Course: Mathematical Physics

Problem Set 1

Due: Sept 25, 2019

1. Consider the set $S \subset \mathbb{C}$ defined by $\{t + i\sin\frac{\pi}{t} \mid t \in (0,1]\}$

   a) Is $S$ open?

   b) Is $S$ closed?

   Give reasons for your answers.

2. The series $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)(\ln(\ln n))^{\alpha}}$, $\alpha \in \mathbb{R}$, is a series of positive terms.

   a) Show that this series either converges or diverges to $+\infty$.

   b) Find the condition on $\alpha$ for convergence or divergence of this series.

3. For $u = (u_1, \cdots, u_k) \in \mathbb{R}^k$, the norm (in fact $l_2$ norm) $\|u\|$ is defined by $\|u\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_k^2}$.

   Consider $\{x_n\}$, a sequence of members of $\mathbb{R}^k$. This sequence converges to $x$ if $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$, s.t. $n > N \Rightarrow \|x_n - x\| < \varepsilon$. This sequence is Cauchy if $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$, s.t., $m, n > N \Rightarrow \|x_m - x_n\| < \varepsilon$. (See Sec. 2.4.1 of Vaughn).

   Prove that $\{x_n\}$ is convergent iff it is Cauchy.

Bonus Problem: We discussed Riemann seres/rearrangemet theorem. You can review the 'algorithm' from the corresponding wikipedia page. Write a program to rearrange $\sum_{k=0}^{\infty} \frac{(-1)^k}{k}$ to converge to $\pi = 3.14159\cdots$. Generate the first 2000 terms or more and plot the partialsums.