Exercises . Physics 603. Quantum Hall Effect (Due Oct 30th)

1. Use the Bohr-Sommerfeld quantization condition that the orbit have a circumference containing an integral number of de Broglie wavelengths to find the allowed orbits of a 2D electron moving in a uniform magnetic field. Show that each successive orbit encloses precisely one additional quantum of flux in its interior. Hint: note the difference between the canonical momentum, which determines the de Broglie wavelength, and the dynamical momentum which determins the velocity. Use the symmetric gauge $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$ in which the vector potential is purely azimuthal and constant in magnitude around the orbit.

2. Express the exact lowest-Landau-level two-body eigenstate

$$\psi(z_1, z_2) = (z_1 - z_2)^3 e^{-\frac{1}{4}(|z_1|^2 + |z_2|^2)} \tag{1}$$

as a sum of all possible two-body Slater determinants.

3. In quantum mechanics, for any single particle wavefunction $\psi(x)$, the particle has an uncertainty in its position x and momentum p set by the uncertainty principle, $\Delta x \Delta p \geq \frac{\hbar}{2}$. Show that for a wave function $\psi(x, y)$ in the nth Landau level, there is a similar uncertainty relation $\Delta x \Delta y \geq (n+1)l^2$, where $l = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length. (Hint: write x and y in terms of the guiding center and mechanical momentum, and use this to evaluate an expression for $\langle x^2 \rangle$ and $\langle y^2 \rangle$, from which you can obtain a lower bound for $\Delta x \Delta y$.)

4. A system of charge e Bosons in a partially filled Landau level at a high magnetic field is observed to develop quantum Hall plateaux.

   (a) Write down the Laughlin wavefunction for a $\nu = 1/(2p)$ filled Landau level composed of hard core bosons. What is the expected Hall constant?

   (b) Adapt Jain's idea of composite fermions to this case: consider each boson in the partially filled Landau level to be bound to an odd number $2p + 1$ flux tubes, forming a composite fermion. Suppose the composite fermions occupy the first q Landau levels, calculate the filling factor ν of the original bosons, to predict the main sequences of fractional Hall constant values ($\rho_{xy} = \frac{1}{\nu}\frac{h}{e^2}$) for hard core bosons.

(c) We can also think of the bosonic Laughlin state as a superfluid in which each boson carries $2p$ flux tubes. Write down the Chern Simons Ginzburg Landau action that describes this situation. What value of k is required to attach the $2p$ flux tubes?

(d) How might an experiment be carried out to confirm your predictions?