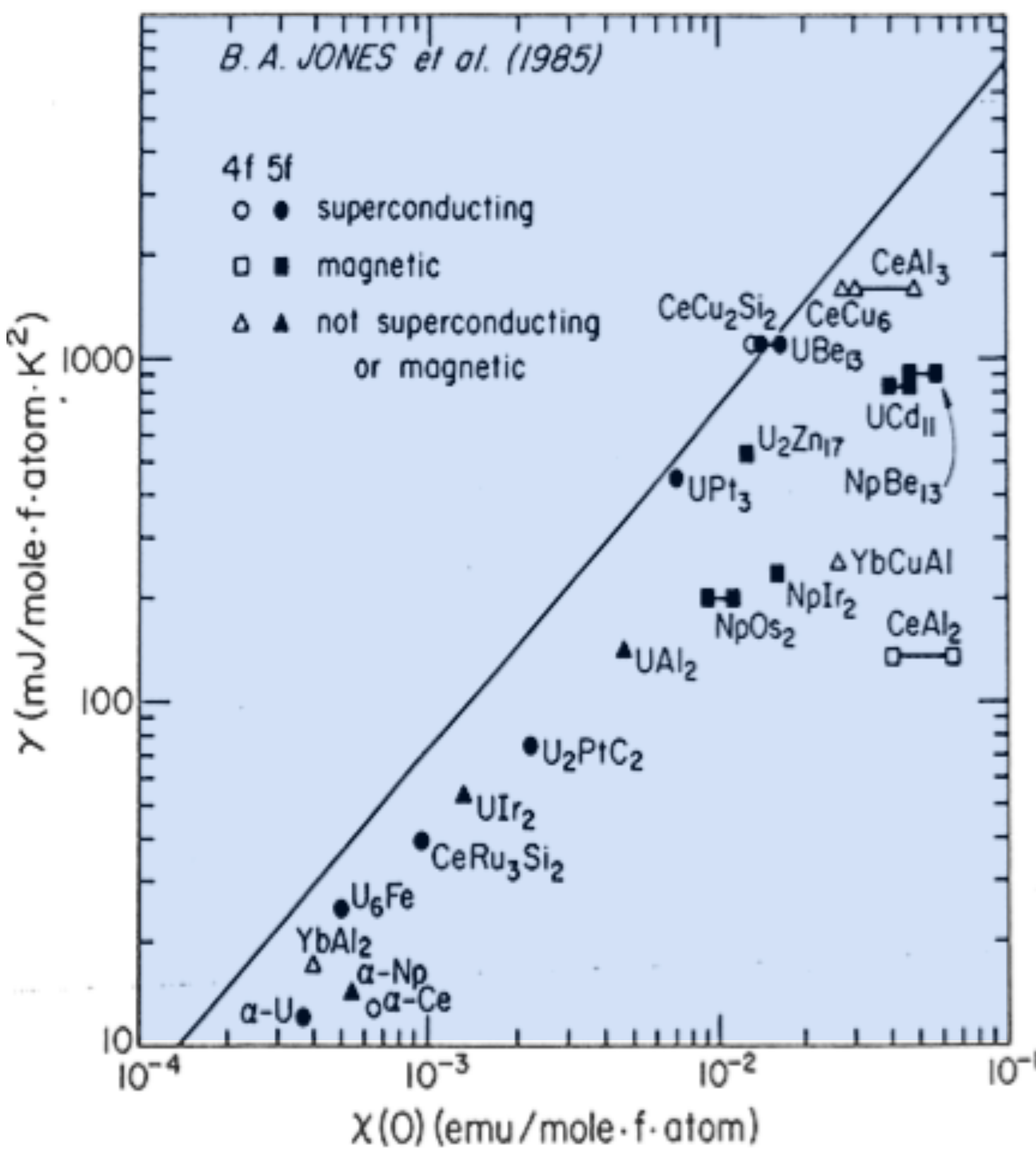

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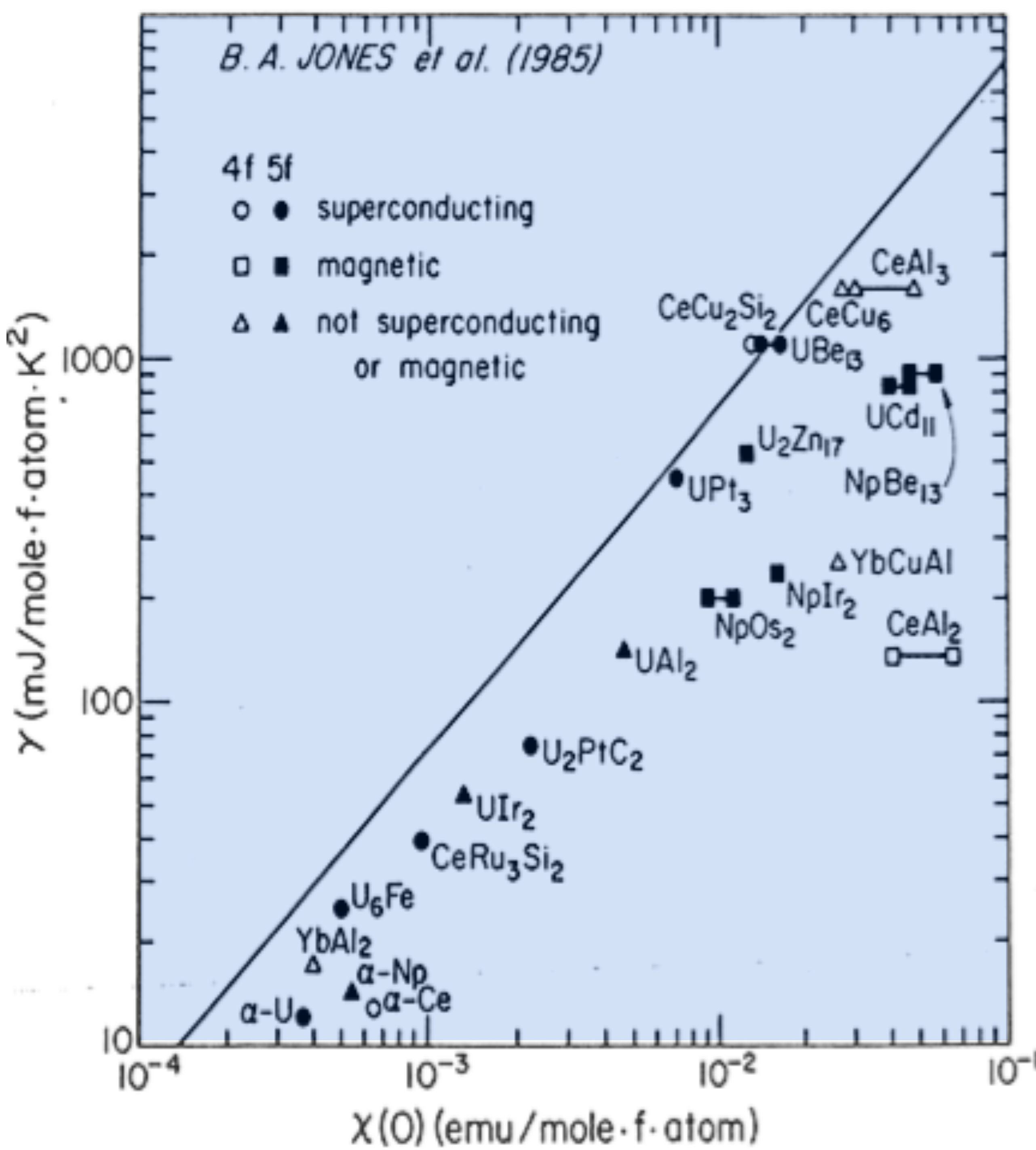
“Wilson” or “Sommerfeld” ratio.



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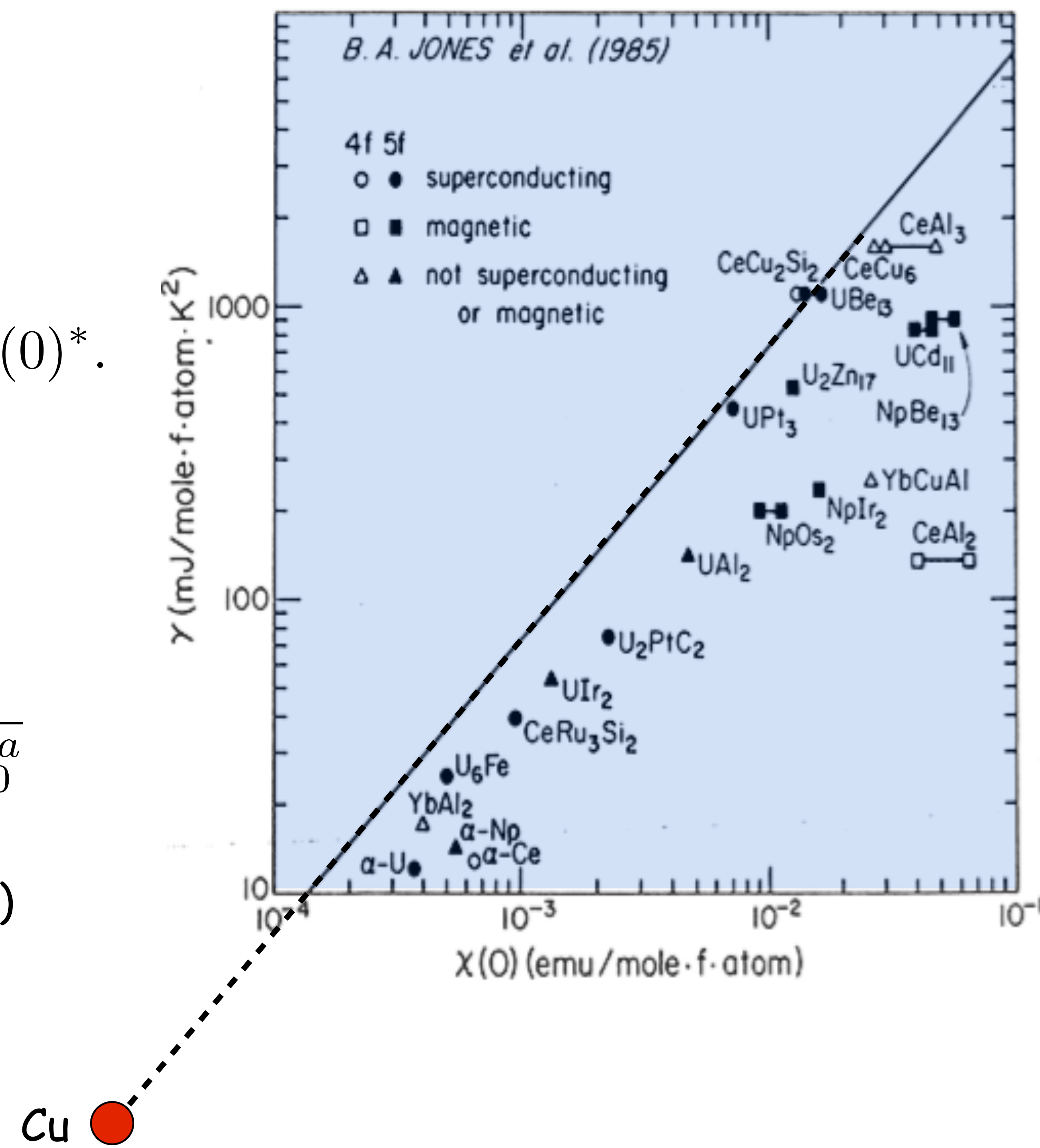
eg Cu vs $CeCu_6$ (copper, spin doped)

γCu ~ 1 mJ/mol/K^2,



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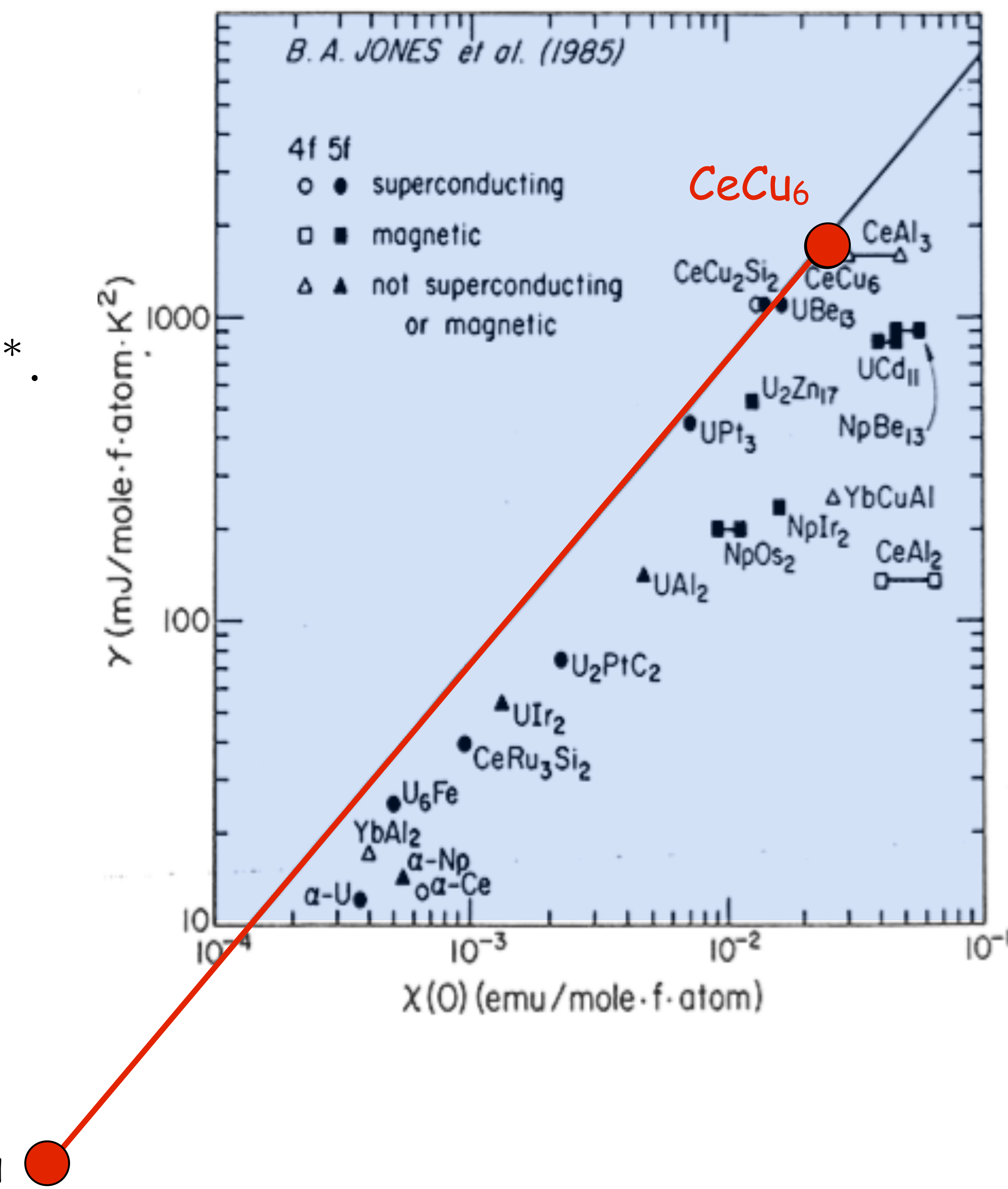

eg Cu vs $CeCu_6$ (copper, spin doped)

$\gamma Cu \sim 1$ mJ/mol/K^2,

$\Upsilon[CeCu_6] \sim 1000$ mJ/mol/K^2,

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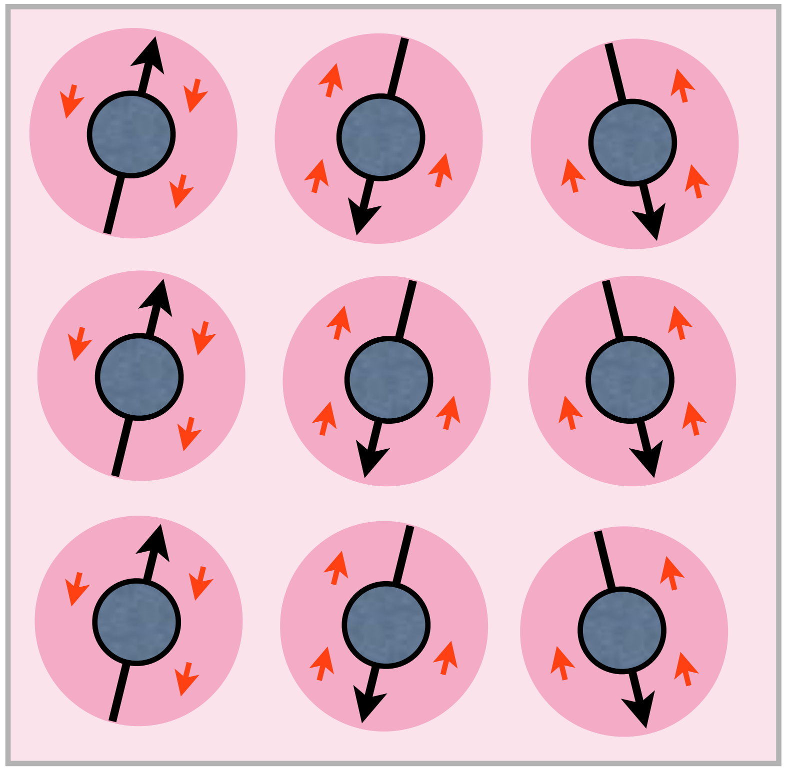
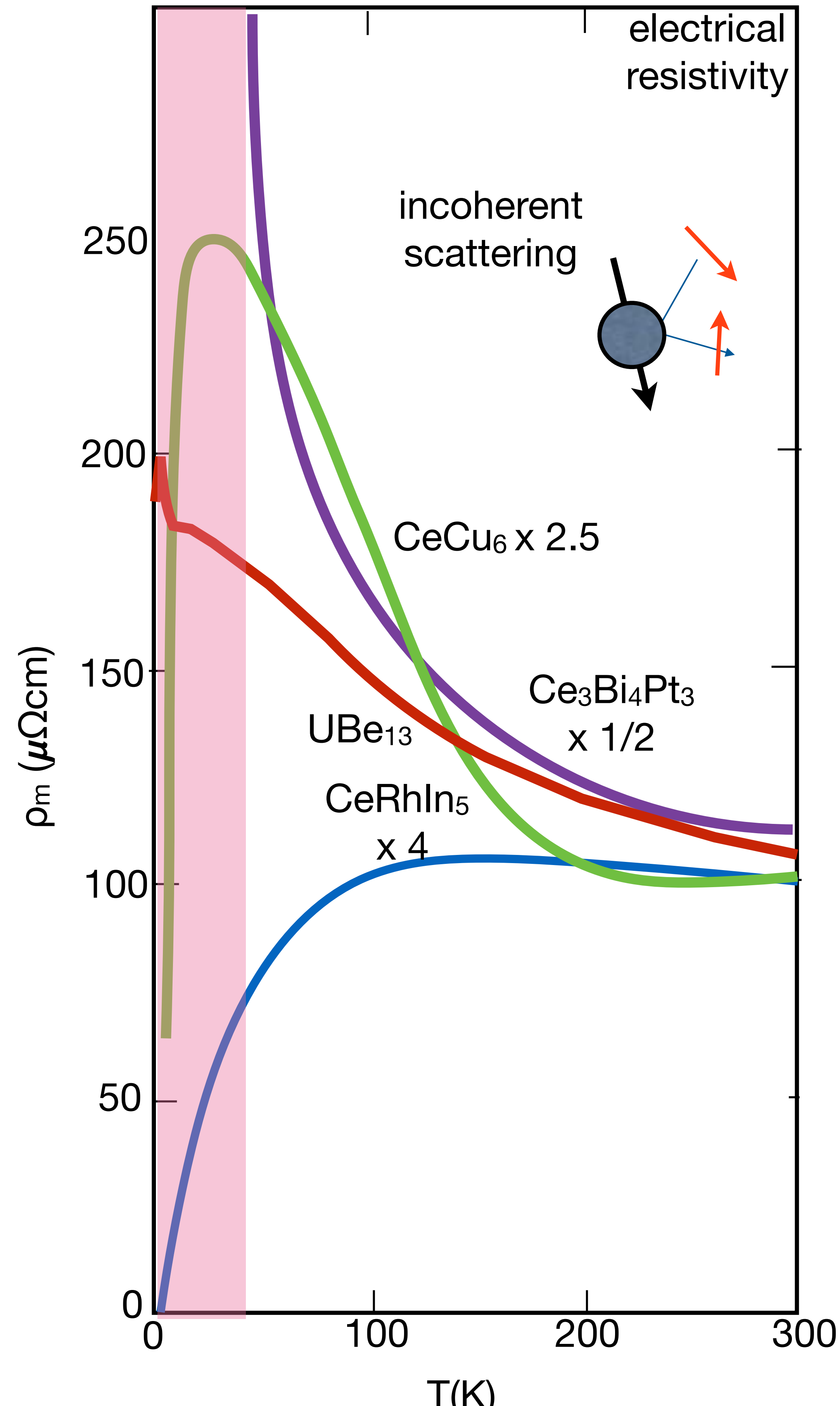


"Kondo Lattice"

Entangled spins and electrons

→ **Heavy Fermion Metals**



$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

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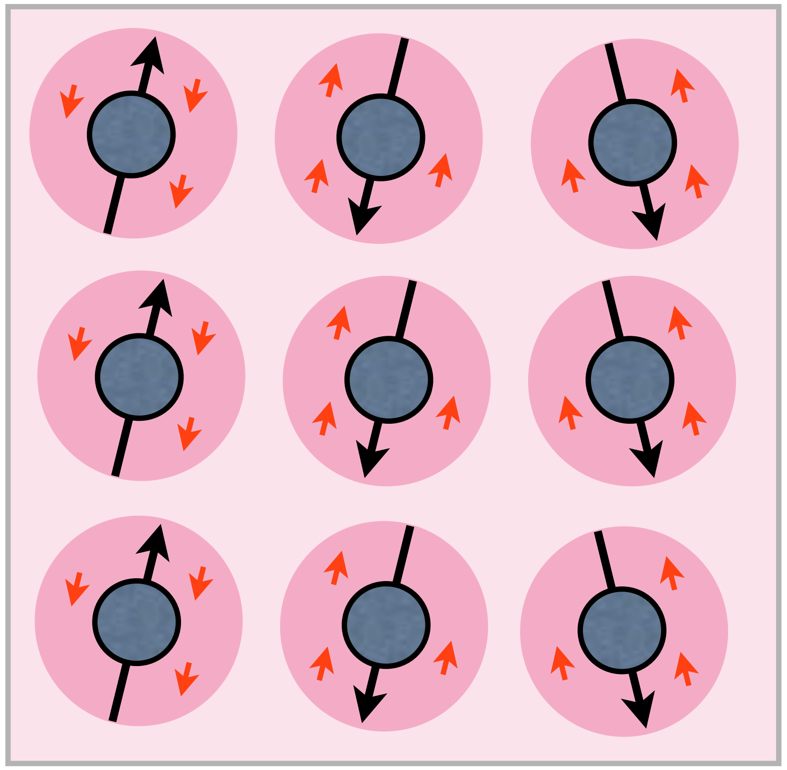
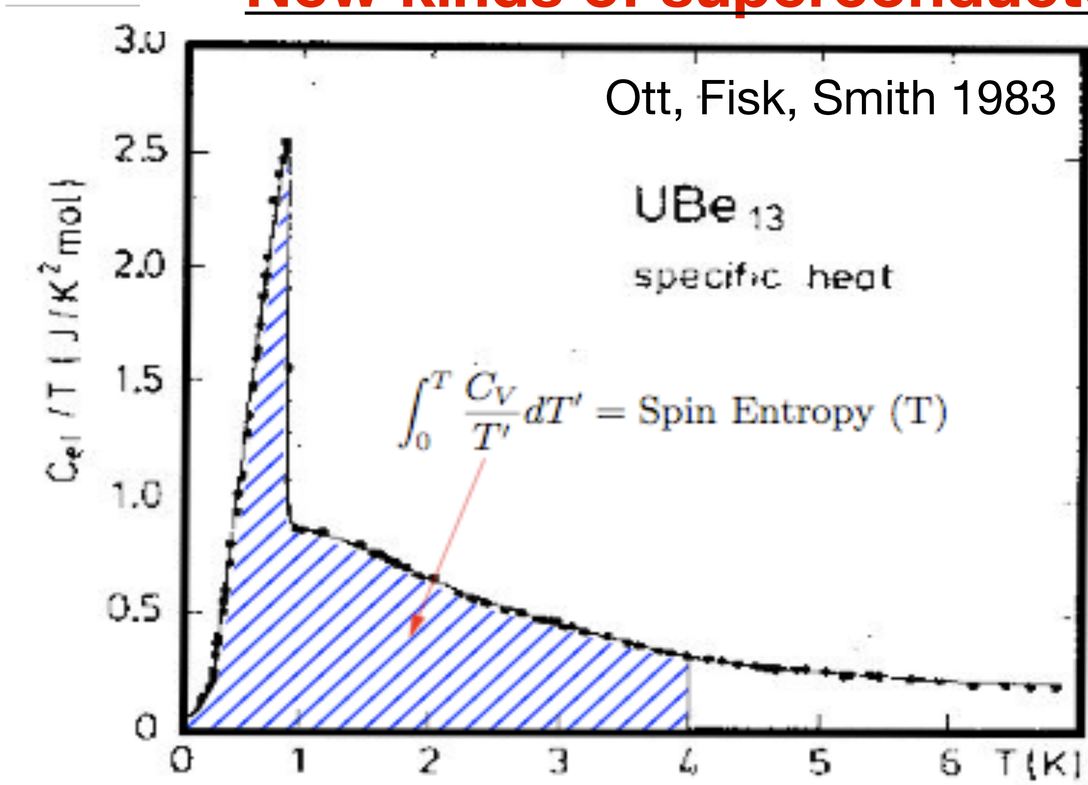
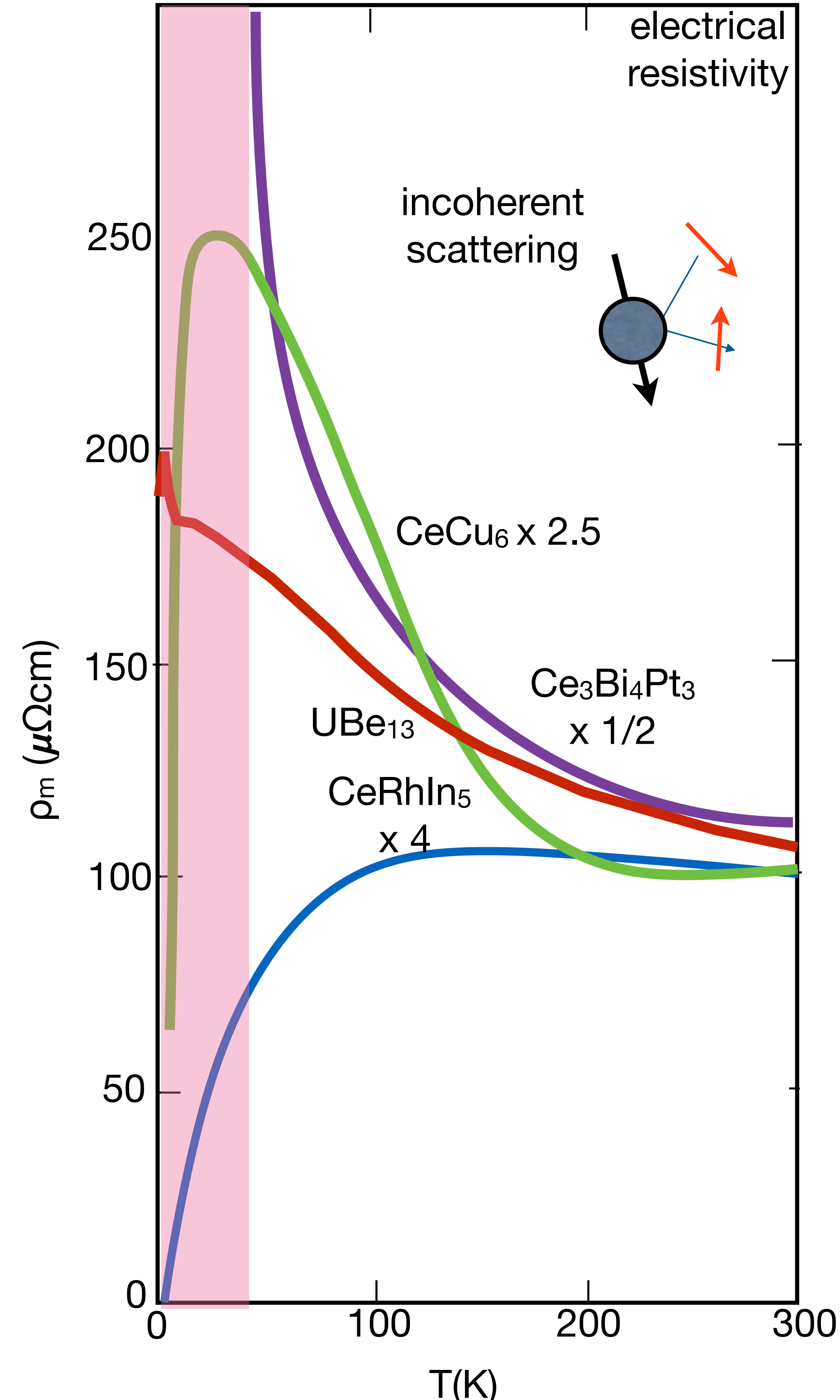


“Kondo Lattice”

Entangled spins and electrons

→ **New kinds of superconductor**





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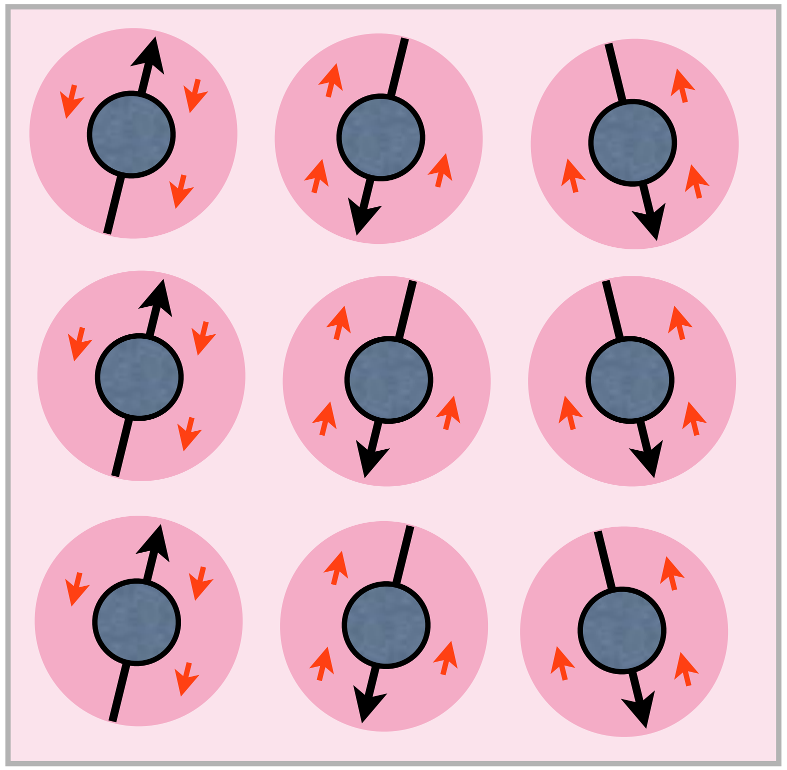

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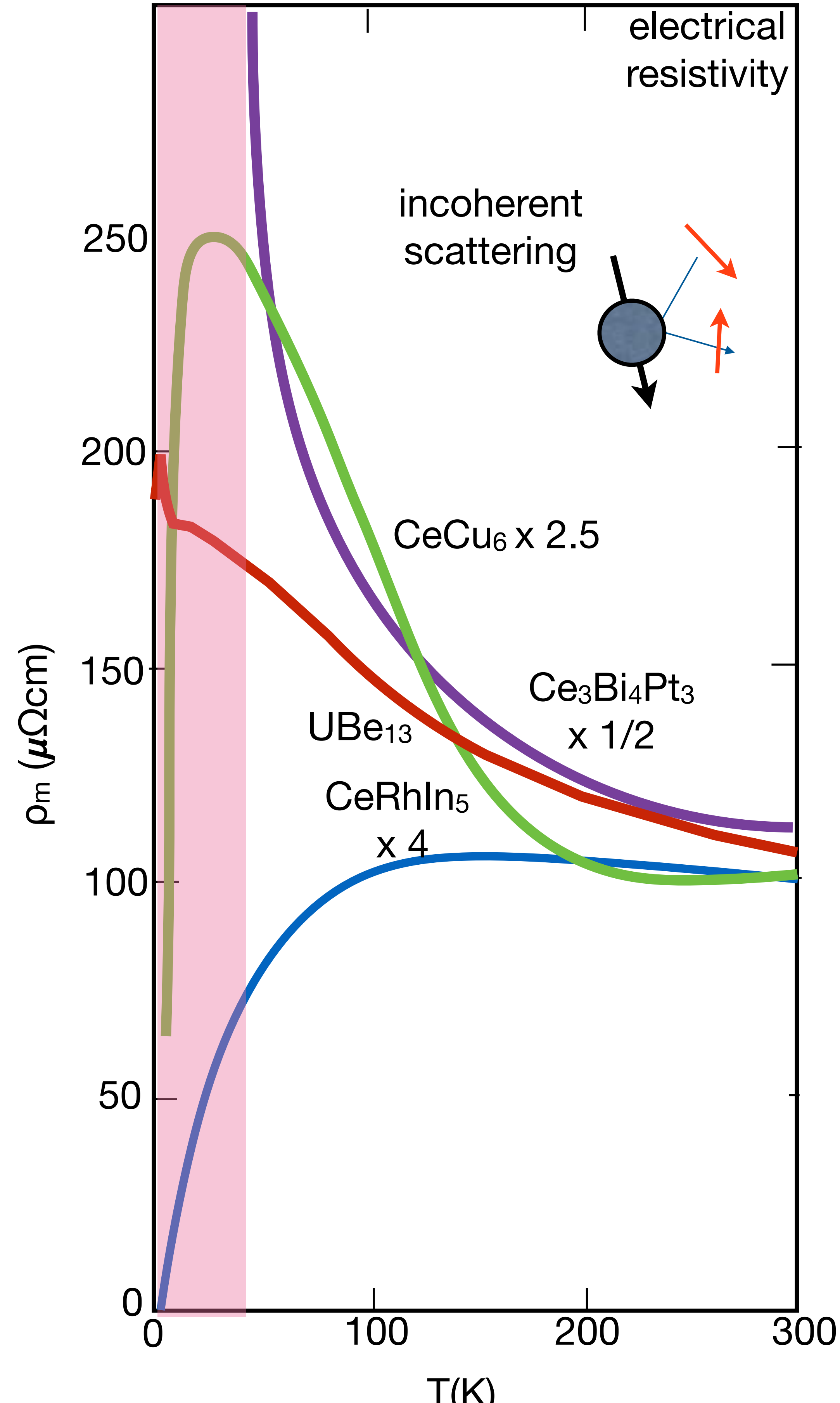

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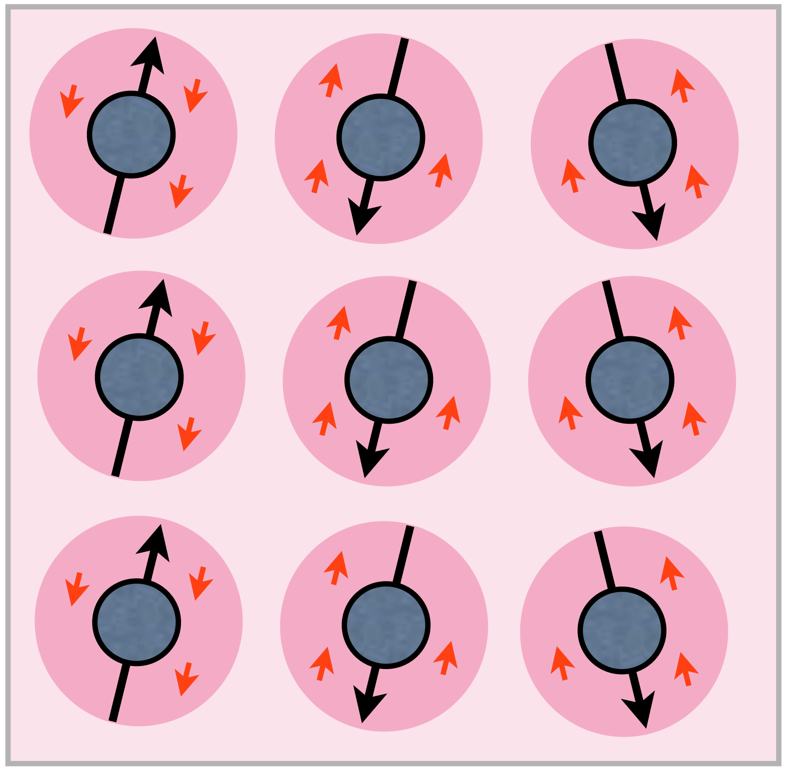
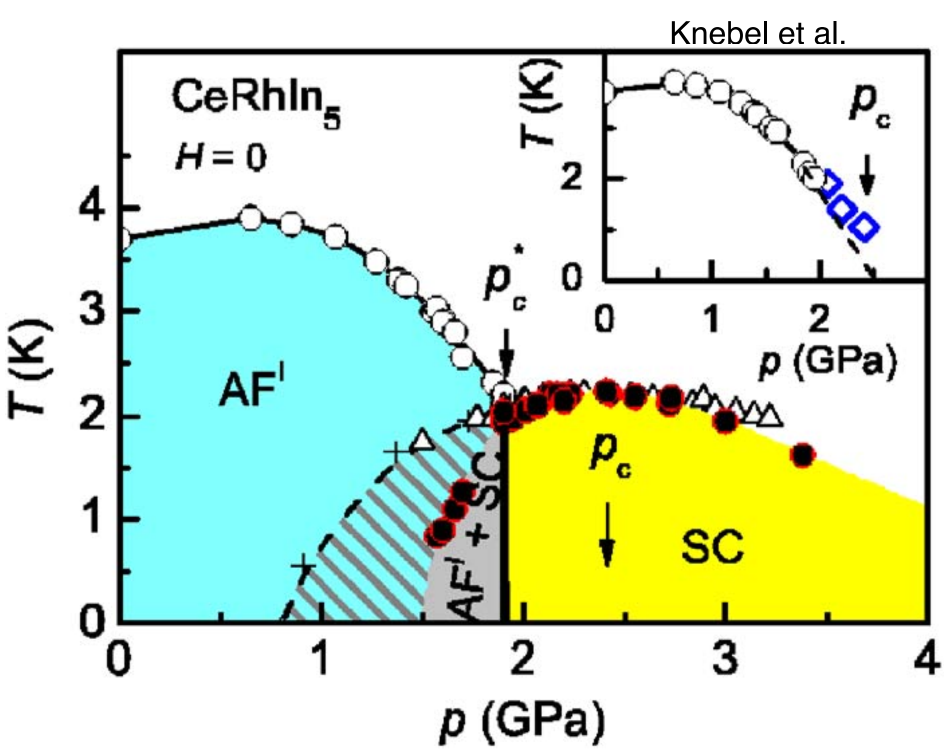
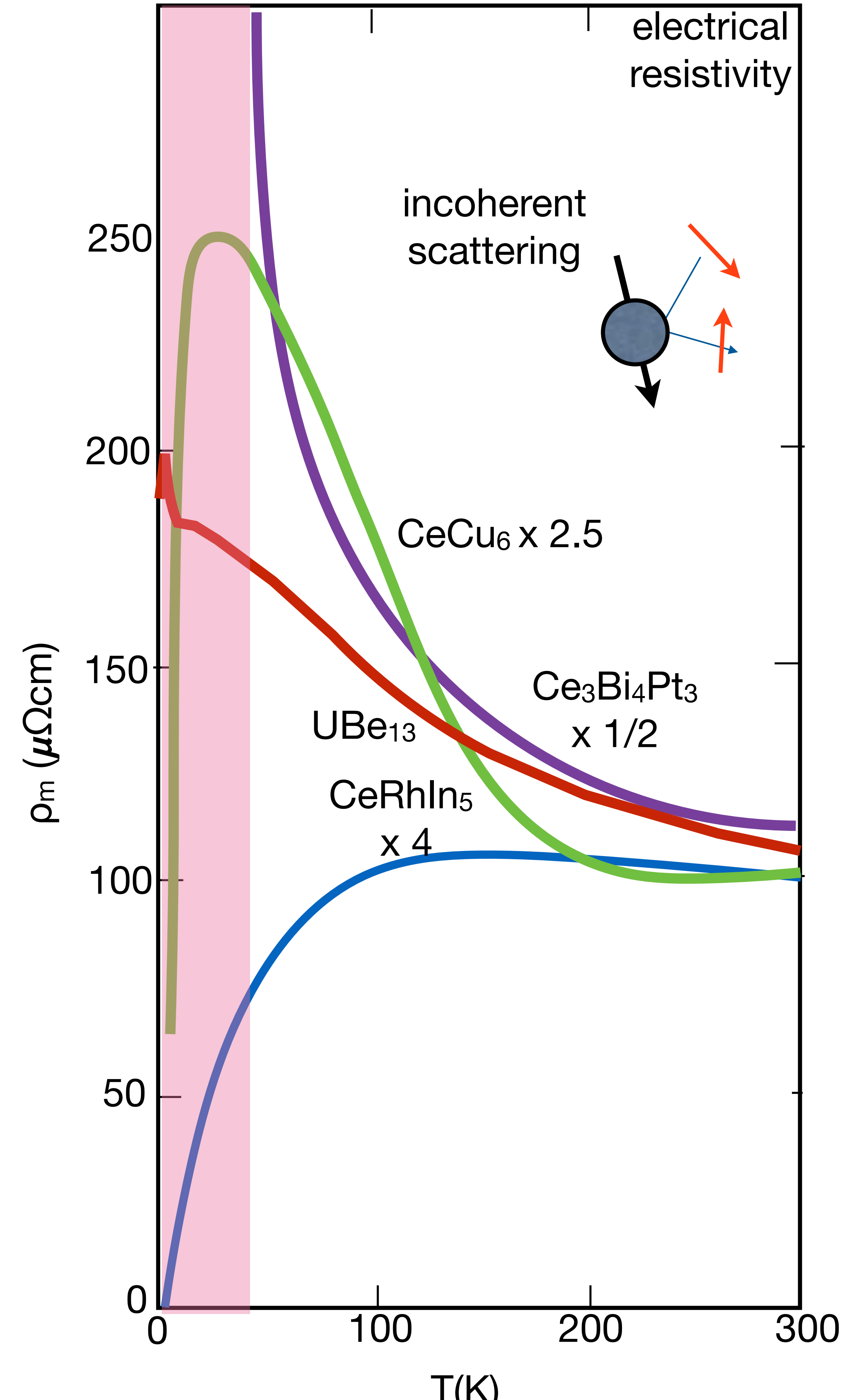


“Kondo Lattice”

Entangled spins and electrons

→ **AFM/Superconductivity**





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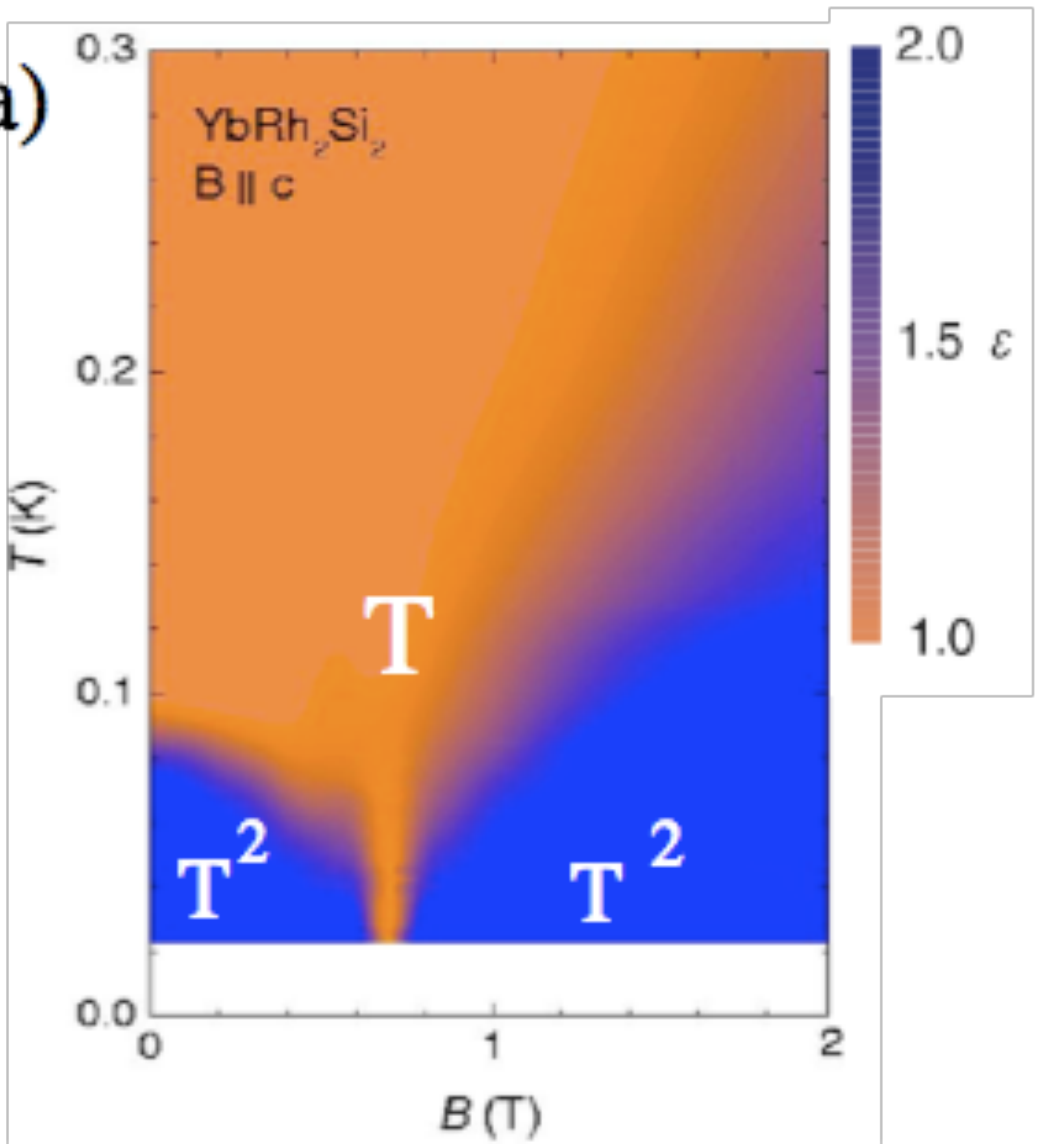
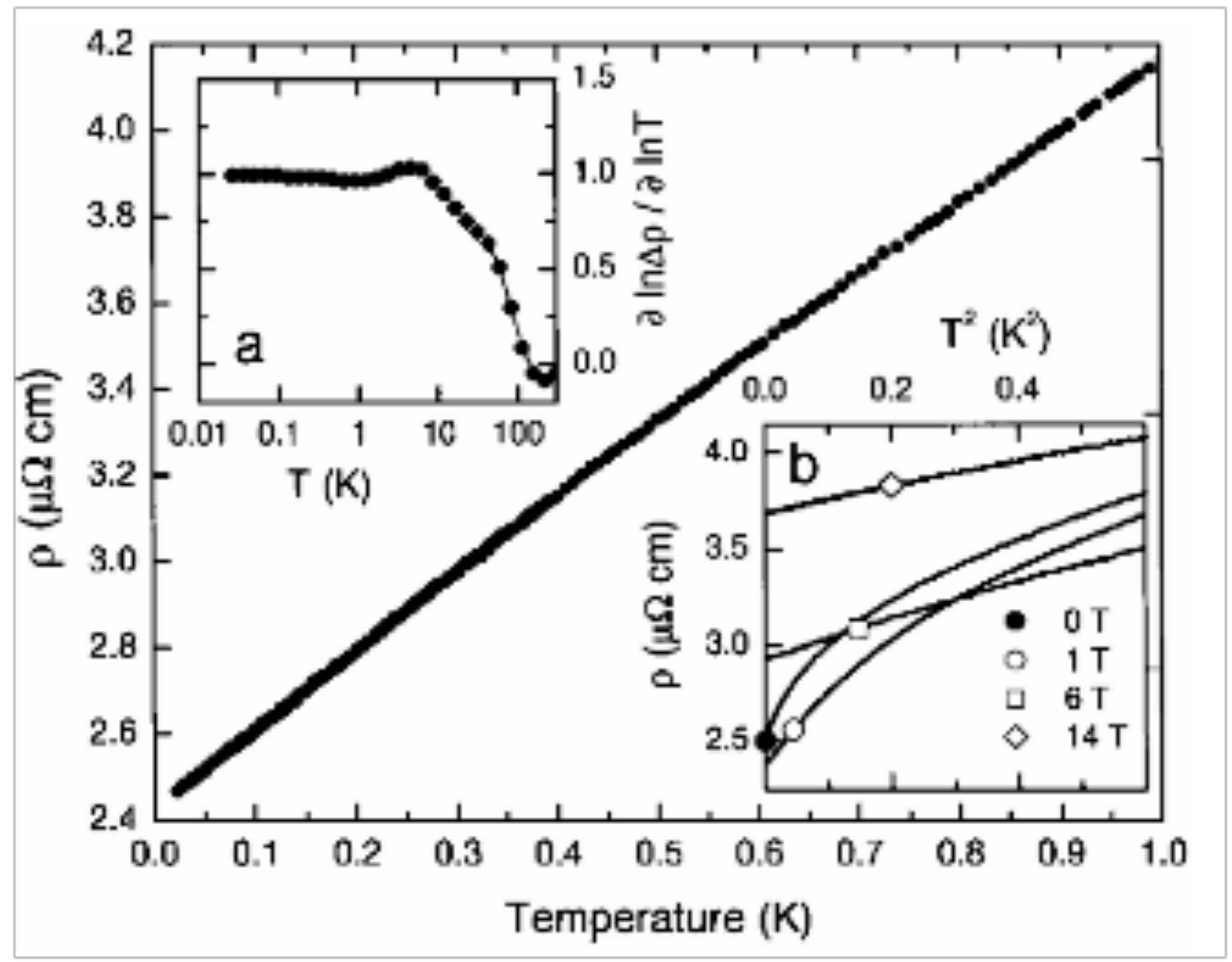
Coherent Heavy Fermions

$YbRh_2Si_2$: Field tuned quantum criticality.

(a)



(b)



Custers et al, (2003)

Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. **<u>From Anderson to Kondo</u>**
4. Kondo Insulators: the simplest heavy fermions.
5. Oshikawa's Theorem.
6. Large N expansion for the Kondo Lattice
7. Heavy Fermion Superconductivity
8. Topological Kondo Insulators
9. Co-existing magnetism and the Kondo Effect.

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Please ask questions!

Rutgers Center for Materials Theory

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Kondo (1962)

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Kondo (1962)

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Anderson 1961

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Kondo (1962)

From Anderson to Kondo

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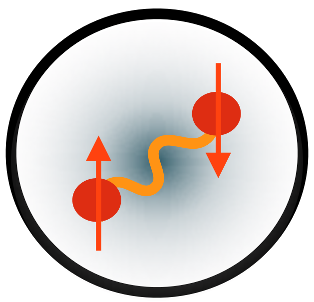
Kondo (1962)

From Anderson to Kondo

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} \left[V c^{\dagger}_{\mathbf{k}\sigma} f_{\sigma} + V^{*} f^{\dagger}_{\sigma} c_{\mathbf{k}\sigma} \right] + E_f n_f + U n_{f\uparrow} n_{f\downarrow}$$

Anderson 1961



$$U = \frac{e^2}{4\pi\epsilon_0} \int_{\mathbf{r},\mathbf{r}'} \frac{1}{|\mathbf{r}-\mathbf{r}'|} \rho_f(\mathbf{r}) \rho_f(\mathbf{r}')$$

$$H = \sum \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma} + J(\psi^{\dagger} \vec{\sigma} \psi) \cdot \vec{S}$$

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From Anderson to Kondo

$$H_{atomic} = E_f n_f + U n_{f\uparrow} n_{f\downarrow}$$

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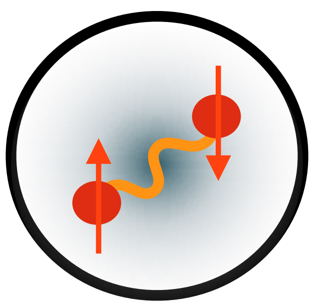

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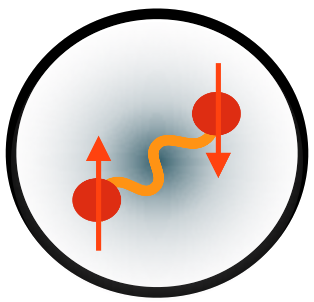

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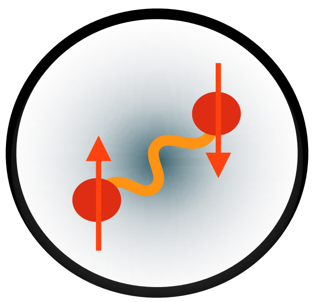

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$$\left.\begin{array}{ll} |f^2\rangle & E(f^2) = 2E_f + U \\ |f^0\rangle & E(f^0) = 0 \end{array}\right\} \text{ non-magnetic}$$

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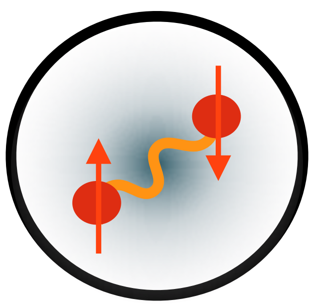

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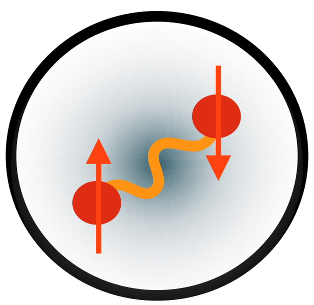

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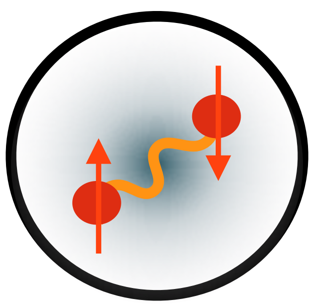

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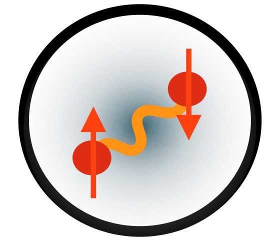
$$U/2 > |E_f + U/2|,$$

Criterion for magnetic ground-state

From Anderson to Kondo



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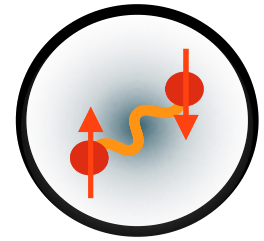

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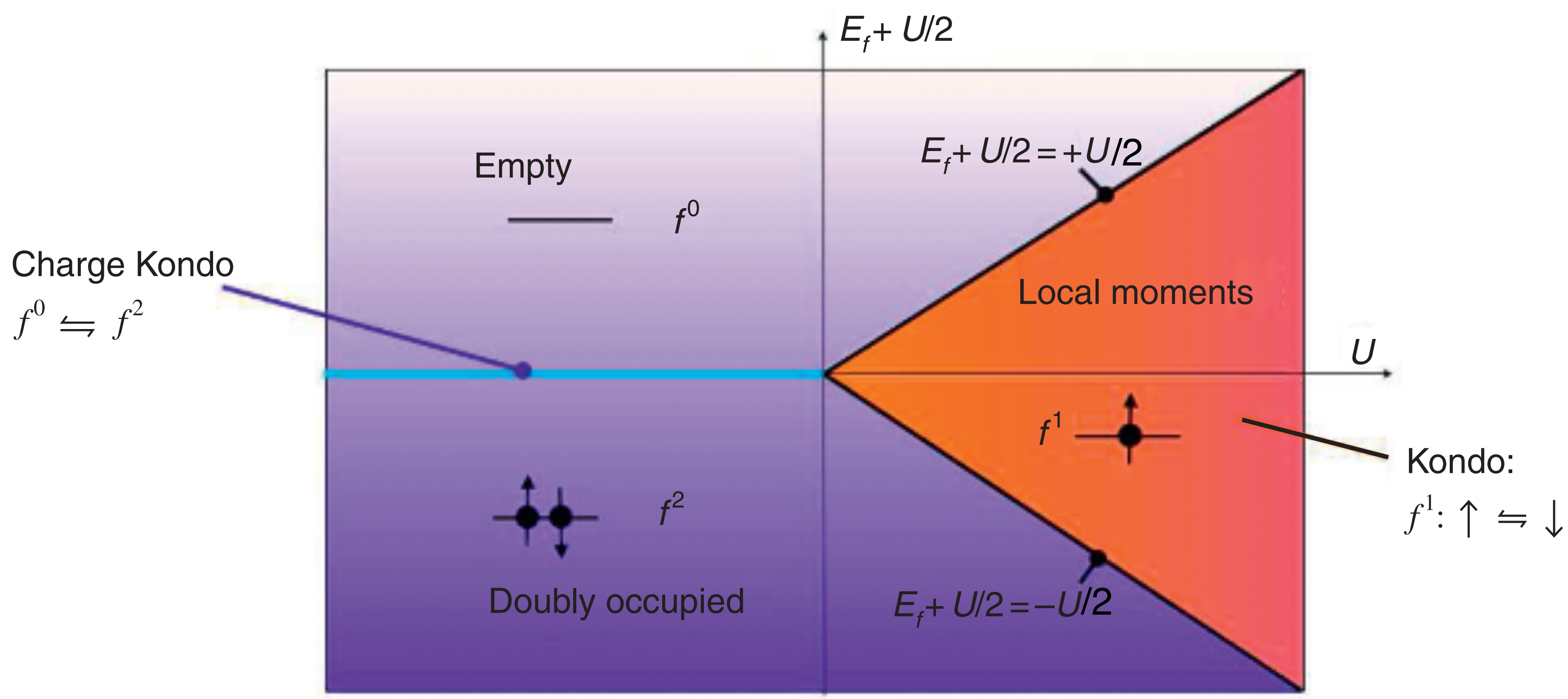

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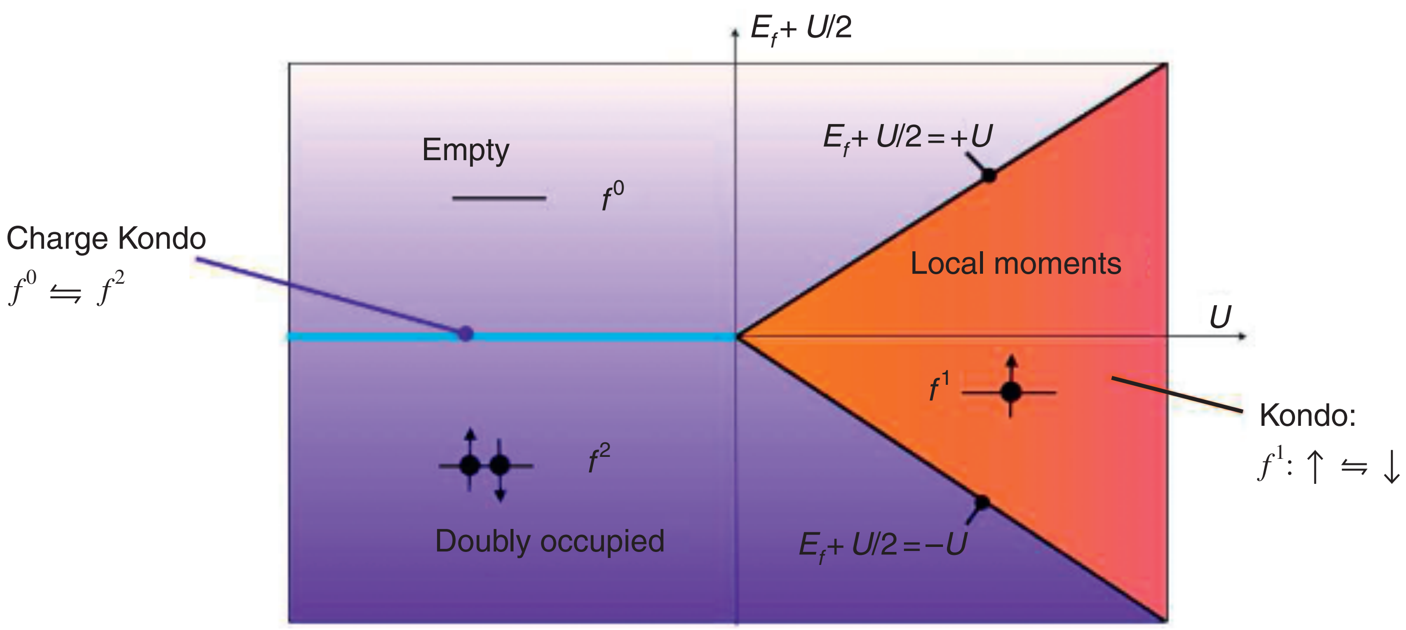

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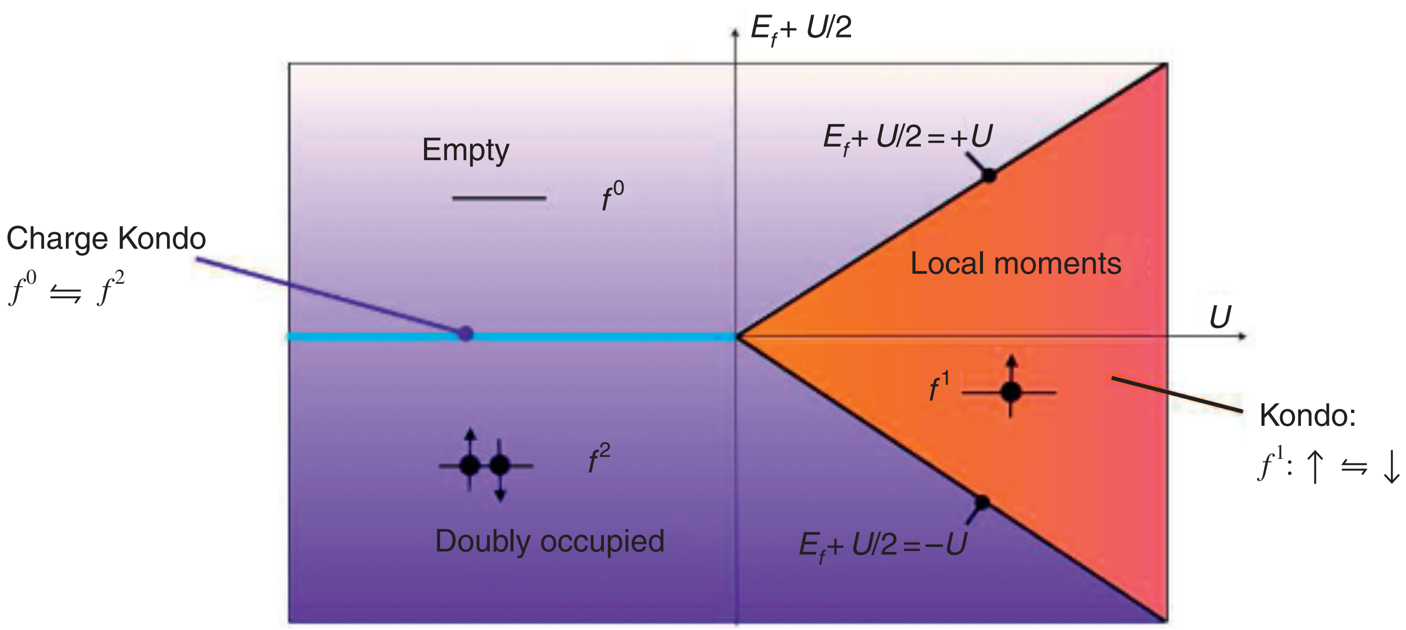

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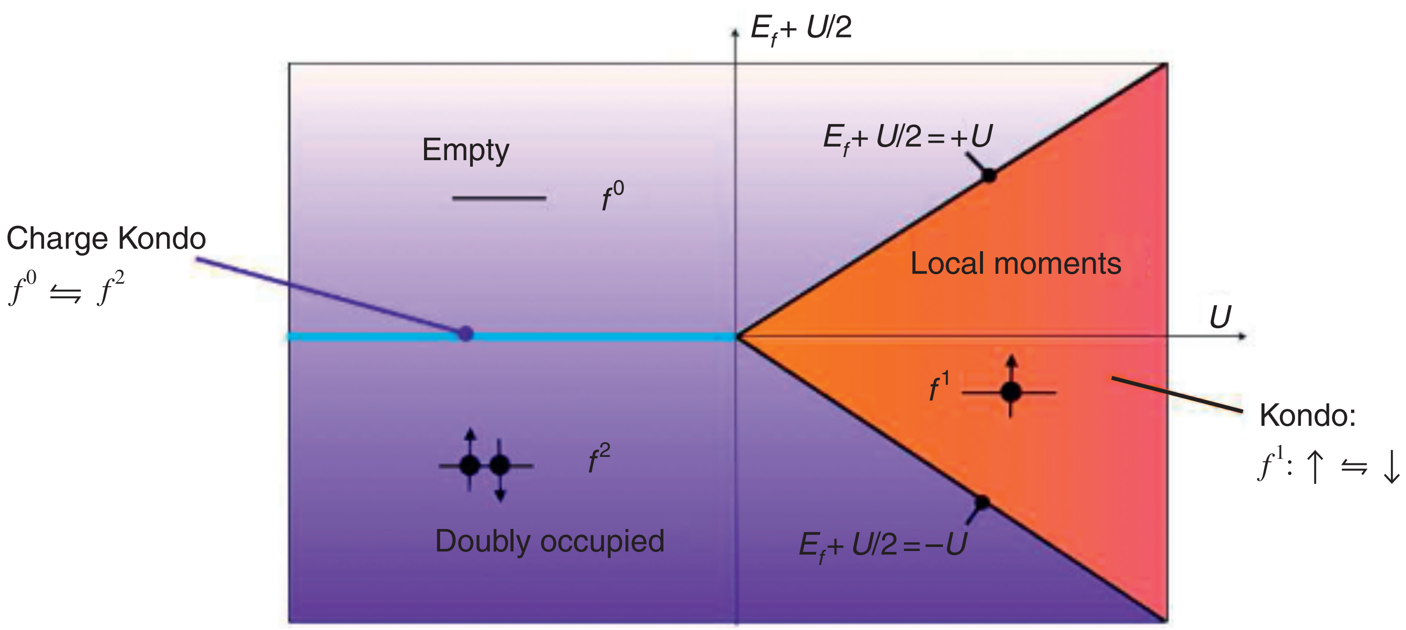

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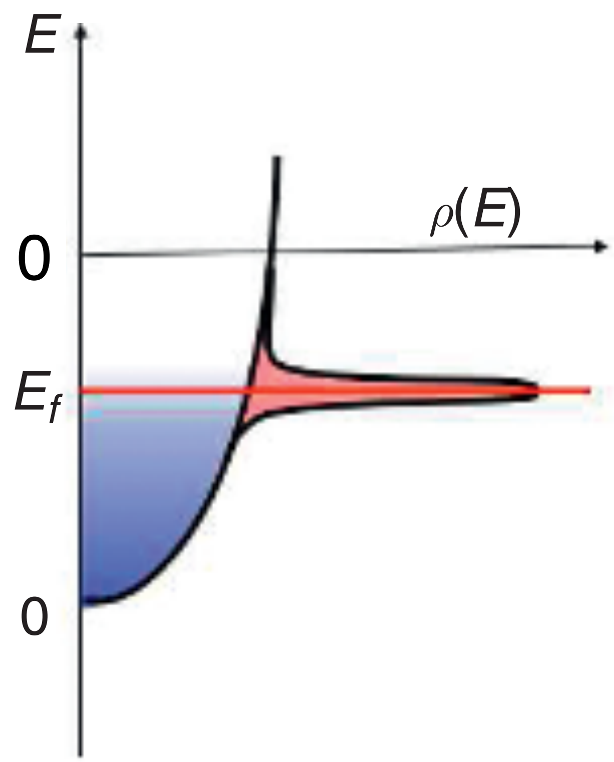

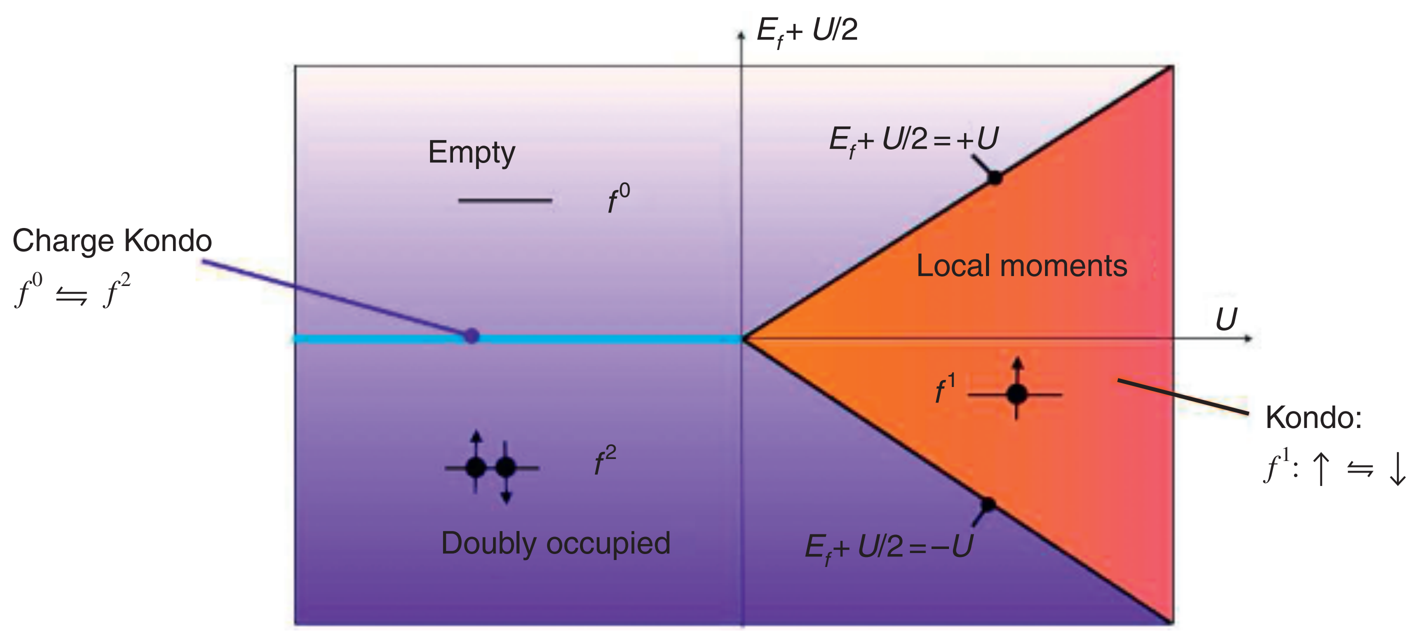

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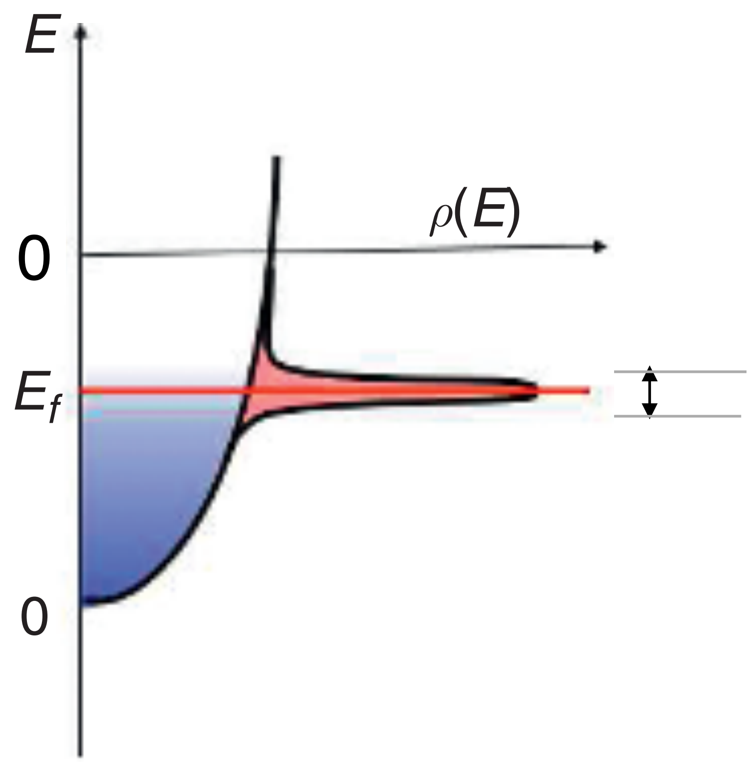
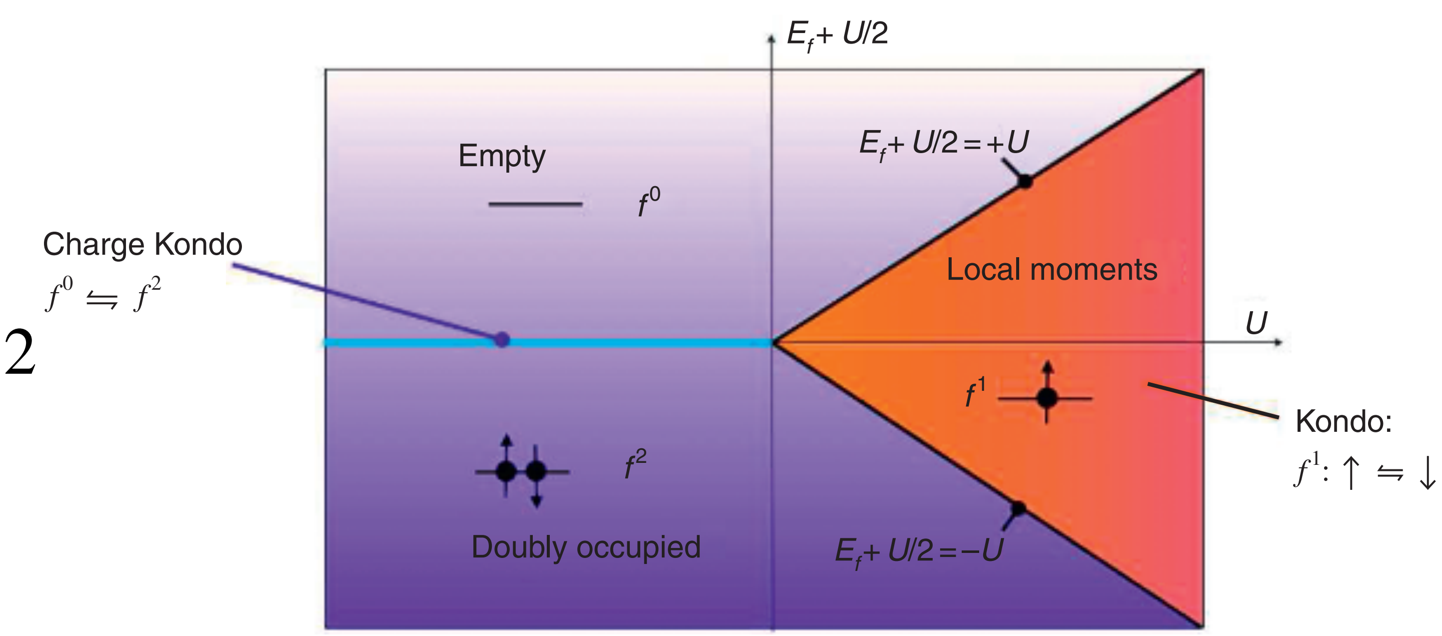


$$\Delta = \pi \rho |V|^2$$



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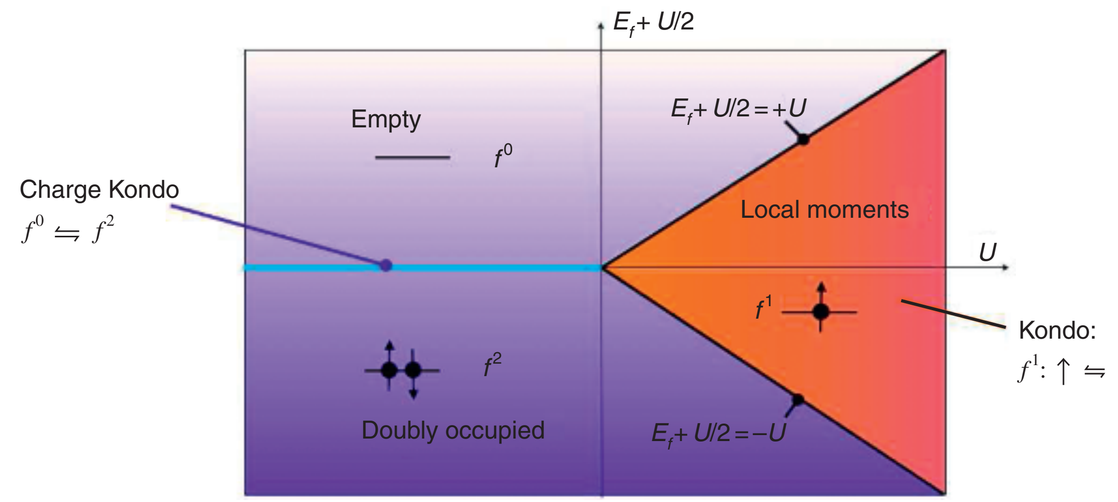

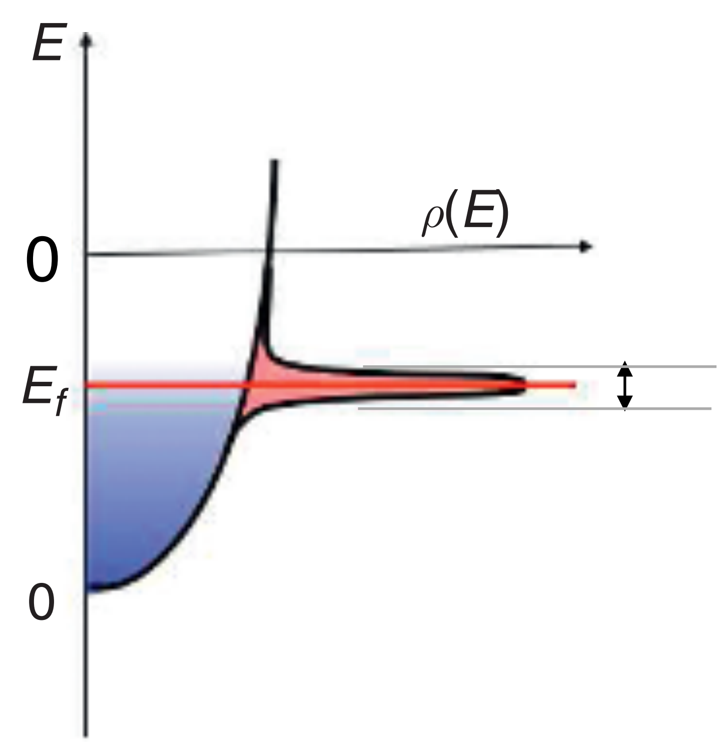

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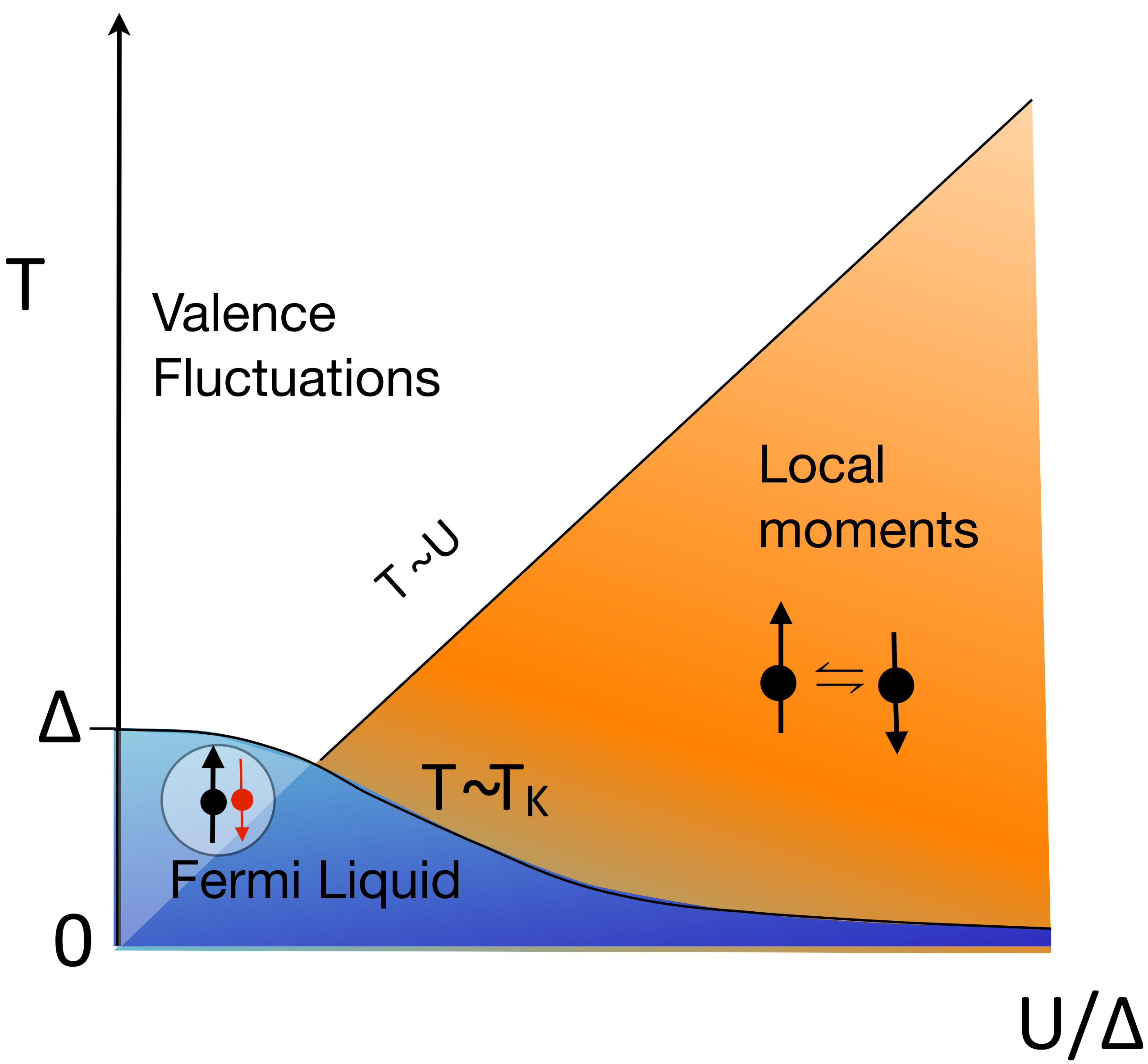

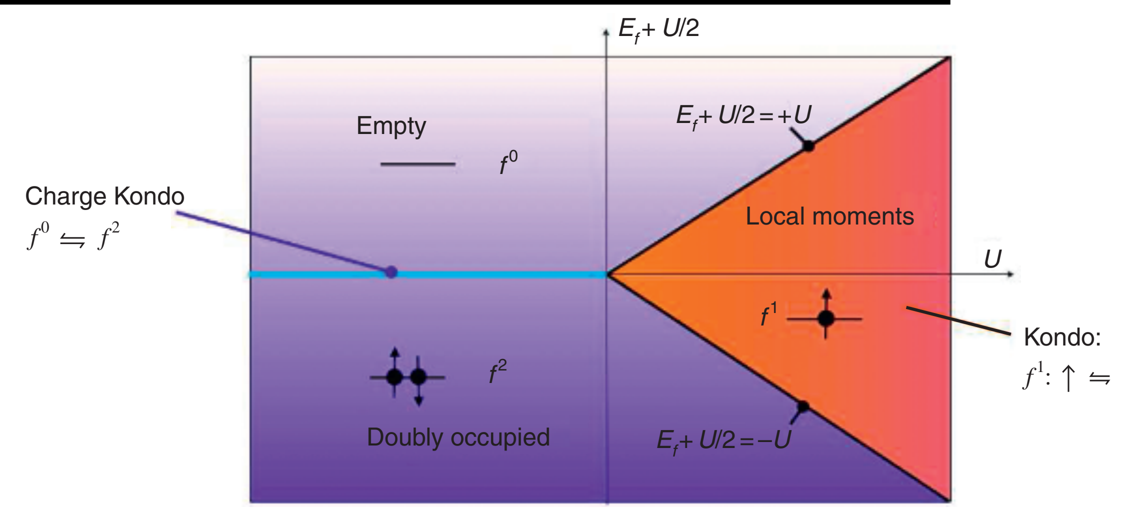

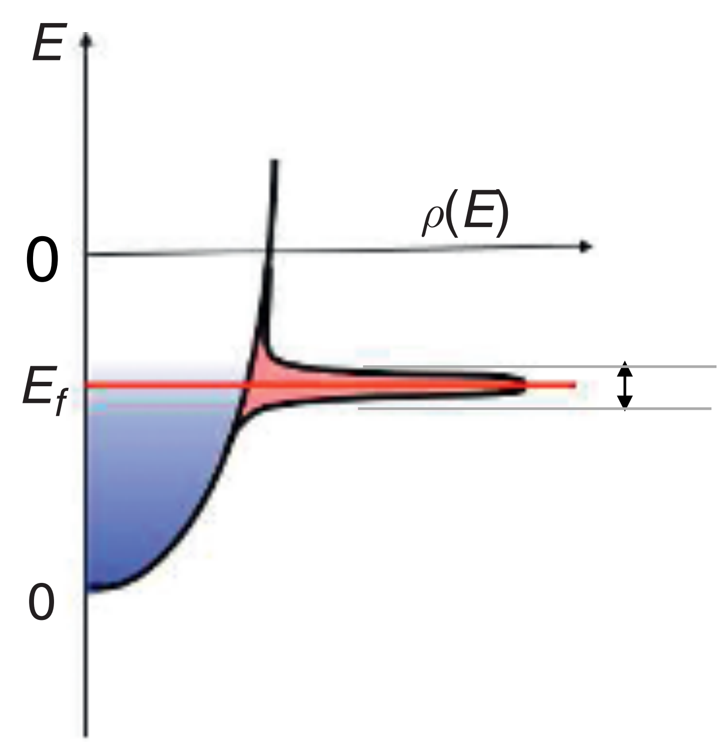

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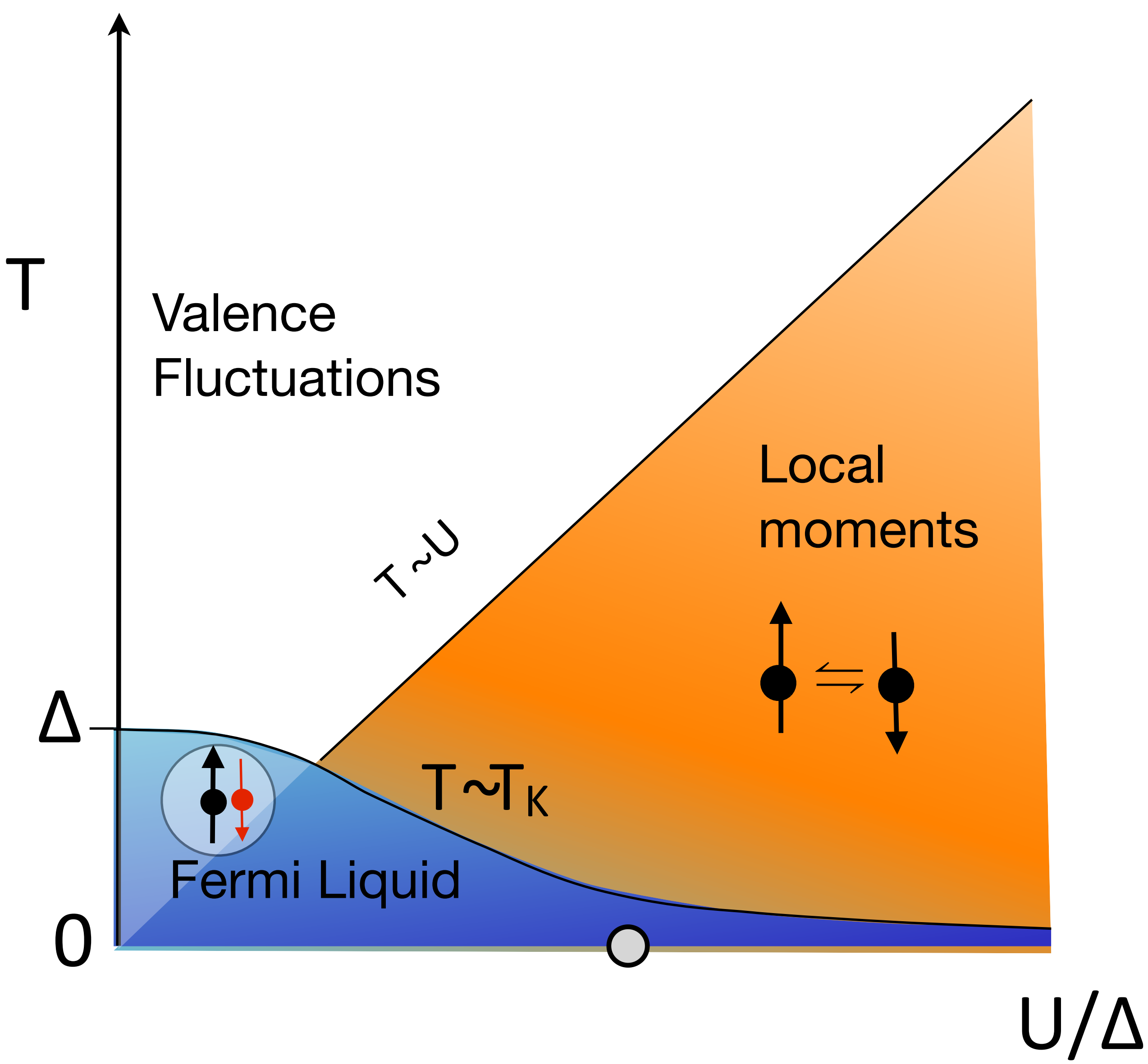

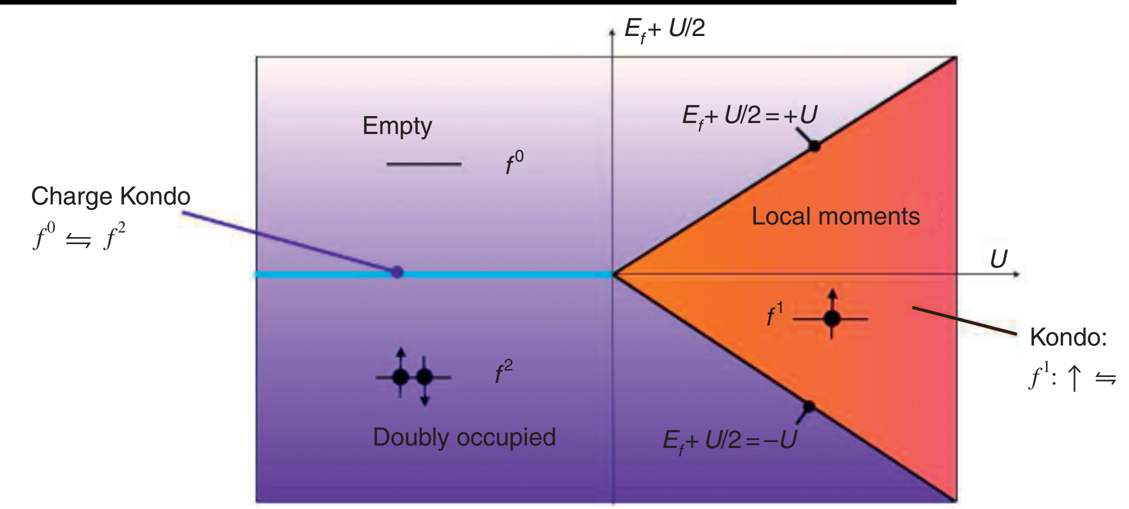

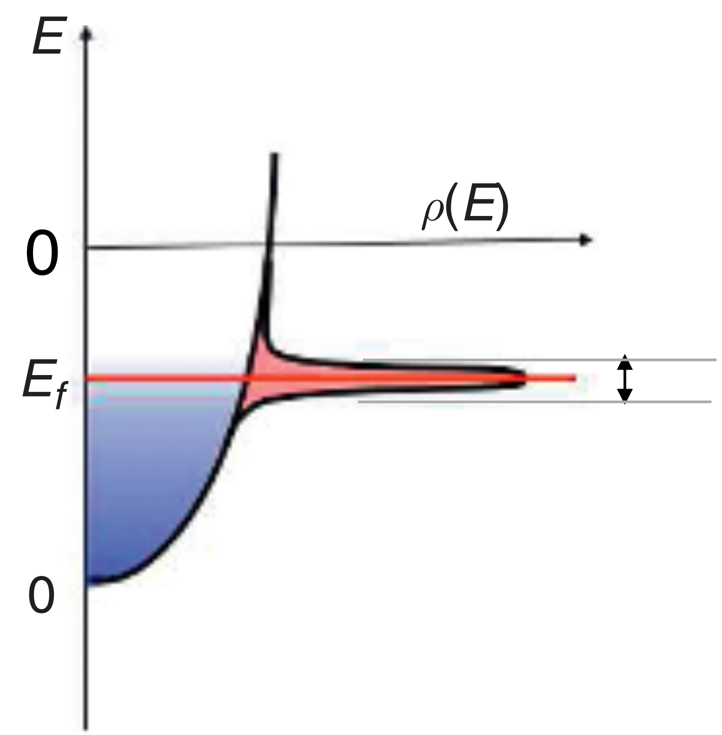

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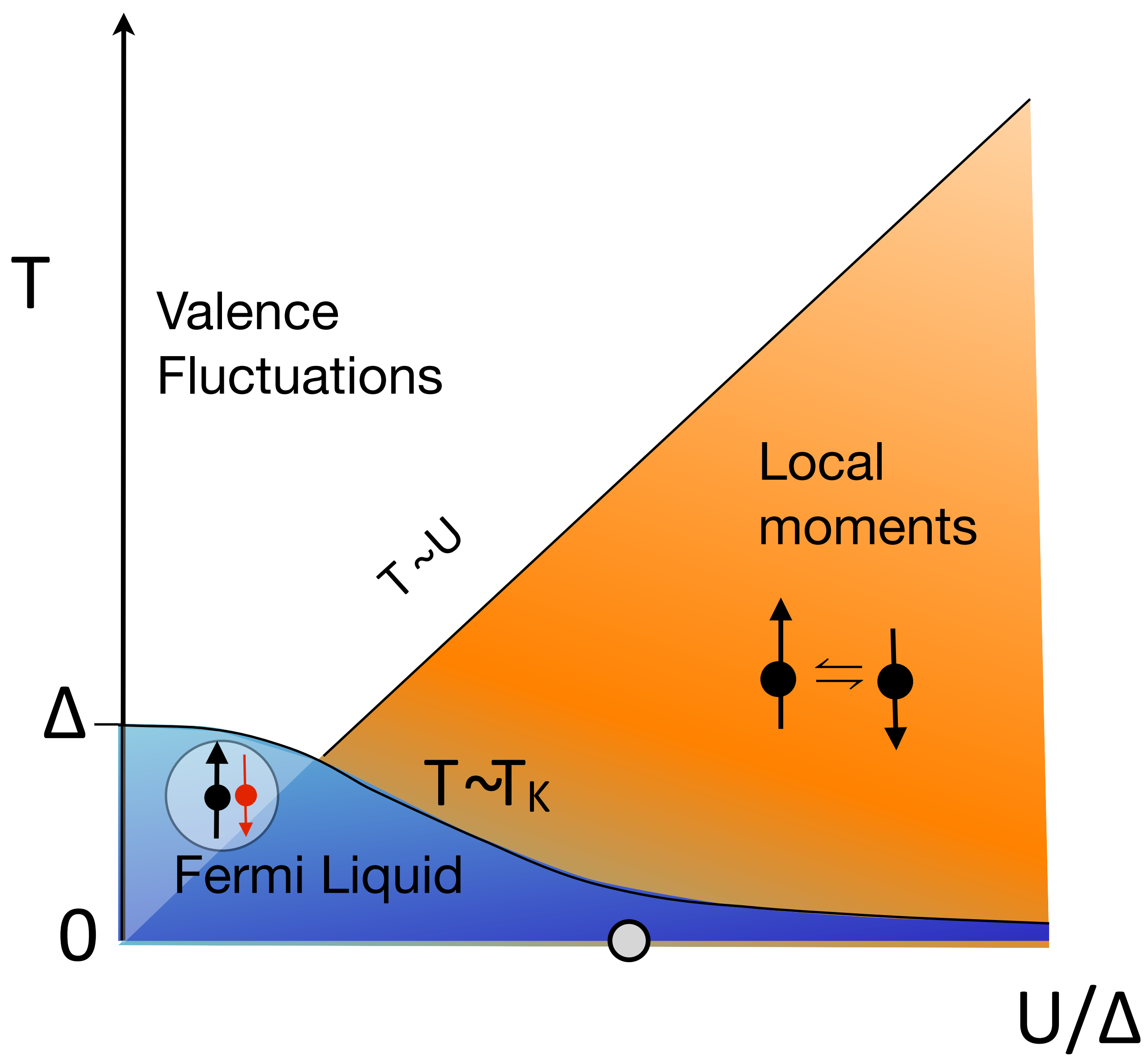

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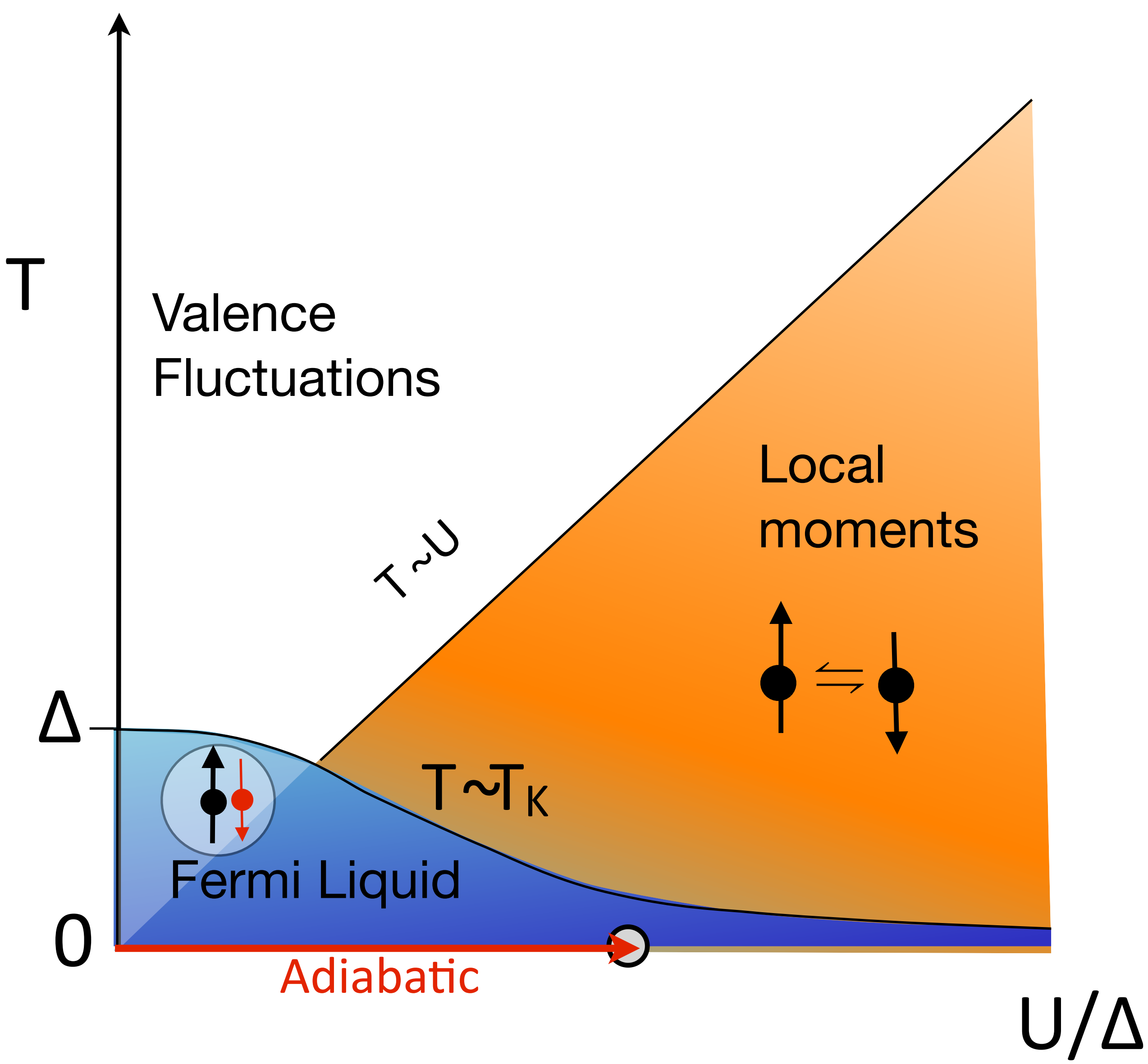

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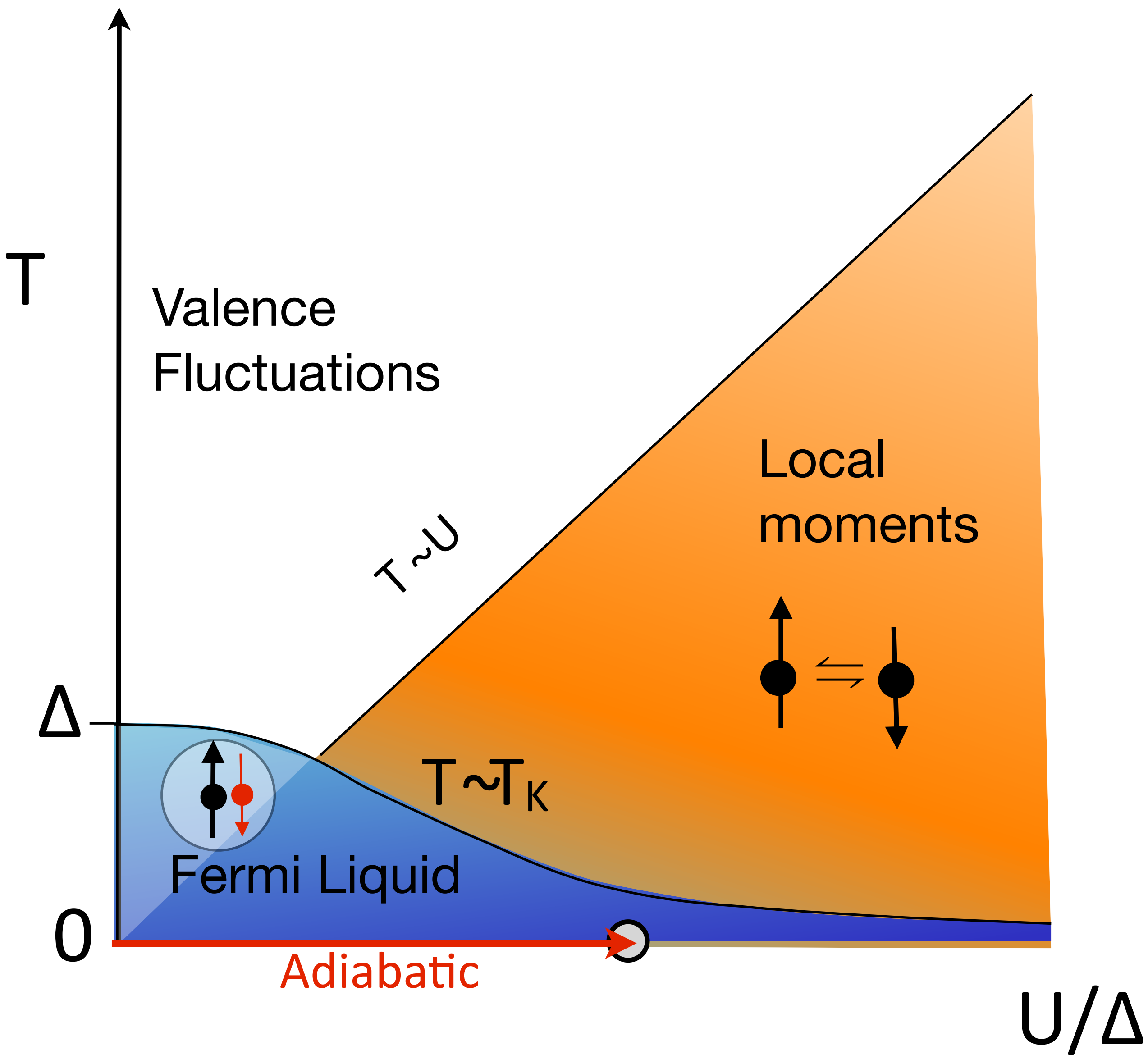

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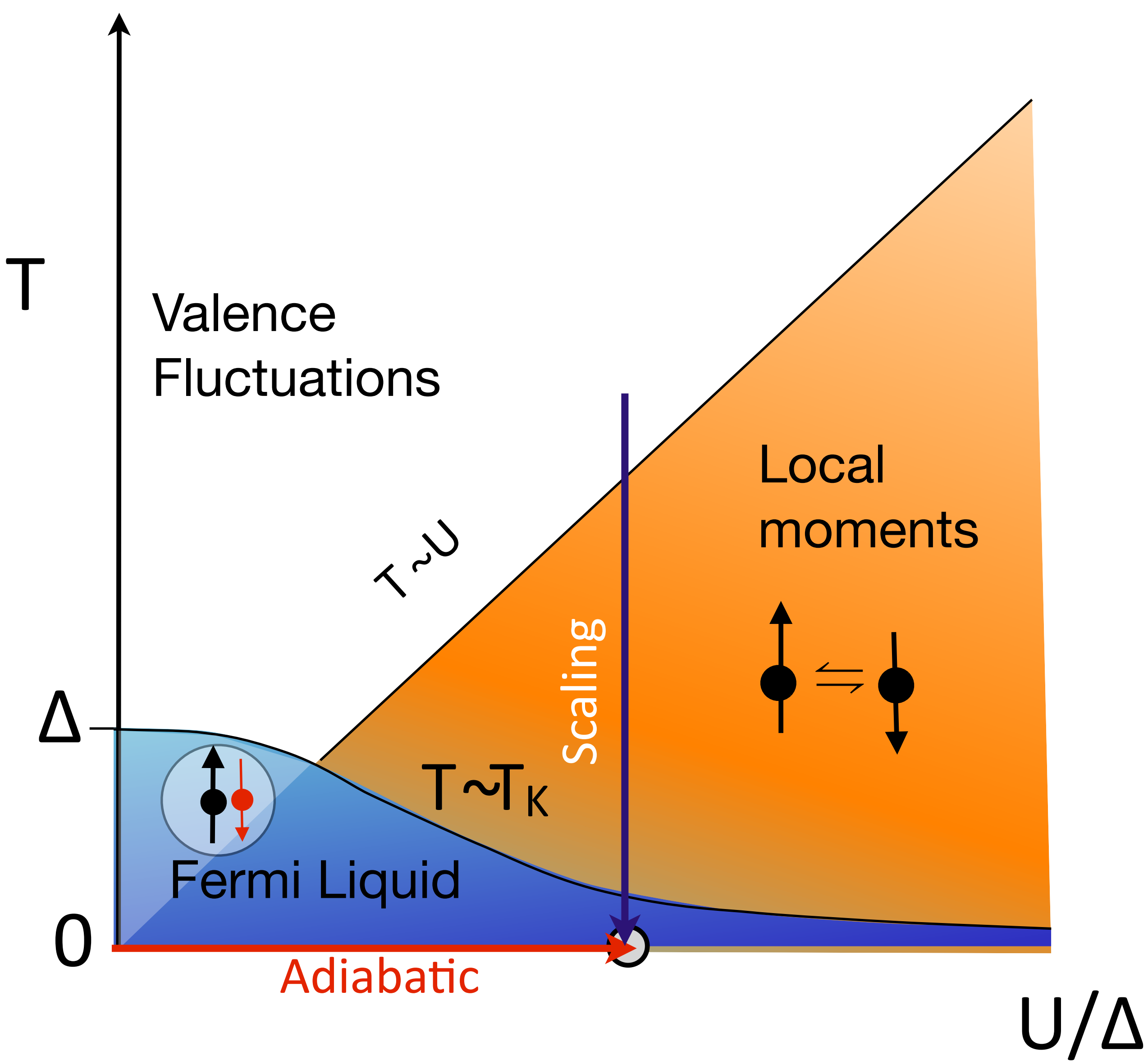

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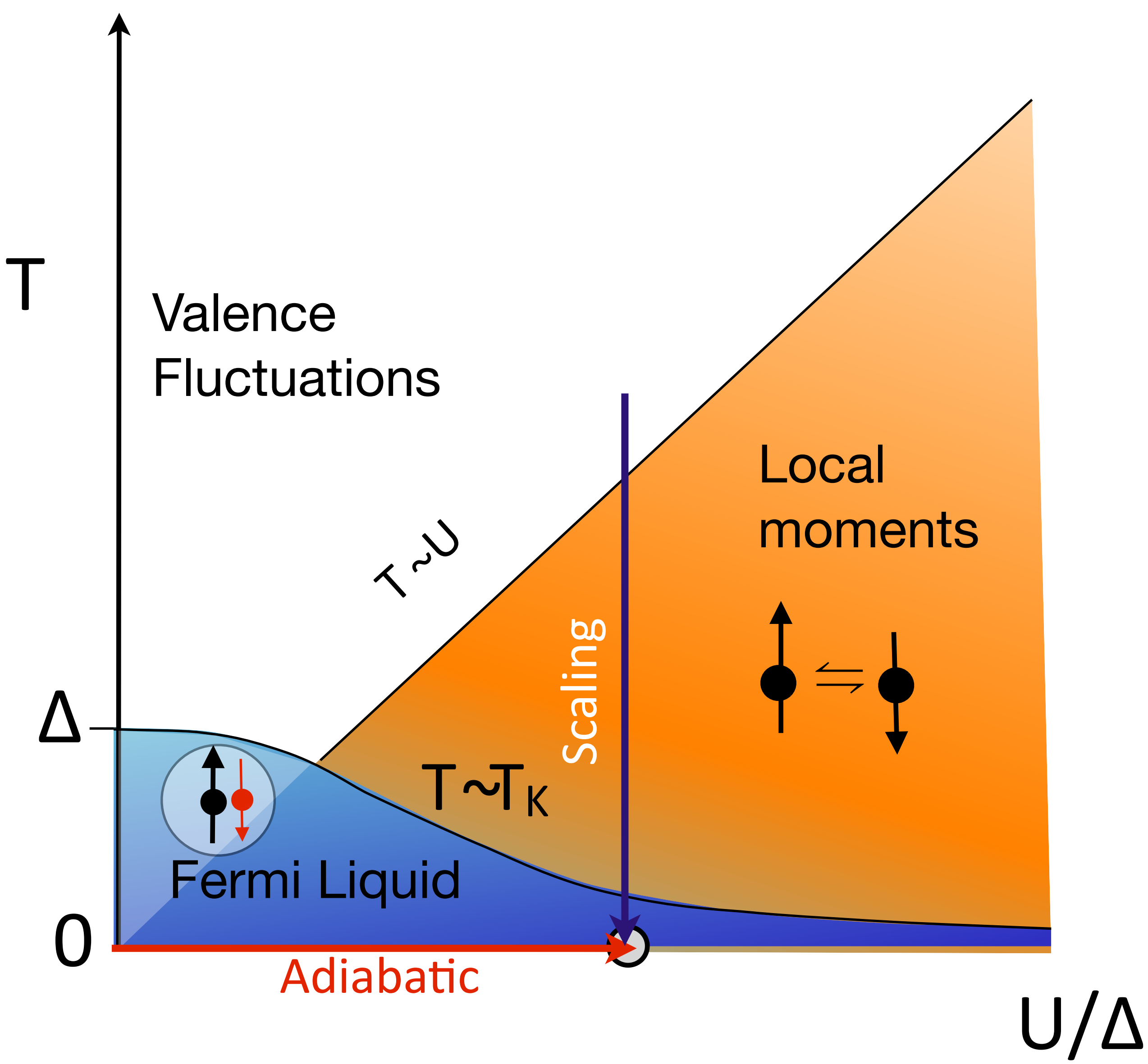
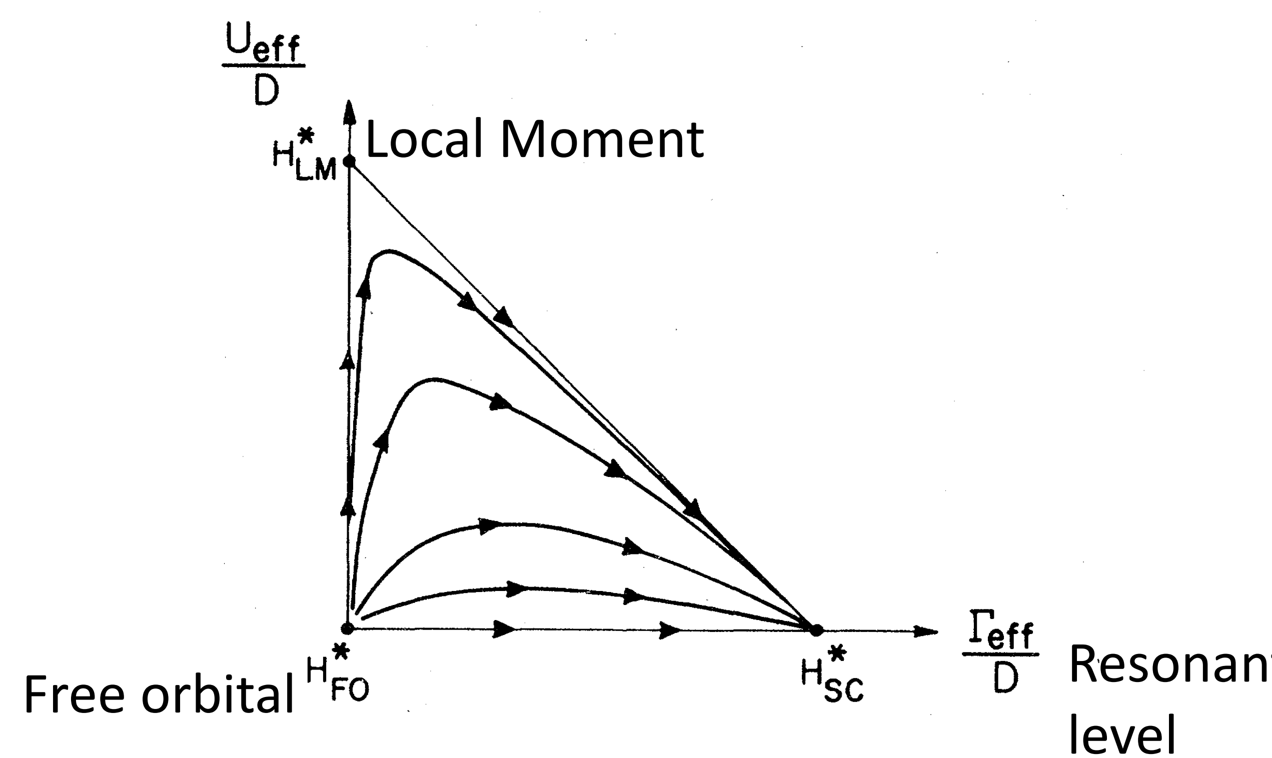


PHYSICAL REVIEW B VOLUME 21, NUMBER 3 1 FEBRUARY 1980

Renormalization-group approach to the Anderson model of dilute magnetic alloys. I. Static properties for the symmetric case

H. R. Krishna-murthy,* J. W. Wilkins, and K. G. Wilson
Physics Department, Cornell University, Ithaca, New York 14853

(Received 10 September 1979)



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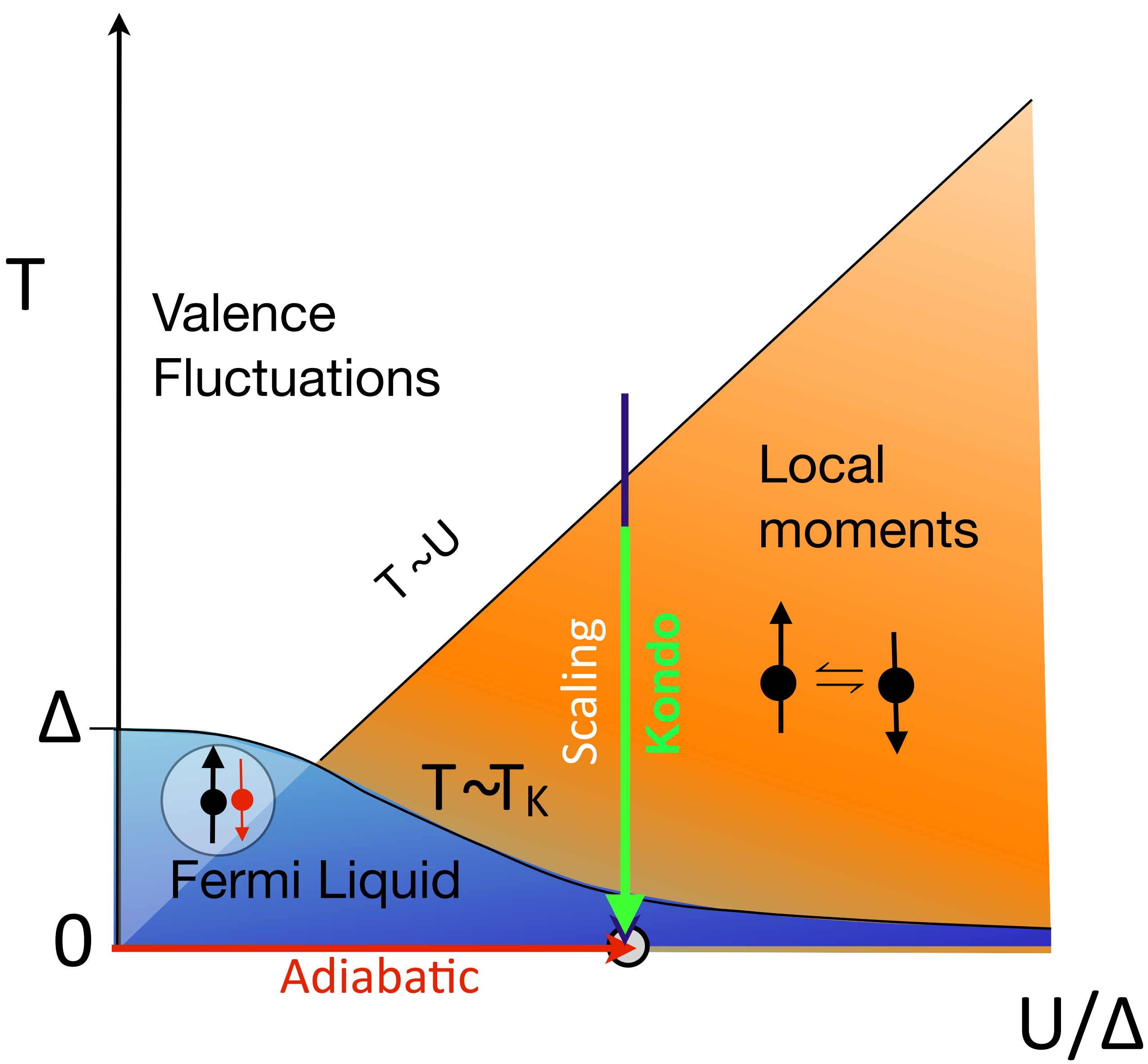

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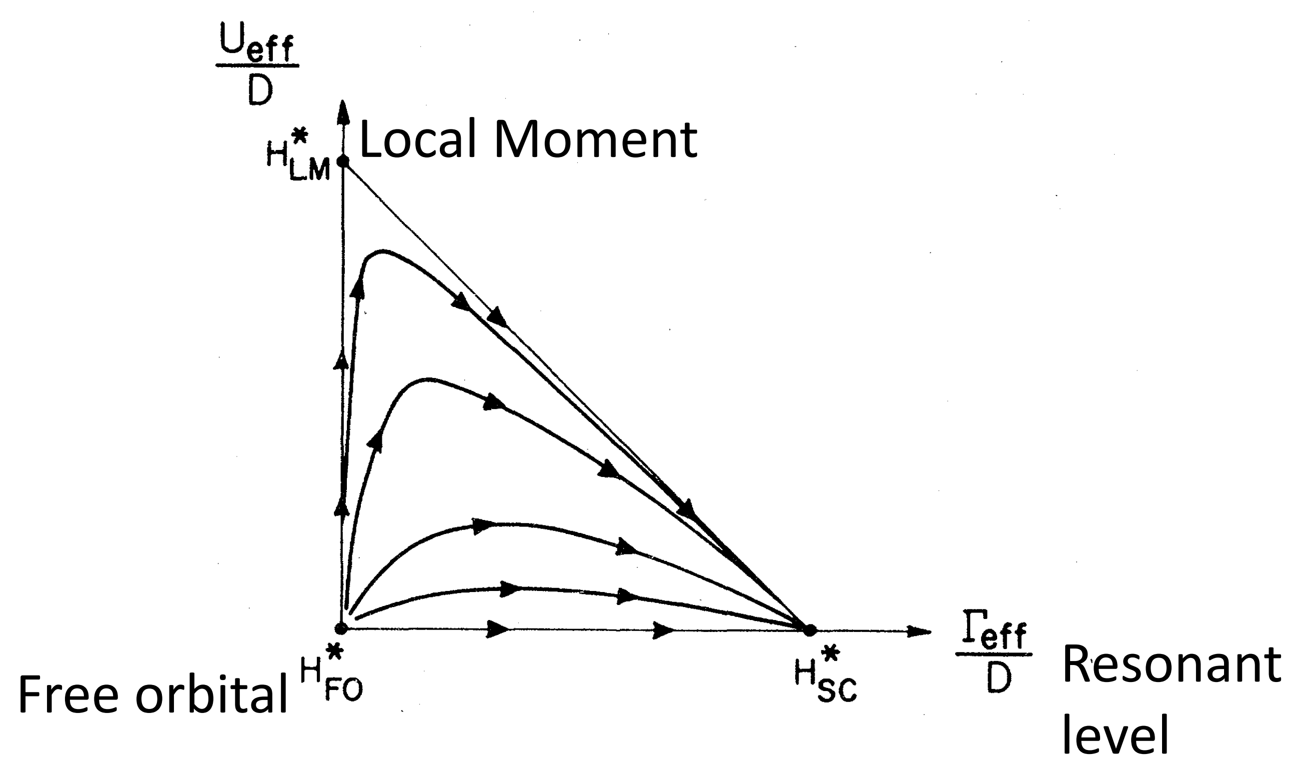

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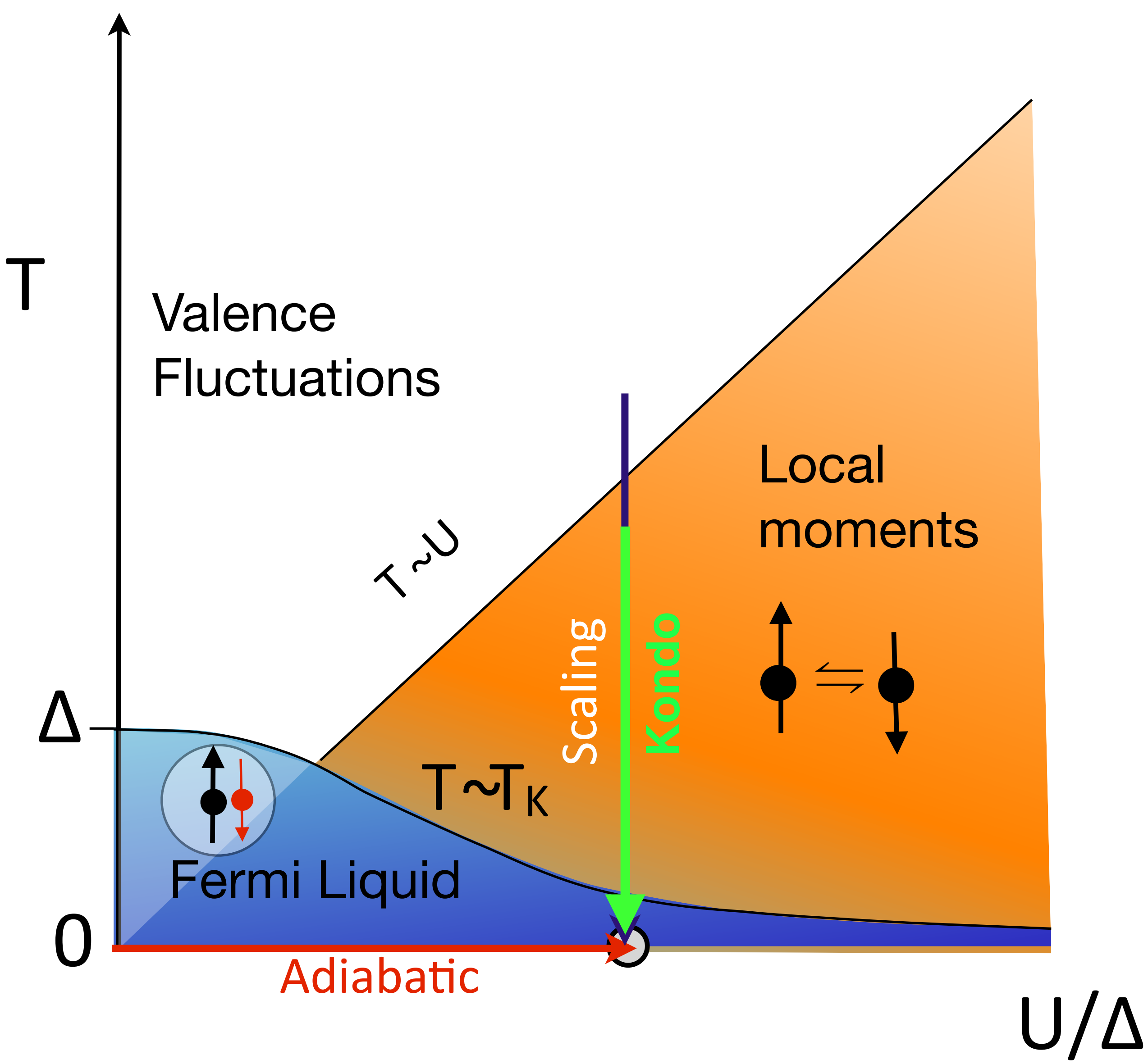

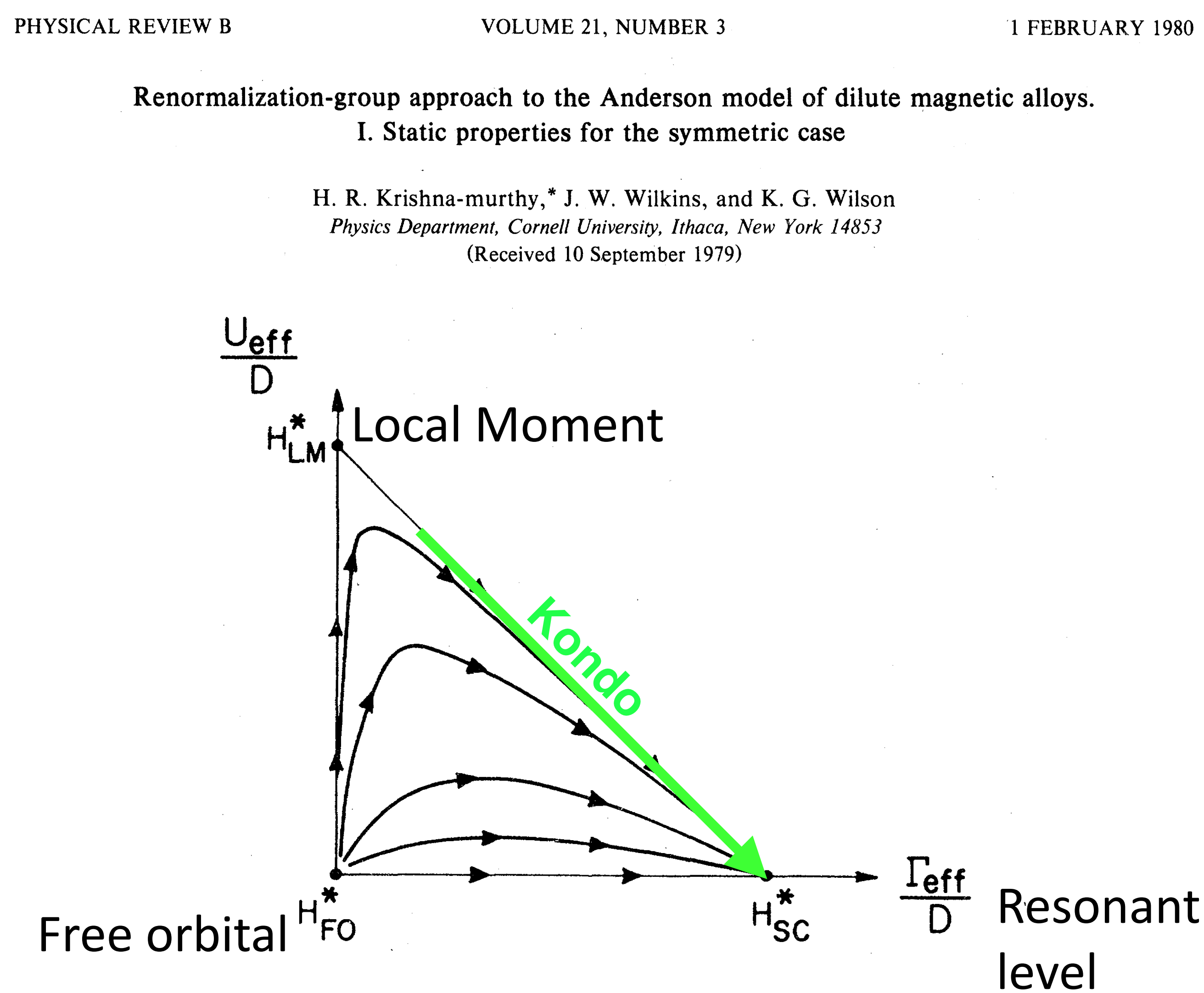

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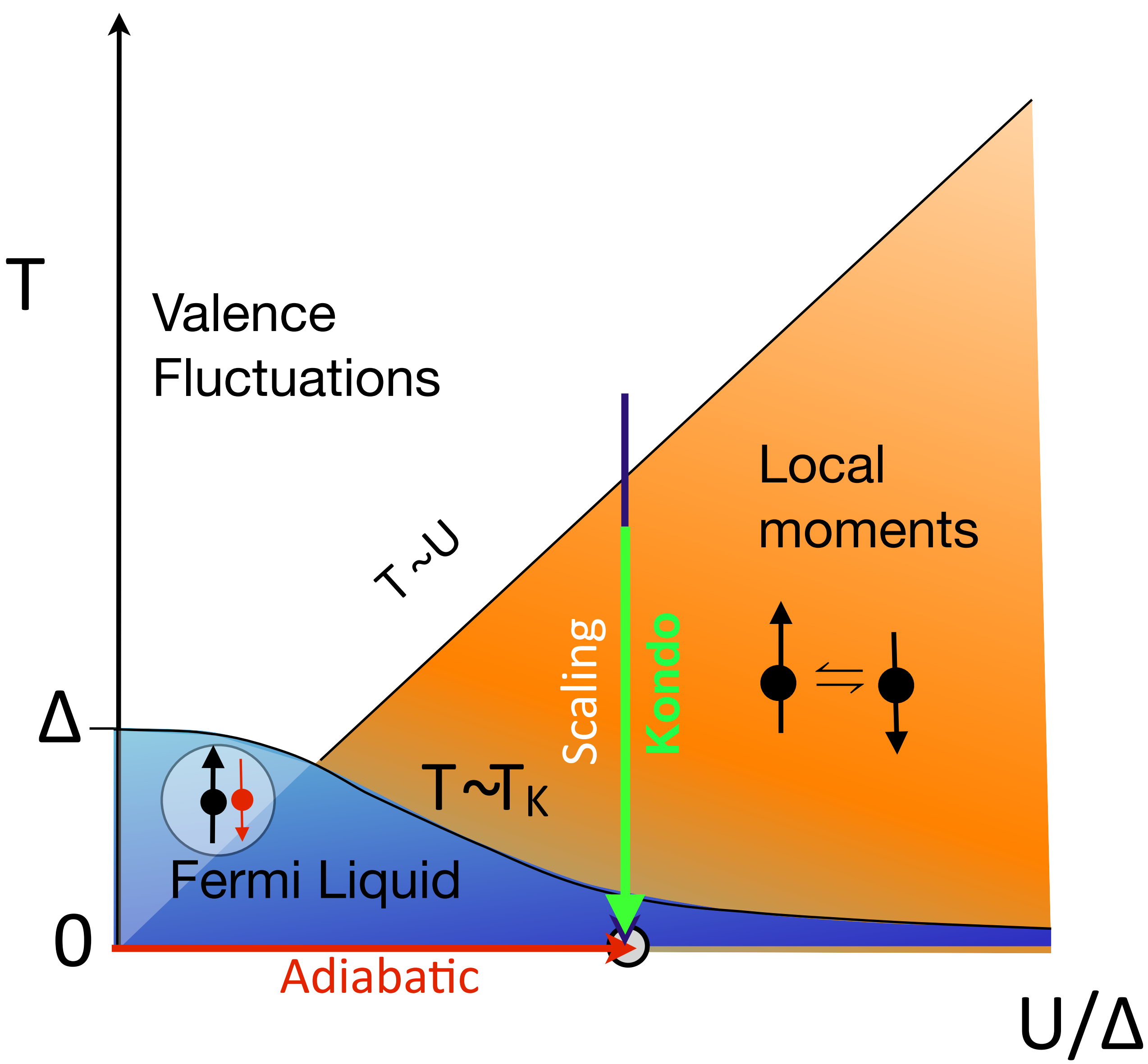

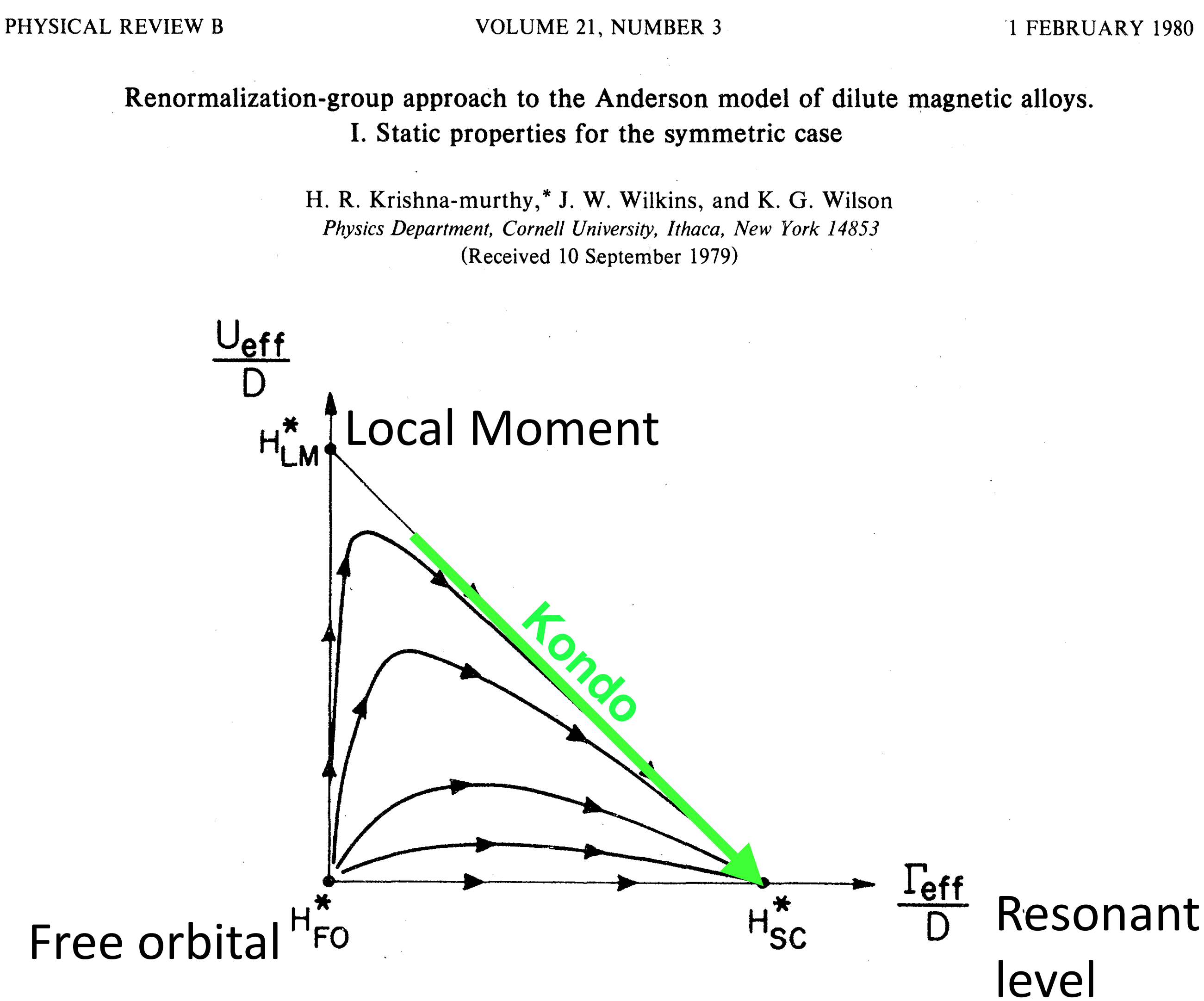

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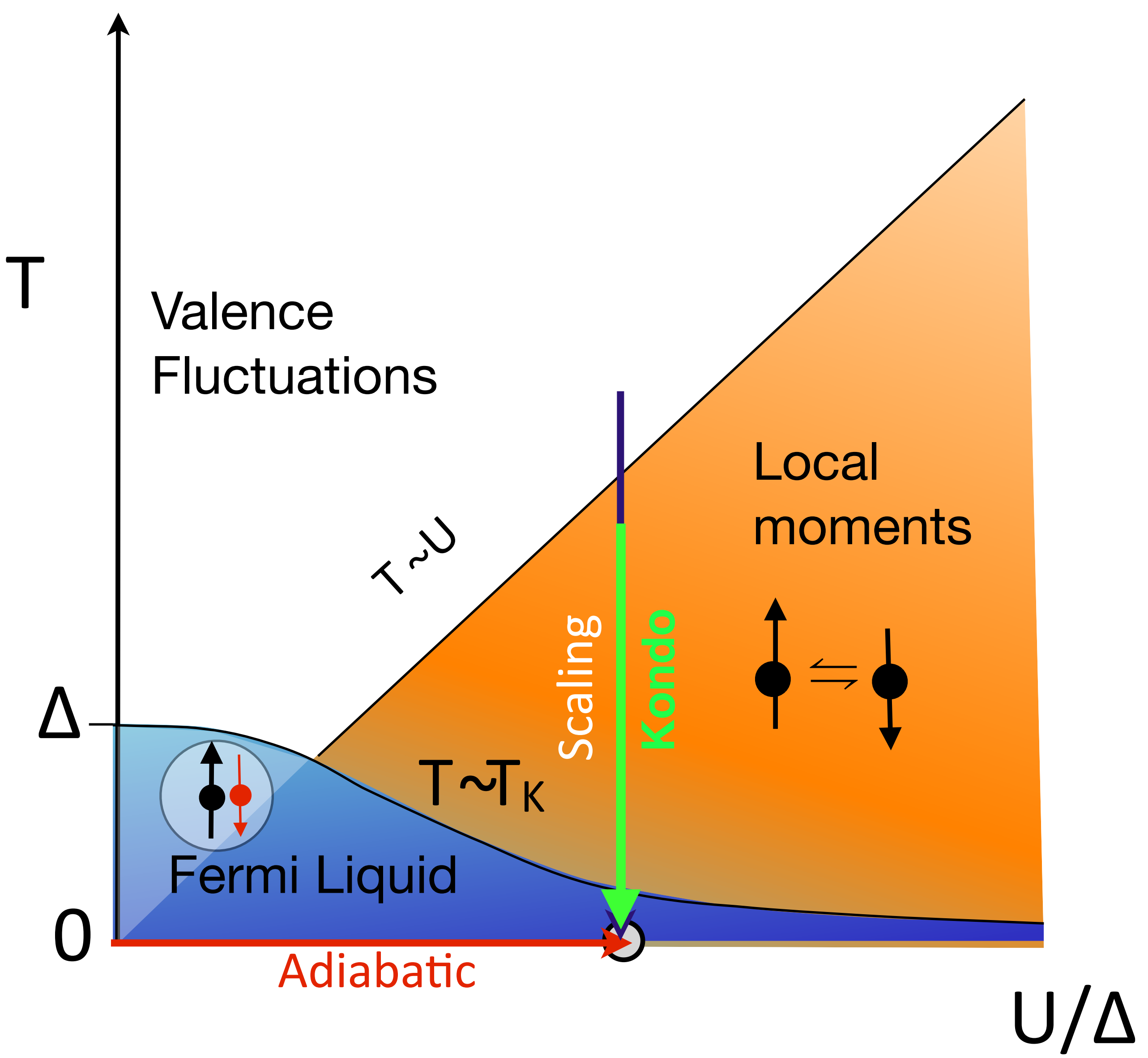
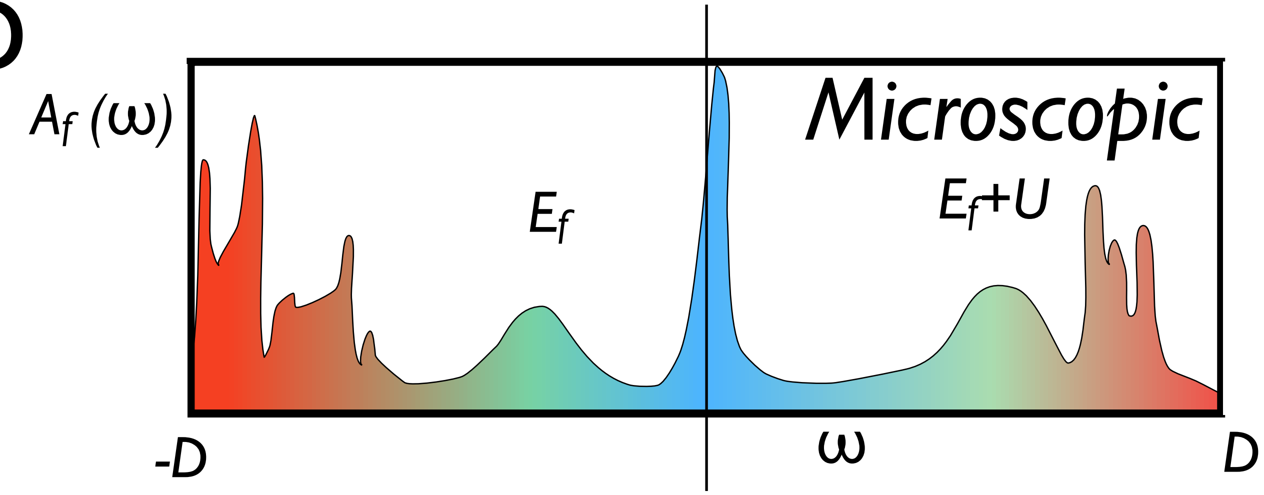
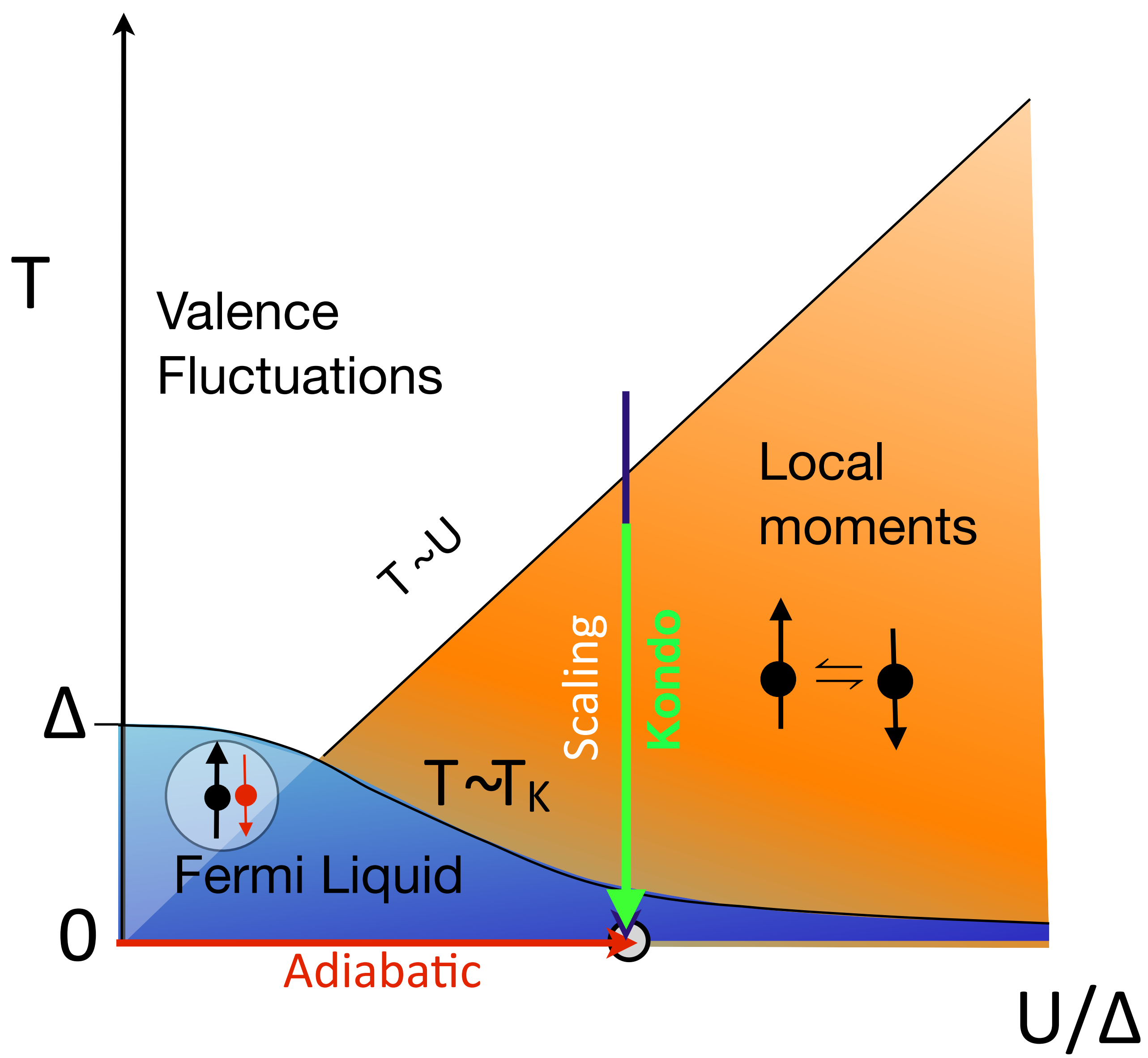


From Anderson to Kondo





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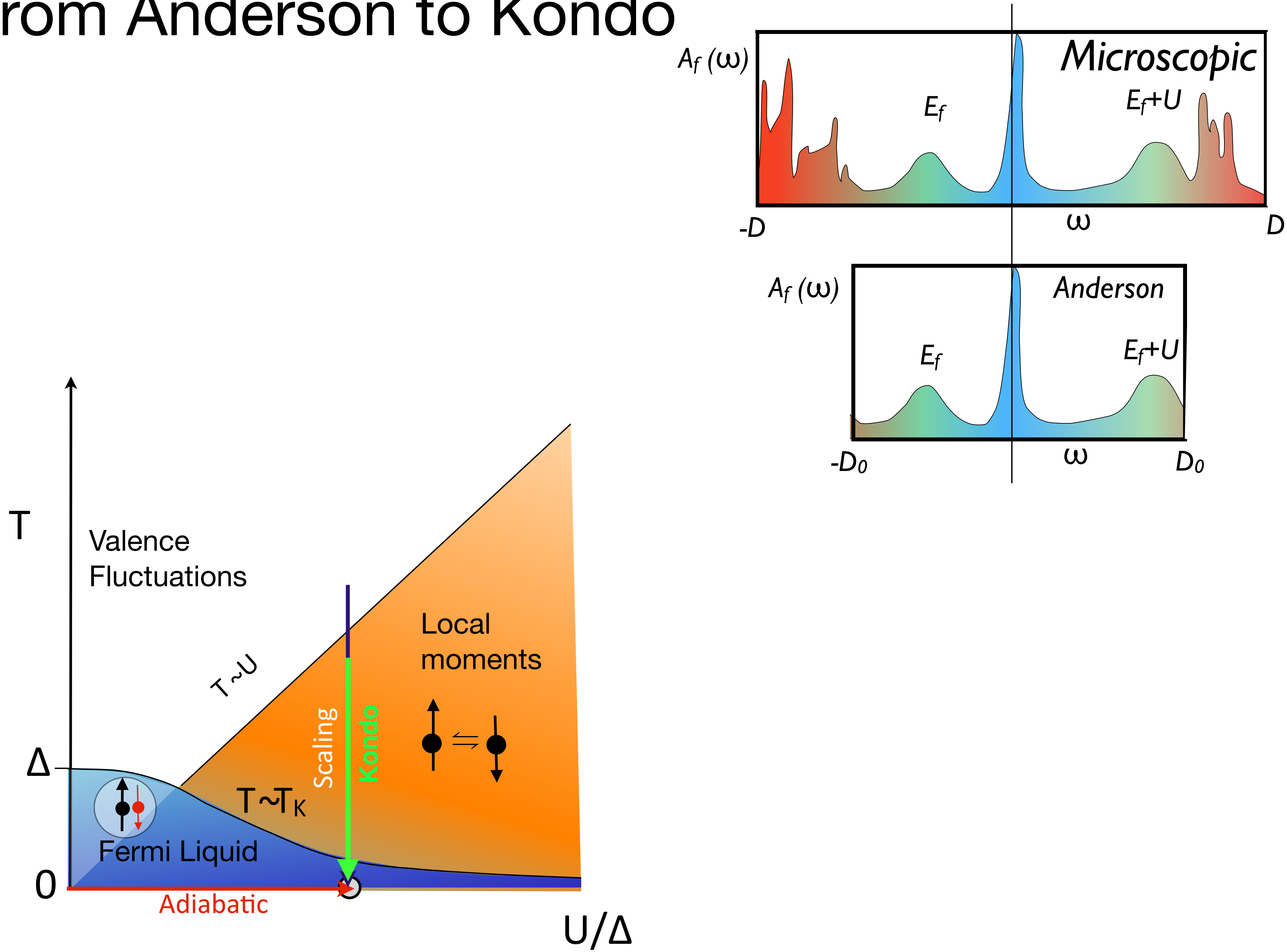

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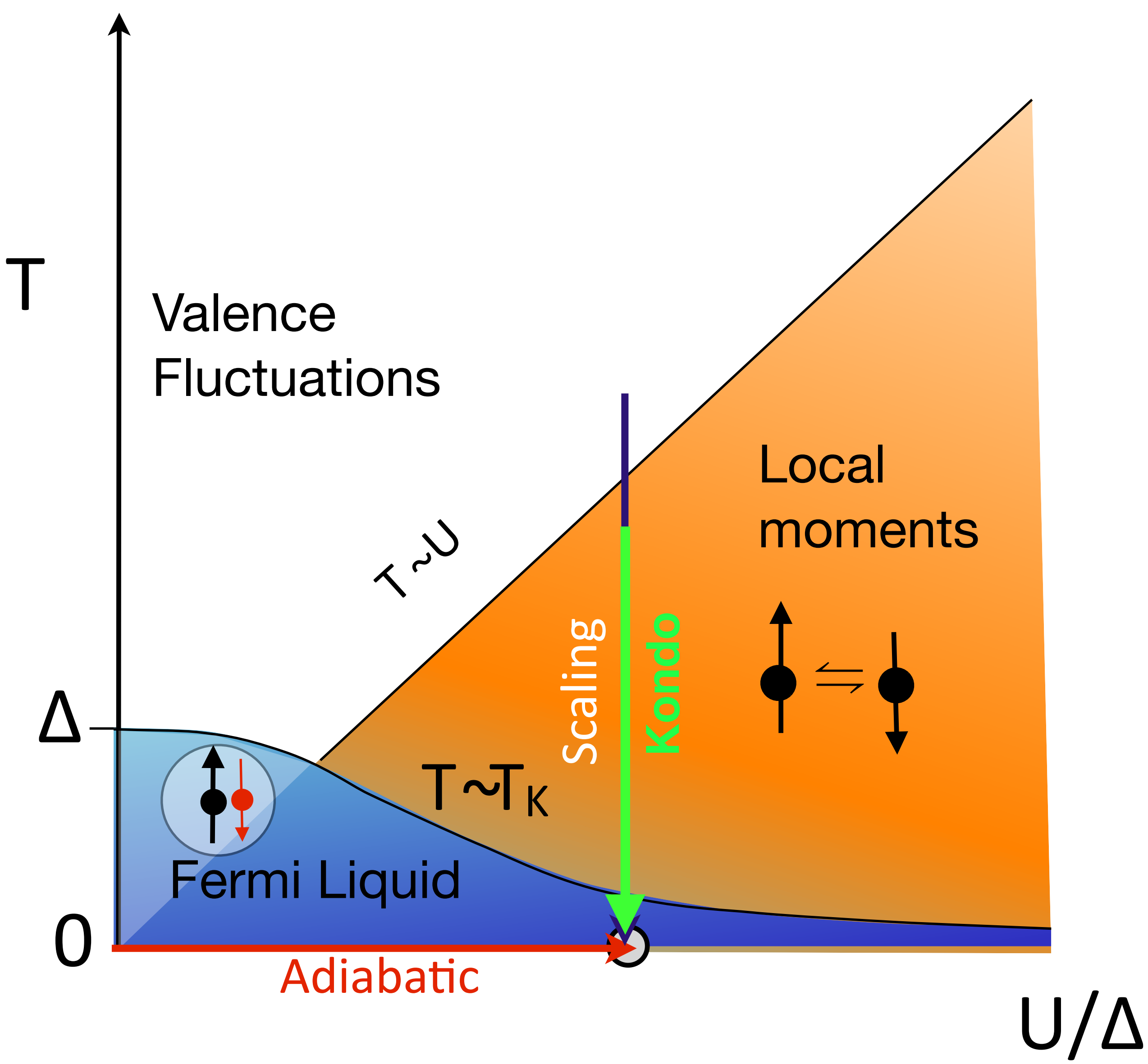
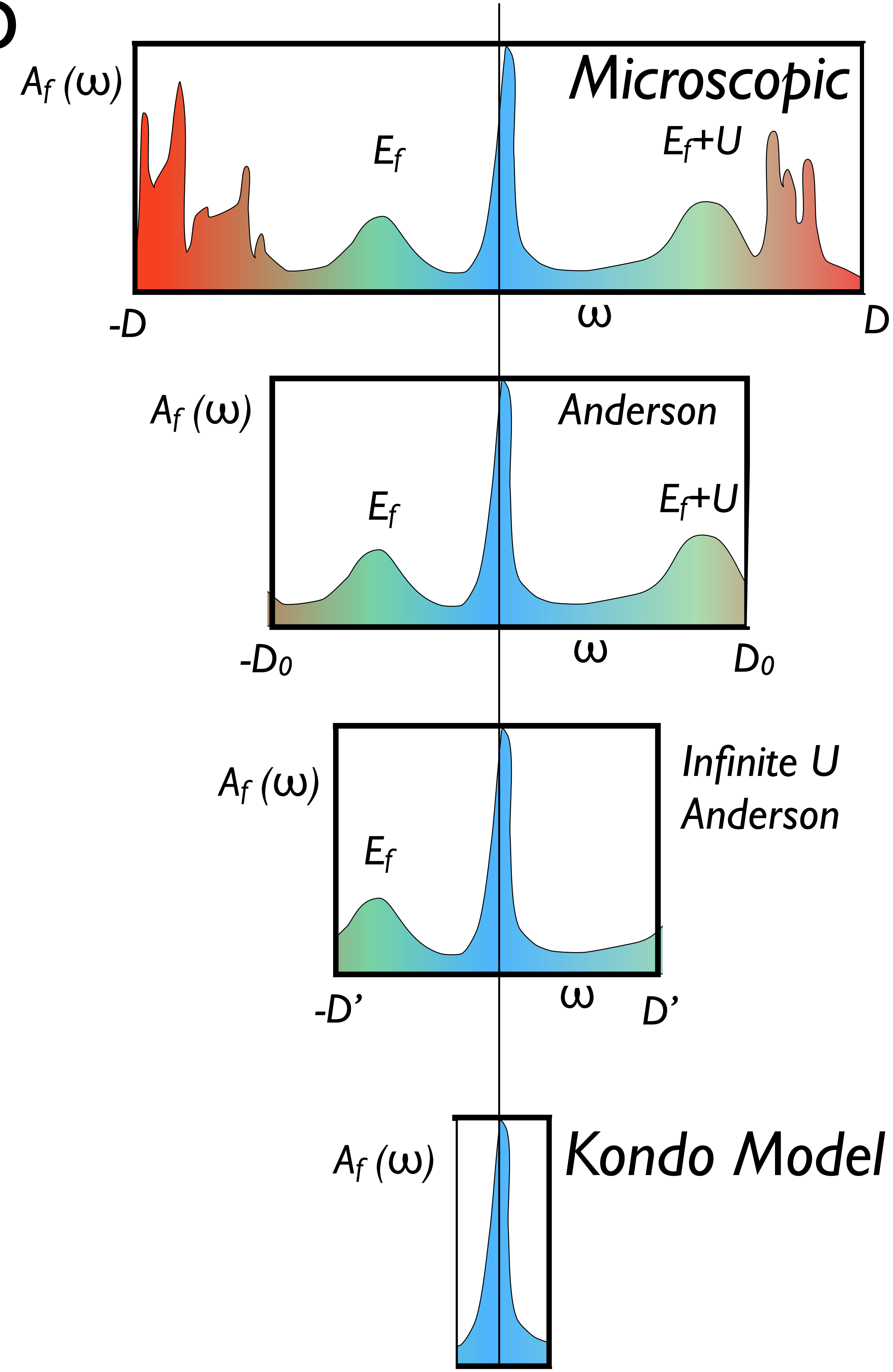
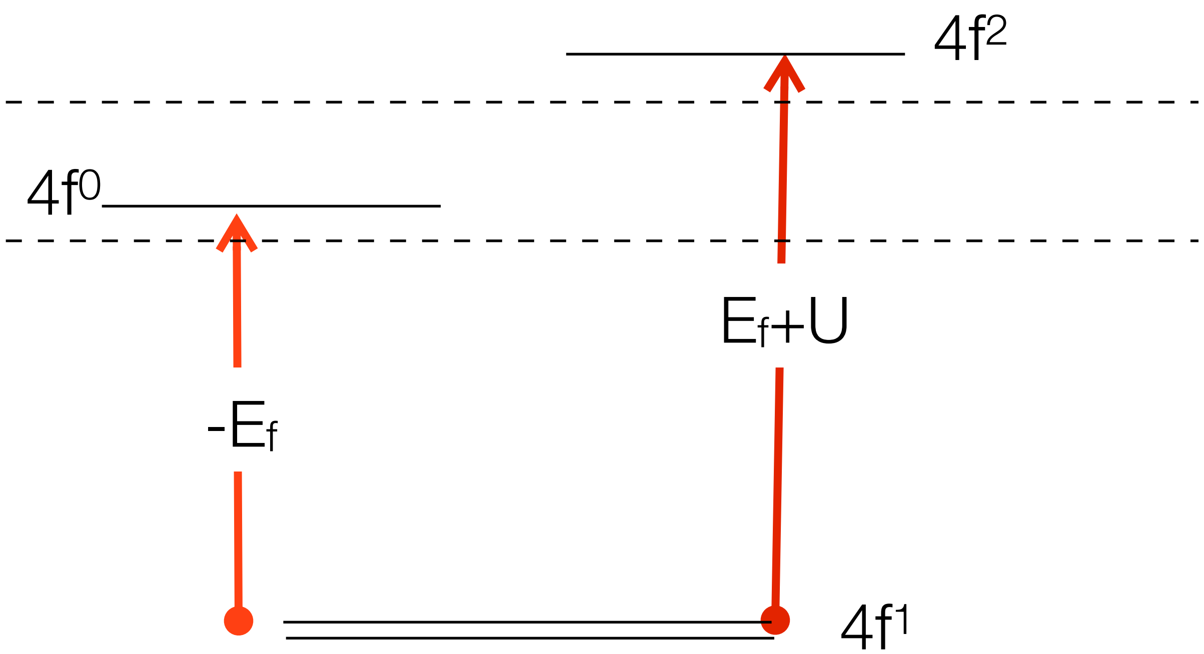




Kondo effect

1. Schrieffer Wolff Transformation



Virtual Valence fluctuations in the singlet channel, induced by hybridization

$$e^-_\uparrow + f^1_\downarrow \leftrightarrow f^2 \leftrightarrow e^-_\downarrow + f^1_\uparrow \qquad \Delta E_I \sim U + E_f$$

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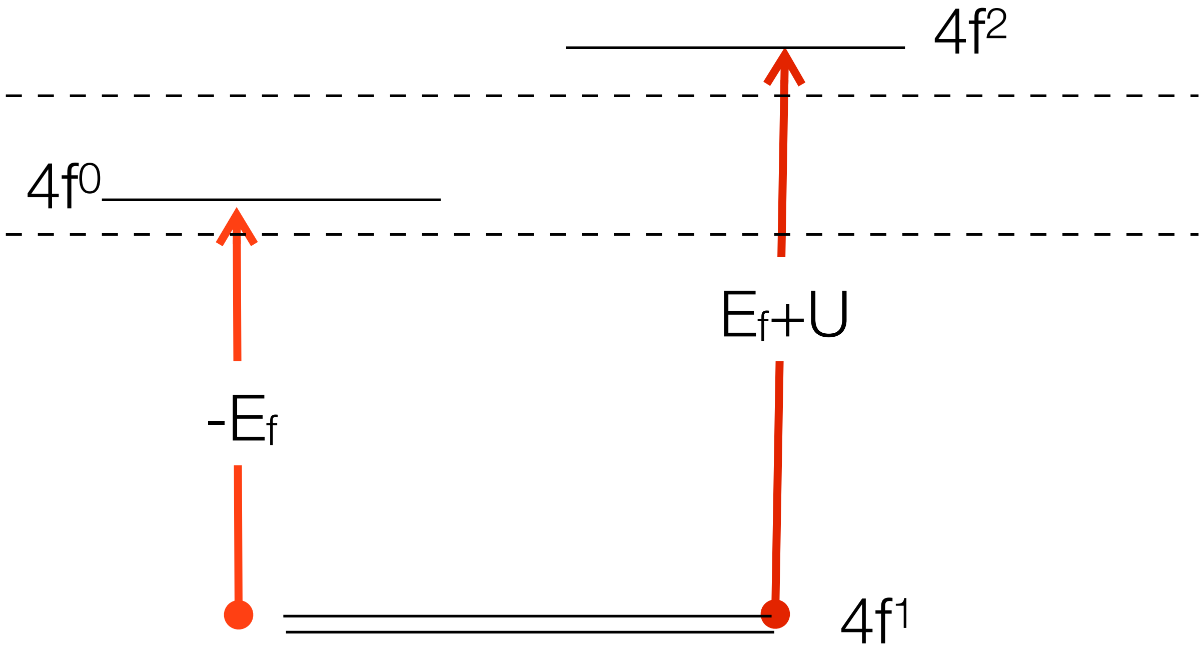

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From second order perturbation theory, the energy of c-f singlets reduces by an amount 2J, where

$$J = V^2 \left[\frac{1}{\Delta E_1} + \frac{1}{\Delta E_2} \right]$$

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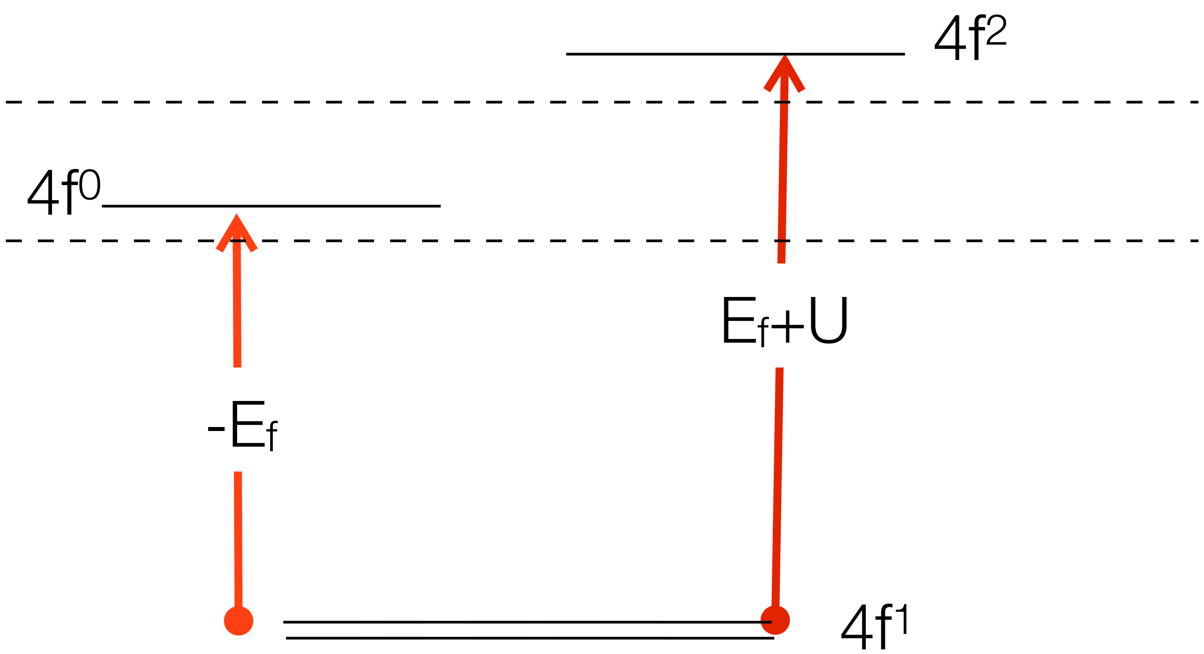

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$$H_K = -2JP_{S=0} = -2J \left[\frac{1}{4} - \frac{1}{2} \vec{\sigma}_c(0) \cdot \vec{S}_f \right] \rightarrow J \vec{\sigma}_c(0) \cdot \vec{S}_f$$

Antiferromagnetic interaction