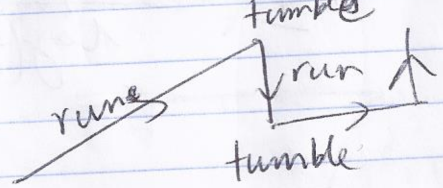
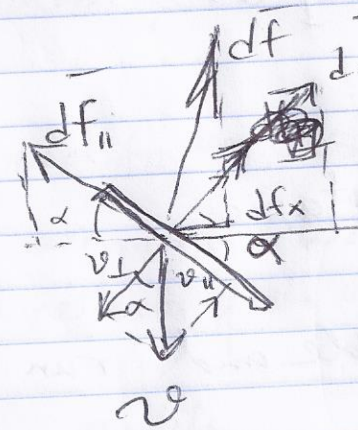
E. coli motion



When thing are getting better (more nutrient / less toxin) motor switches less often to clockwise (CW) rotation, resulting in less tumbles.



$$v_\perp = v\cos\alpha$$

$$v_\parallel = v\sin\alpha$$

$$(d\vec{f}_\parallel)_x = -df_\parallel \cos\alpha = -c_\parallel v_\parallel \, ds \cos\alpha$$

$$= -c_\parallel v \sin\alpha \cos\alpha \, ds$$

$$(d\vec{f}_\perp)_x = df_\perp \sin\alpha = c_\perp v_\perp \, ds \sin\alpha$$

$$= c_\perp v \cos\alpha \sin\alpha \, ds$$

$$(d\vec{f})_x = (c_\perp - c_\parallel) v \sin\alpha \cos\alpha \, ds$$

$$= \frac{1}{2}\left(C_\perp - C_\parallel\right) v \sin 2\alpha \; ds$$

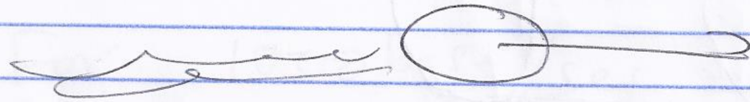
Note $C_\perp \neq C_\parallel$ is essential for motion

If the radius of the helix is R, the angular velocity, ω, summing over the helix only leaves the x component.

$$\vec{F} = \frac{1}{2}\left(C_\perp - C_\parallel\right) v \sin 2\alpha \; L$$

length along the helix

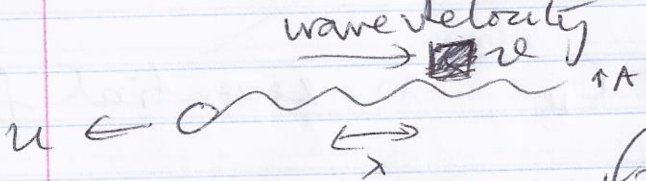
This force has to be ~~mad~~ matched by drag due to horizontal motion.



F = (drag on body of the cell
+ " " flagella)

In some limits drag or flagella could be ~~as impo~~ more important.

Motion of sperm



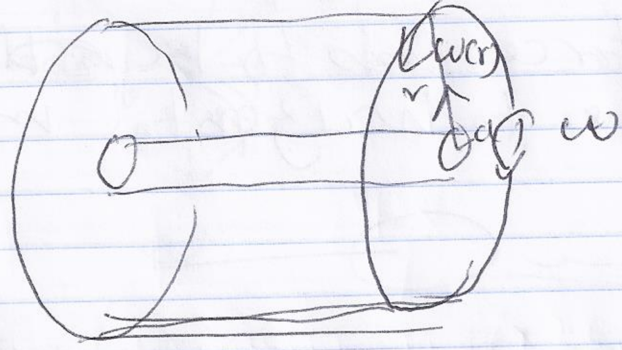
Amplitude

$$u = \frac{1}{2}\left(\frac{2\pi A}{\lambda}\right)^2 v$$

wavelength

for small $A \ll \lambda$ and $C_\perp \approx 2C_\parallel$

Drag on a rotating body



Torque on the body

$$\tau(r) = \underbrace{(2\pi r L)}_{\text{Area}}\ \underbrace{r\ \underbrace{\eta\ \overbrace{r\frac{d\omega(r)}{dr}}^{\text{Strain}}}_{\text{force per unit area}}}_{\text{Torque per unit area}}$$

$\tau(r)$ needs to be a constant for steady motion

$$2\pi\eta r^3 L \frac{d\omega(r)}{dr} = \tau$$

$$\Rightarrow \quad \omega(r) = -\frac{\tau}{4\pi\eta L r^2}$$

with $\omega(a) = \omega$ $\qquad \tau = -4\pi\eta L a^2\omega$

Work done per turn $= 2\pi \times 4\pi\eta L a^2\omega$

$$8\pi^2\eta L a^2\omega$$

When DNA is replicated, DNA ahead rotates

For 1000 bases/sec rate and 10 bases to a full rotation

$\omega = \frac{2\pi}{(100)^{-1}\sec}$ $\qquad a = 1nm$

$2\pi\tau = 8\pi^2\eta a^2\omega L \approx 8\pi^2 \times 10^{-3} \times (10^{-9})^2 \times 2\pi \times 100 \times L = 16\times\pi^3\times10^{-19}\times L$

$\approx 5\times10^{-17}\times L$

Energy from 1 ATP $\quad 20 k_B T$

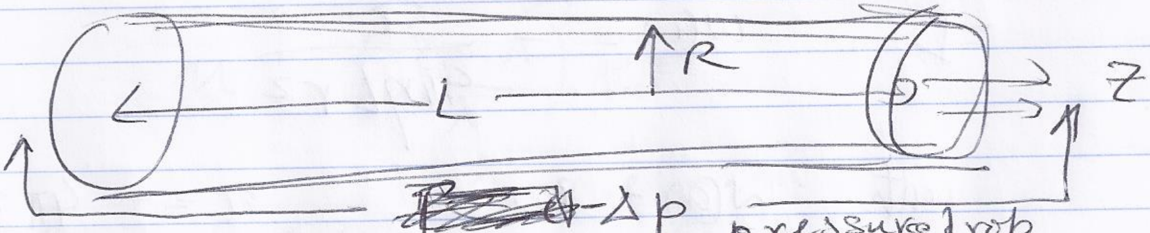
$= 20 \times 4.1 \times 10^{-21}$ J

If all energy went to viscous drag $\approx 0.8\times10^{-19}$ J

$L \approx \frac{0.8\times10^{-19}}{5\times10^{-17}}$ m $= 1.6\times10^{-3}$ m ~ 1.6 mm.

Drag is not a big deal in this context.

Flow in tubes/pipes



$\vec{f} = 0 \quad -\vec{\nabla} p + \eta \nabla^2 \vec{u} = 0$

$\bar{\nabla} \cdot \bar{u} = 0$

Since $\bar{\nabla} \cdot \bar{u} = 0$

Use cylindrical coordinates

$\nabla^2 p = \eta \nabla^2 \bar{\nabla} \cdot \bar{u} = 0$

$$p = p_0 - \frac{\Delta p x}{L}$$

$\bar{u}$ has only $u_z(r)$ component

$$\eta \nabla^2 u_z = -\frac{\Delta p}{L}$$

$u_z(R) = 0$

$$u_z(r) = \frac{\Delta p}{4\eta L}\left(R^2 - r^2\right)$$

Total flow

$$Q = 2\pi \int_0^R dr\, r\, u_z(r) = \frac{\pi \Delta P}{2\eta L} \int_0^R dr\, r (R^2 - r^2)$$

$$= \frac{\pi \Delta P}{2\eta L} \left[\frac{r^2 R^2}{2} - \frac{r^4}{4} \right]_{r=0}^{r=R} = \frac{\pi \Delta P}{2\eta L} \left(\frac{R^4}{2} - \frac{R^4}{4} \right)$$

$$= \frac{\pi R^4}{8 \eta L} \Delta P$$

Hagen-Poiseuille Formula!

Few percent change in R produces a large change in Q.

Used in regulating blood flow by slighty altering the vessel diameter.