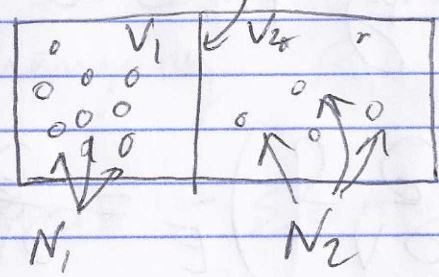
Applications of Entropic Forces

Osmotic pressure:

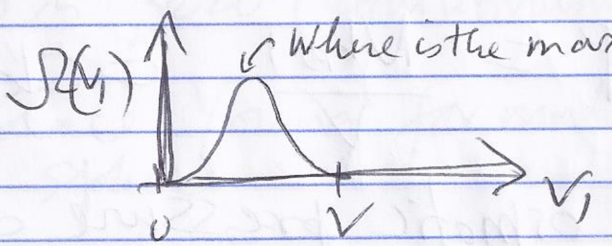
Semipermeable [redacted] Membrane: Lets the fluid to pass but not particles



$V_1 + V_2 = V$

$N_1 + N_2 = N$

$$\Omega(V_1) \propto V_1^{N_1} V_2^{N_2} = V_1^{N_1} (V - V_1)^{N_2}$$



$$\frac{\partial}{\partial V_1} \ln \Omega(V_1) = \frac{\partial}{\partial V_1} \left[N_1 \ln V_1 + N_2 \ln(V - V_2) \right]$$

$$= \frac{N_1}{V_1} - \frac{N_2}{V - V_2} = \frac{N_1}{V} - \frac{N_2}{V_2}$$

So the best position of the membrane is when it has moved to make the concentrations equal.

For small number of particles, we could have some fluctuations in V, which can be estimated by taking second ~~deriv~~ derivative of $\ln \Omega(V,)$

By the way pressure is given by

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_E = \frac{\partial}{\partial V}\left(k_B N \ln V\right)$$

$$= \frac{N k_B}{V}$$

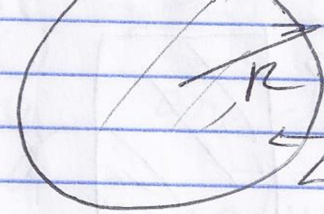
Or $P = \frac{N k_B T}{V} = c k_B T$

Extra osmotic pressure due to particles $P_{osm} = c k_B T$

van't Hoff Relation.

Osmotic pressure can make or break a cell.

For a sphere of radius R, and surface tension Σ



Δp pressure diff

Changing R [redacted] has virtual work [redacted]

$$\Delta p \, dV - \Sigma \, dA = \Delta p \, d\left(\frac{4\pi}{3} R^3\right) - \Sigma \, d\left(4\pi R^2\right)$$

$$= 4\pi\left[\Delta p \, R^2 dR - 2\Sigma R \, dR\right]$$

This is zero (equilibrium) $\boxed{\Delta p = \frac{2\Sigma}{R}}$

Laplace Young formula for sphere.

For $C \approx$ [redacted] $1\,mM$ [Concentraion of many molecules, etc]

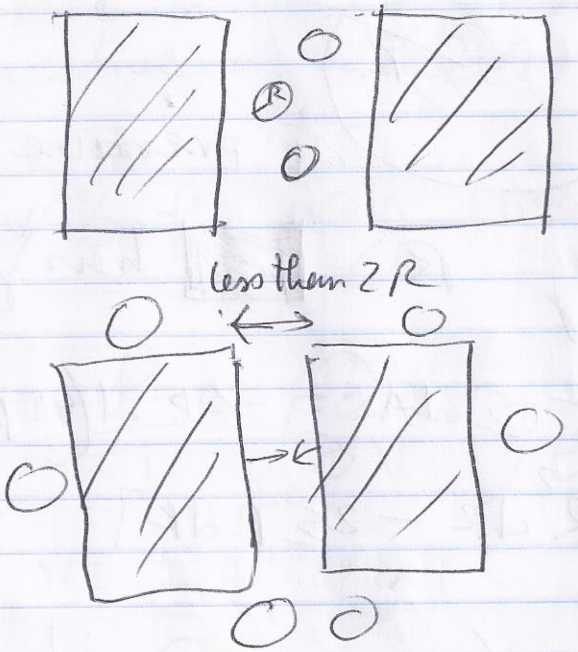
$$\Delta p \approx \frac{10^{-3} \times 6 \times 10^{23}}{10^{-3}\, m^3} \times 4 \times 10^{-21} \times N m$$

$$= 24 \times 10^2 \, N m^{-2} = 2.4 \times 10^3 \, Pa$$

$$\Sigma = \frac{\Delta P \, R}{2} \quad \text{with } R = 10\,\mu m$$

$$= 1.2 \times 10^{-3} \, N/m$$, often good enough to rupture a cell!

Depletion force



Force $A\,c\,k_B T$ when separation less than $2R$.

$$\Delta F = -A(2R)\,c\,k_B T$$

Missed entropy of space in between.

If there is good fit between surfaces

$AR \sim 10^4 (nm)^3$ If $c \sim 0.1\,mM$

$= 6\times 10^{22}\, m^{-3}$

$= 6\times 10^{-5} (nm)^{-3}$

then $\Delta F \sim -k_B T$