Solution to Prob Set 1

1. $$c\int\left[1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right]^{-\left(\frac{\nu+1}{2}\right)} dx = 1$$

$$\int_{-\infty}^{\infty}\frac{dx}{\left[1+\frac{1}{\nu}\left(\frac{x-\mu}{\sigma}\right)^2\right]^{\frac{\nu+1}{2}}} = \sigma\int_{-\infty}^{\infty}\frac{dt}{\left[1+\frac{t^2}{\nu}\right]^{\frac{\nu+1}{2}}}$$ Using $t=\frac{x-\mu}{\sigma}$

$$= 2\sigma\int_0^{\infty}\frac{dt}{\left[1+t^2/\nu\right]^{(\nu+1)/2}}$$

$$= \sigma\sqrt{\nu}\int_0^{\infty}\frac{s^{-1/2}ds}{\left[1+s\right]^{\frac{\nu+1}{2}}}$$ $\frac{t^2}{\nu}=s$

$$= \sigma\sqrt{\nu}\ B\left(\frac{1}{2},\frac{\nu}{2}\right)$$

$$= \sigma\sqrt{\nu}\ \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}$$

$$= \sigma\sqrt{\pi\nu}\ \frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}$$

So $$c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\sqrt{\pi\nu}\ \Gamma\left(\frac{\nu}{2}\right)}$$

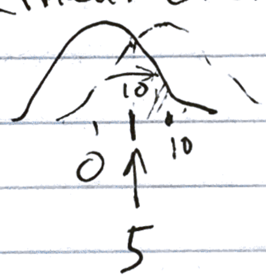
2. I did 500 runs and got the following statistics

	Mean misclass rates	Std of misclass rates	95% conf. int. for mean
1-NN	47.5%	5.1%	[47.1%, 47.9%]
5-NN	47.0%	5.0%	[46.5%, 47.4%]
Linear disc.	30.9%	4.6%	[30.5%, 31.4%]

Since $\sum_{i=1}^{100} X_i \sim N(0, 10^2)$

$\sum_{i=1}^{100} Y_i \sim N(10, 10^2)$

Linear discriminator



The misclassification rate $\text{Prob}(Z > \frac{1}{2}) = 30.9\%$

standard normal variate

3. a) For the volume of 4d ball with unit radius we need, 1% accuracy with 95% confidence

If p was the probability the points $x \in [-1,1]^4$ satisfy $\|x\|_2 \leq 1$,

then, with, N draws, the number k of points inside the ball has a binomial distribution.

$$E[k] = Np \qquad Var[k] = Npq$$

$$\hat{p} = \frac{k}{N}$$

$$E[\hat{p}] = p \qquad Var[\hat{p}] = \frac{pq}{N}$$

The 95% confidence interval is $\left[\hat{p} - 1.96\sqrt{\frac{\hat{p}\hat{q}}{N}}, \hat{p} + 1.96\sqrt{\frac{\hat{p}\hat{q}}{N}}\right]$

for large N. So, we are asking for $1.96\sqrt{\frac{\hat{p}\hat{q}}{N}} / \hat{p} = 0.01 \Rightarrow \frac{\hat{q} \times 10^{+4} \times (1.96)^2}{\hat{p}} = N$

Of course, we do not know $\hat{p}$ yet. We can do a preliminary run to find $\hat{p}$.

There are 3 runs of 100 steps each. The $\hat{p}$ values I get are 0.31, 0.29 and 0.35. So $\hat{p} \simeq 0.32 \pm .04$

$$N \simeq \frac{0.68}{0.32} \times 10^4 \times (1.96)^2 = 8 \times 10^4$$

In two different runs of length I got $\hat{p} = 0.3100$ and 0.3108.

$$\text{Volume} = 16 \times \hat{p} = 4.96, 4.9732$$

Consistent with our expectation of 1% variation.

Final report $4.96 \pm .05$

~~5c~~

b) Ans. $\pi^2/2 = 4.9348$

This within our predicted range.