Course: Classical Mechanics

Problem Set 1

Due: Oct 4, 2018

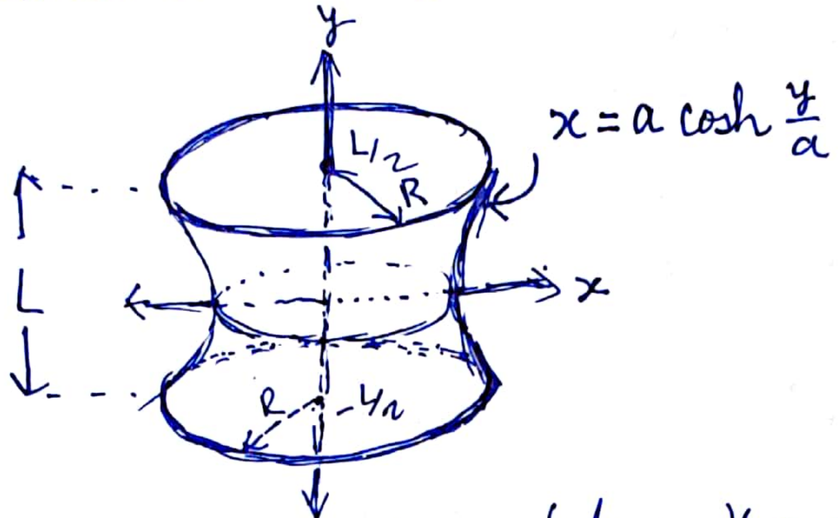
1. Using generalized coordinates q_j, we express kinetic energy $T = \sum_i \frac{1}{2} m_i \vec{v}_i^{\,2} = T(q_1, \cdots, q_n, \dot{q}_1, \cdots, \dot{q}_n, t)$.
Note the $\frac{dT}{dt}$ could be expressed in terms of q_j, $\dot{q}_j$, $\ddot{q}_j$'s and time t. Lagrange's equation is $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j$, Q_j being the generalized force.

Show that $\frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Q_j$.

$$\left[\frac{\partial \dot{T}}{\partial \dot{q}_j} = \frac{\partial \dot{T}}{\partial q_j} \bigg|_{\{q_k\}, \{\dot{q}_\ell\}_{\ell \neq j}, \{\ddot{q}_k\}, t} \right.$$

Namely this partial derivative is calculated holding q's, other $\dot{q}$'s, $\ddot{q}$'s at t fixed.]

2. Consider a soap film spanning two identical coaxial circular rings of radius R, spaced by distance L. Using the solution from class,



the surface is made by rotating the curve $x = a\cosh\frac{y}{a}$, with the boundary conditions

$x = R, \quad y = \pm\frac{L}{2}$.

So $\quad R = a\cosh\frac{L}{2a} \Rightarrow \frac{R}{a} = \cosh\frac{L}{R}\frac{R}{2a}$

$$\Rightarrow Z = \cosh \alpha Z/2$$

with $\alpha = L/R$, determined by boundary conditions, and $Z = R/a$, to be determined from the equation above.

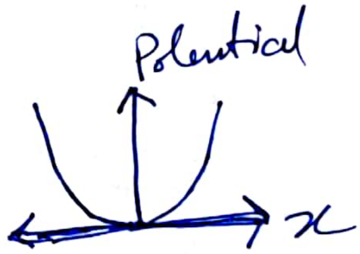
a) Show that for $\alpha < \alpha_c$, a critical value, there are two solutions of Z, using graphical method.

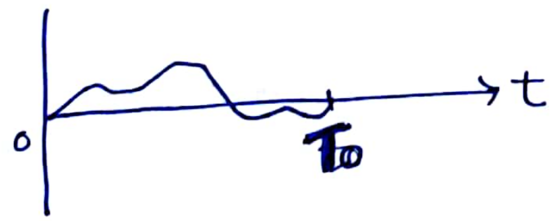
b) Show that these two solutions have have two different areas. Which one corresponds to the lower area, the larger or the smaller z solution?

c) Determine α_c numerically.

3. Consider a harmonic oscillator: $L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2$. The action is given by

$$S = \int_0^{T_0} \left(\frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 \right) dt .$$





Consider $x(t)$ such that $x(0) = x(T_0) = 0$

One solution is $x_0(t) = 0$. Now we take perturbation $x(t) = 0 + \eta(t)$

with $\eta(0) = 0$, $\eta(T_0) = 0$.

$$S = \int_0^{T_0} \left(\frac{1}{2} m \dot{\eta}^2 - \frac{1}{2} m \omega^2 \eta^2\right) dt$$

Express $\eta(t)$ as an expansion in sinusoids, satisfying boundary conditions

$$\eta(t) = \sqrt{\frac{2}{T_0}} \sum_{n=1}^{\infty} a_n \sin \frac{\pi n t}{T_0}$$

a) Show that $S = \frac{m}{2} \sum_{n=1}^{\infty} \left(\frac{\pi^2 n^2}{T_0^2} - \omega^2\right) a_n^2$

b) When $T_0 < \frac{\pi}{\omega}$, is $x(t) = 0$ a minimum for the action S?

c) What happens when $T_0 > \pi/\omega$?