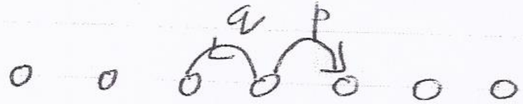
Random Walk and Diffusion



Random walk
on a lattice

Walks

r l r r

R = # right moves
L = # left moves

Displacement on lattice

$$X = R - L$$

Number of hops

$$T = R + L$$

$$\text{Prob}(R \text{ right hops}) = \binom{T}{R} p^R q^L$$

Binomial distr$\underline{\text{n}}$

$$R = \frac{T+X}{2} \qquad L = \frac{T-X}{2}$$

So $$P(X) = \frac{T!}{\left(\frac{\cancel{T}+X}{2}\right)!\left(\frac{T-X}{2}\right)!}(pq)^{T/2}\left(\frac{p}{q}\right)^{X/2}$$

We could use ~~[illegible]~~ Stirlings formula, but we already know central limit theorem.

$$X = \sum_{i=1}^{\cancel{T}} k_i \qquad k_i = 1 \text{ with prob } p$$
$$= -1 \text{ '' '' } q$$

$$\langle X \rangle = T(p-q)$$

$$\sigma_X^2 = T \times Var(k)$$

$$Var(k) = \cancel{[illegible]} \langle k^2 \rangle - \langle k \rangle^2$$
$$= 1 - (p-q)^2 = 4pq$$

So $\sigma_X^2 = 4Tpq$

$$P(X) \propto e^{-\frac{\left(X - T(p-q)\right)^2}{8Tpq}}$$

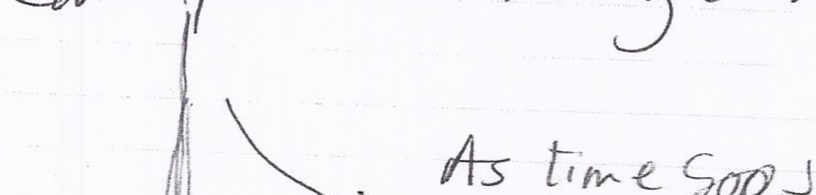
Defining

$$Xa = x$$
$$T\tau = t$$

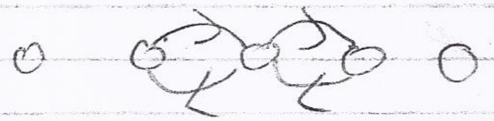
$$v = \frac{(p-q)a}{\tau} \qquad D = \frac{2pqa^2}{\tau}$$

$$p(x) = \frac{e^{-\frac{(x-vt)^2}{4Dt}}}{\sqrt{8\pi D t}}$$

Came from normalization.



Diffusion Equation



$$P(X, T+1) - P(X,T) = J_{X-1,X}(T) - J_{X,X+1}(T)$$

$$J_{X,X+1}(T) = p\,P(X,T) - q\,P(X+1,T)$$

$$\tau \frac{\partial}{\partial t} P(x,t) = -a\tau \frac{\partial}{\partial x} J(x)$$

$$J_{X,X+1} = p\,P(X,T) - q\,P(X+1,T)$$

$$= \frac{p-q}{2}\left(P(X,T) + P(X+1,T)\right) - \frac{p+q}{2}\left(P(X+1,T) - P(X,T)\right)$$

$$J(x) = v\,P(x,t) - D\frac{\partial}{\partial x} P(x,t)$$