28th Jan 2014

Course: Biophysics

Problem Set: 1

Due: 4th Feb 2014

observation.

1.6 *Atomic sizes, again*

In 1858, J. Waterston found a clever way to estimate molecular sizes from macroscopic properties of a liquid, by comparing its surface tension and heat of vaporization.

The surface tension of water, Σ, is the work per unit area needed to create more free surface. To define it, imagine breaking a brick in half. The two pieces have two new surfaces. Let Σ be the work needed to create these new surfaces, divided by their total area. The analogous quantity for liquid water is the surface tension.

The **heat of vaporization** of water, Q_{vap}, is the energy per unit volume we must add to liquid water (just below its boiling point) to convert it completely to steam (just above its boiling point). That is, the heat of vaporization is the energy needed to separate every molecule from every other one.

Picture a liquid as a cubic array with N molecules per centimeter in each of three directions. Each molecule has weak attractive forces to its six nearest neighbors. Suppose it takes energy ϵ to break one of these bonds. Then the complete vaporization of 1 cm^3 of liquid requires that we break all the bonds. The corresponding energy cost is $Q_{vap} \times (1\,cm^3)$.

Next consider a molecule on the *surface* of the fluid. It has only five bonds—the nearest neighbor on the top is missing (suppose this is a fluid–vacuum interface). Draw a picture to help you visualize this situation. Thus, to create more surface area requires that we break some bonds. The energy needed to do that, divided by the new area created, is Σ.

a. For water, $Q_{vap} = 2.3 \cdot 10^9\,J\,m^{-3}$ and $\Sigma = 0.072\,J\,m^{-2}$. Estimate N.

b. Assuming the molecules are closely packed, estimate the approximate molecule diameter.

c. What estimate for Avogadro's number do you get?

Remember that the density of water is about $\frac{1g}{(cm)^3}$ and one mole of water is about 18 g.