24th April, 2014

Course: Biophysics

Problem Set: 6

Due: 1st May, 2014

7.10 [T2] *Charged surfaces*

Use some numerical software to solve Equation 7.29 for β as a function of plate separation $2D$ for fixed charge density σ_q. For concreteness, take σ_q to equal $e/(20\,\mathrm{nm}^2)$. Now convert your answer into a force by using Equation 7.31 and compare your answer with Figure 7.11. Repeat with other values of σ_q to find (roughly) the one that best fits the upper curve in the figure at separation greater than 2 nm. If this surface were fully dissociated, it would have one electron charge per 7 nm^2. Is it fully dissociated?

Relevant parts of the text book!

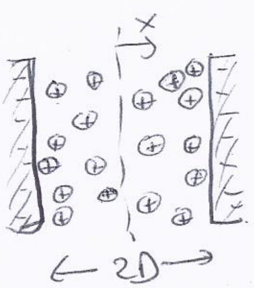

Fig 7.8b (slightly modified)

The rest of the procedure is familiar. Substituting the trial solution into the Poisson–Boltzmann equation (Equation 7.23) gives $A = 2$ and $\beta = \sqrt{2\pi \ell_B c_0}$. The boundary condition at $x = -D$ is again Equation 7.24. Imposing the boundary conditions on our trial solution gives a condition fixing β:

$$4\pi \ell_B(\sigma_q/e) = 2\beta \tan(D\beta). \qquad (7.29)$$

Given the surface charge density $-\sigma_q$, we solve Equation 7.29 for β as a function of the spacing $2D$; then the desired solution is

$$\overline{V}(x) = 2\ln\cos(\beta x), \quad \text{or} \quad c_+(x) = c_0(\cos\beta x)^{-2}. \qquad (7.30)$$

$$f/(\text{area}) = c_0 k_B T. \qquad \text{repulsion of like-charged surfaces, no added salt} \qquad (7.31)$$

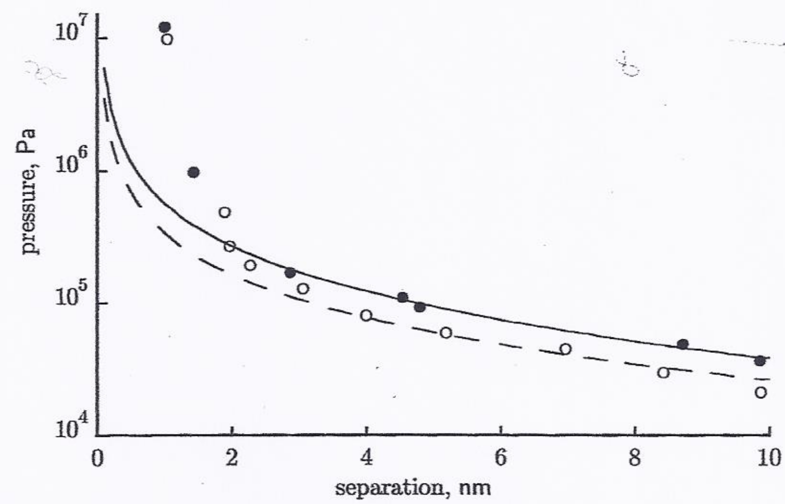

Figure 7.11: (Experimental data with fits.) The repulsive pressure between two positive[ly] charged surfaces in water. The surfaces were egg lecithin bilayers containing 5 mole% [or] 10 mole% phosphatidylglycerol (*open and filled circles*, respectively). The curves show one-parameter fits of these data to the numerical solution of Equations 7.29 and 7.31. The f[it] parameter is the surface charge density σ_q. The *dashed line* shows the solution with one prot[on] charge per 24 nm^2; the *solid line* corresponds to a higher charge density (see Problem 7.10). A[t] separations below 2 nm, the surfaces begin to touch and other forces besides the electrosta[tic] one appear. Beyond 2 nm, the purely electrostatic theory fits the data well, and the membra[ne] with a larger density of charged lipids is found to have a larger effective charge density, [as] expected. [Data from Cowley et al. 1978.]