LECTURE 7.
SPEED OF SOUND.

SPEED OF SOUND IN AIR
SOUND IN AIR IS A LONGITUDINAL WAVE
IT DEPENDS OF PRESSURE & THE
DENSITY OF AIR.

P = PRESSURE = FORCE PER UNIT AREA.

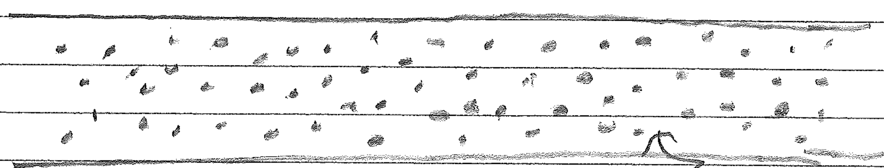
$$\text{FORCE} = kg\cdot\frac{m}{s^2} \qquad \text{AREA} = m^2$$

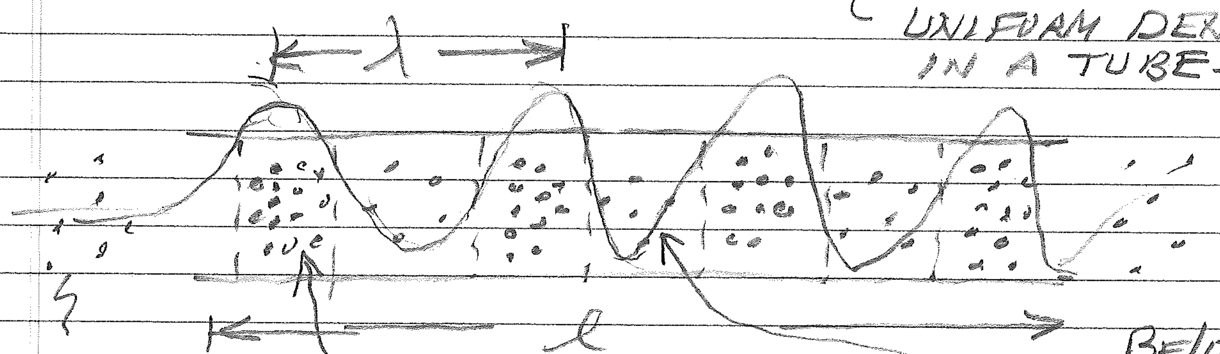
$$P = \frac{kg\,\frac{m}{s^2}}{m^2} = kg\,\frac{1}{m\cdot s^2}$$

$$\text{DENSITY}\quad \rho = \frac{kg}{m^3}. \qquad \frac{P}{\rho} = \frac{m^2}{s^2}$$

$$v_{\text{SOUND}} = v \propto \sqrt{\frac{P}{\rho}}.$$

A SOUND WAVE INVOLVES PRESSURE VARIATIONS OR DENSITY VARIATIONS IN REGIONS OF SPACE. THESE VARIATIONS ARE ABOVE - COMPRESSION - OR BELOW - RAREFIED - NORMAL ATMOSPHERIC PRESSURE





SPEED OF SOUND.

v_{SOUND} MUST INVOLVE SOMETHING MORE THAT JUST PUTTING $\sqrt{P}$

SPEED OF SOUND INVOLVES A DENSITY WAVE OR PRESSURE VARIATIONS $\Rightarrow$

THE CORRECT QUANTITY FOR v MUST CONTAIN

$\frac{\partial P}{\partial V}$ ← PRESSURE VARIATIONS WITH VOLUME

$$\underbrace{-V}_{\text{NEED THIS SO UNITS ARE OK}}\frac{\partial P}{\partial V} \equiv B \quad \text{BULK MODULUS.}$$

HOOKE'S LAW $F=-kx$

$$v = \sqrt{\frac{B}{\rho}}$$

BULK MODULUS IS A STRESS STRAIN RELATION (LIKE $F=-kx$)

STRESS $\frac{F}{A} = \Delta P$ STRAIN $\frac{\Delta V}{V}$

$\frac{F}{A} = \text{CONSTANT} \frac{\Delta V}{V}$

THE B IS RELATED TO THE COMPRESSIBILITY

$$K_{COMPRESSIBILITY} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)$$

NOTE PRESSURE, ρ DENSITY, T (TEMP) ARE CONNECTED BY EQUATION OF STATE

IDEAL GAS $PV = nRT$ ← IN KELVIN

n ← # OF MOLES.

R = GAS CONSTANT $= 8.314\ J/({}^{0}K \cdot \text{MOLE})$ PER MOLE

$= \frac{PV}{nR}$

J ← PV IS AN ENERGY

SPEED OF SOUND.

$PV = N k_B T$ N = NUMBER OF ATOMS.

$k_B = R/N_A$

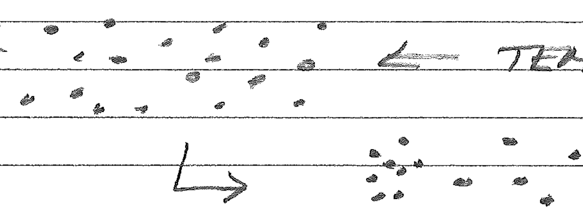
N_A = AVOGADRO CONSTANT $= 6.022 \times 10^{23}$ / MOLE

$k_B = 1.38 \times 10^{-23}$ J/°K

$= 8.617 \times 10^{-5}$ eV/°K.

WHAT DO WE HOLD FIXED WHEN WE EVALUATE $B = -V \dfrac{\partial P}{\partial V}$ OR $K = -\dfrac{1}{V}\dfrac{\partial V}{\partial P}$

NORMAL DENSITY



← TEMP. T

SAME T ?

ISOTHERMAL COMPRESSION   ISOTHERMAL EXPANSION

NO.

VARIATIONS ARE DONE WITH NO HEAT LEAVING OR ENTERING – CALLED ADIABATIC

TRANSFER OF HEAT IS A SLOW DIFFUSION LIKE PROCESS.

DENSITY VARIATION IN SOUND IS FAST

$\omega, f = \dfrac{1}{T}$ $\omega \sim 20 - 20000$ Hz.

$T \sim \dfrac{1}{f} = \dfrac{2\pi}{\omega} \sim \dfrac{6}{600\,\text{Hz}} = .01$ sec.

SPEED OF SOUND

$$B = -V\left(\frac{\partial P}{\partial V}\right)_{\Delta Q = 0}$$

ADIABATIC

NEED ADIABATIC CONNECTION BETWEEN

P, V $\quad PV^{\gamma} = \text{CONSTANT}.$

$\gamma = C_p/C_v \quad C_p = C_v + k_B \text{ or } R$

DEPENDING ON USING. $PV = Nk_BT$ $= nRT$ or EVEN 1

$dU = dQ - dW$

INTERNAL ENERGY

HEAT ADDED + SUBTRACTED − → 0

ADIABATIC PROCESS.

WORK DONE BY SYSTEM $= pdV$

$dU = -pdV$ IDEAL GAS CASE - MONATONIC GAS.

$U = \frac{3}{2}Nk_BT \quad dU = \frac{3}{2}Nk_B dT$

$p = \frac{Nk_BT}{V}$

$\frac{3}{2}\cancel{Nk_B}\, dT = -\cancel{Nk_B}\, T\frac{dV}{V}$

$\frac{3}{2}\frac{dT}{T} = -\frac{dV}{V} \quad \frac{3}{2}\ln T = \ln T^{3/2} = -\ln V + \text{CONSTANT}$

$\ln T^{3/2} + \ln V = \text{CONSTANT} = \ln T^{3/2}V$

$T^{3/2}V = \text{ANOTHER CONSTANT}$

$T^{3/2}V = \text{CONSTANT} \qquad TV^{2/3} = \text{CONSTANT}$

$$T = \frac{PV}{Nk_B}$$

$$PV \cdot V^{2/3} = \text{CONSTANT} \cdot Nk_B = PV^{1+2/3}$$
$$= PV^{5/3}$$

FOR AN IDEAL GAS - MONATOMIC

$C_p = C_V + k_B \qquad C_V = \frac{3}{2}k_B$ PER PARTICLE

$C_p = \frac{5}{2}k_B \qquad = \left(\frac{dU}{dT}\right)_V - \left(\frac{dQ}{dT}\right)_V \quad p\,dV = 0$ AT CONSTANT V

$$\frac{C_p}{C_V} = \frac{5/2}{3/2} = 5/3$$

IN GENERAL $\quad pV^{\gamma} = \text{CONSTANT} \quad \gamma = C_p/C_V$

γ IS SOME NUMBER THAT HAS TO BE GIVEN.

$$P = \frac{\text{CONSTANT}}{V^{\gamma}} \qquad \left.\frac{\partial P}{\partial V}\right|_{\Delta Q=0} = \frac{d}{dV}\left(\frac{\text{CONST}}{V^{\gamma}}\right)$$

$$\left(\frac{\partial P}{\partial V}\right)_{\Delta Q=0} = -\gamma\frac{\text{CONST}}{V^{\gamma+1}}$$

$$-V\left(\frac{\partial P}{\partial V}\right)_{\Delta Q=0} = +\gamma\frac{\text{CONST}}{V} = \gamma P.$$

$$v_{\text{SOUND}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}.$$

$\rho_{AIR} = 1.29\ kg/m^3$. ($0°C$ $273°K$)

$P_{ATMOSPHERIC} = 1.00 \times 10^5\ N/m^2 = 1.00 \times 10^5$ PASCALS ($0°C$)

$\gamma_{AIR} = 1.4$ (NOT AN IDEAL MONATOMIC GAS)

$$v = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}}\ m/s = 331.\ m/s$$

TEMPERATURE DEPENDENCE

$$PV = N_{ATOMS}\, k_B T = nRT \quad T \text{ in } °K$$

$$P = nRT/V$$

$$\frac{v(T°K)}{v(0°K)} = \sqrt{\frac{T}{273°}} = \sqrt{\frac{273° + (T - 273°)}{273°}}$$

$$= \left(1 + \frac{(T°C)}{273}\right)^{1/2} \simeq 1 + \frac{1}{2} \frac{T°C}{273}$$

USED $(1 + x_{SMALL\#})^{1/2} = 1 + \frac{x_{SMALL\#}}{2}$

$$v(T°K) = v(0°K)\left(1 + \frac{T°C}{546}\right) = 331\,m/s\left(1 + \frac{T°C}{546}\right)$$

$$= 331\,m/s + .6\,T°C$$

$\frac{331}{546} \simeq \frac{3}{5} = .6 \qquad \frac{331}{546} = .606$

SPEED OF SOUND INCREASES WITH T