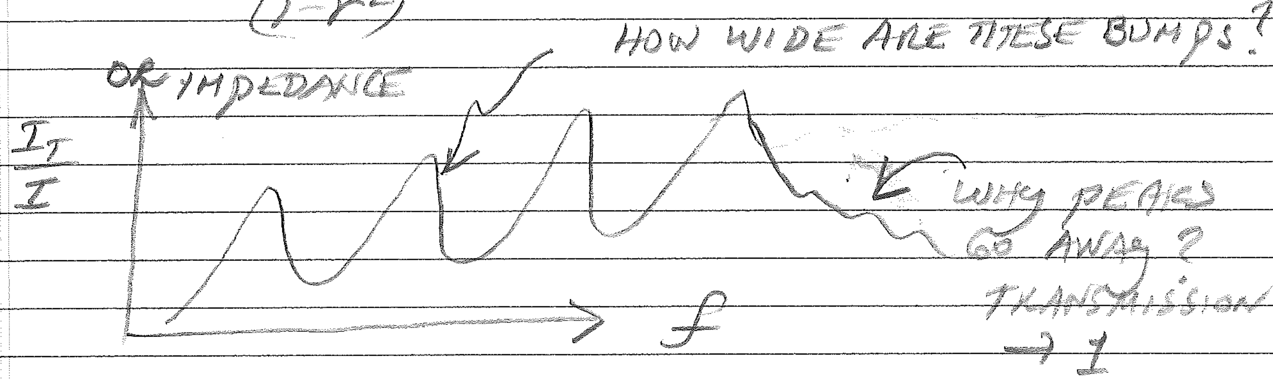
LECTURE 21

REMINDER – EXAM WED APRIL 16.
ONE SHEET (BOTH SIDES) OF EXPRESSIONS.
EXAM PARALLELS HOMEWORK

NEXT MONDAY – REVIEW OF HW ETC

TOPICS TODAY
WHAT SOUNDS GOOD & HOW TO BE A COMPOSER
FINISH $I_T/I_0 = \dfrac{1}{1+F\sin^2\frac{\delta}{2}}$.

$F = \dfrac{4r^2}{(1-r^2)^2}$



WHAT IS IMPEDANCE?
~~POWER SPECTRUM – EXAMPLE BLACK BODY RADIATION~~

COMBINATION TONES – CHAPTER 8
IMPORTANT FOR LAB.

AM – AMPLITUDE MODULATION &
FM – FREQUENCY MODULATION.

MISSING HANDOUT—LECTURE 26

Birkhoff's Aesthetic Theory

A partial answer may come from the "*theory of aesthetic value*" propounded by the American mathematician George David Birkhoff (1884–1944). Birkhoff's theory, in a nutshell, says that for a work of art to be pleasing and interesting it should neither be too regular and predictable nor pack too many surprises. Translated to mathematical functions, this might be interpreted as meaning that the power spectrum of the function should behave neither like a boring "brown" noise, with a frequency dependence f^{-2}, nor like an unpredictable white noise, with a frequency dependence of f^{0}.

$f(t) = \sum_n a_n \sin 2\pi n f_1 t + b_n \cos 2\pi n f_1 t$

POWER IN MODE nf_1 $= a_n^2 + b_n^2$ · $f_n = nf_1$

In a white-noise process, every value of the process (e.g., the successive frequencies of a melody) is completely independent of its past—it is a total surprise (see Figure 4A). By contrast, in "brown music" (a term derived from Brownian motion), only the *increments* are independent of the past, giving rise to a rather boring tune (see Figure 4B). Apparently, what most listeners like best, and not only in Bach's time, is music in which the succession of notes is neither too predictable nor too surprising—in other words, a spectrum that varies according to f^{α}, with the exponent α between 0 and -2. As Richard Voss discovered, the exponents found in most music are right near the middle of this range: $\alpha = -1$, giving rise to the hyperbolic power law f^{-1} (see Figure 4C) [VC 78]. Or, as Balthazaar van der Pol once said of Bach's music, "It is great because it is inevitable [implying $\alpha < 0$] and yet surprising [$\alpha > -2$]." (I found this quotation in Marc Kac's captivating autobiography, *Enigmas of Chance* [Kac 85].)

HOW TO BE A COMPOSER →

if $a_n \sim \frac{1}{n}$

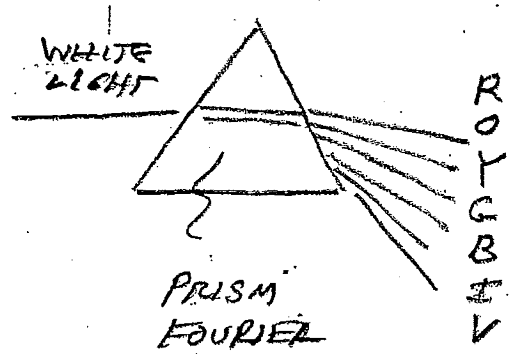
$a_n^2 \sim \frac{1}{f_n^2} \sim \frac{1}{f^2}$

(A)

(B)

(C)

WHITE NOISE = WHITE LIGHT



PRISM FOURIER ANALYZES LIGHT

RANDOM (EQUALITY for All KEYS)

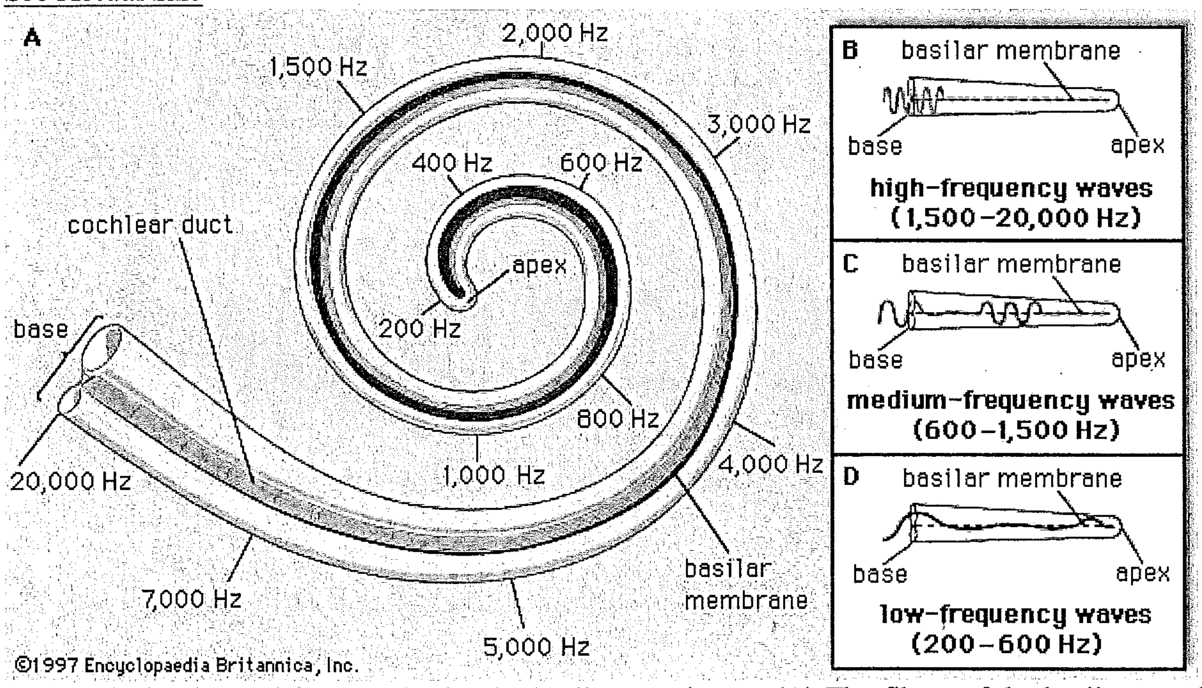
Figure 4 (A) "White" music produced from independent notes; (B) "brown" music produced from notes with independent increments in frequency; and (C) "pink" music—frequencies and durations of notes are determined by $1/f$ (pink) noise [VC 78].

US INCOME DISTRIBUTION — $\boxed{\text{INEQUALITY}}$ FOR All

400 PEOPLE HAVE MORE WEALTH THAN ENTIRE BOTTOM 50%

basilar membrane: analysis of sound frequencies

See Actual Size



The analysis of sound frequencies by the basilar membrane. (A) The fibres of the basilar membrane become progressively wider and more flexible from the base of the cochlea to the apex. As a result, each area of the basilar membrane vibrates preferentially to a particular sound frequency. (B) High-frequency sound waves cause maximum vibration of the area of the basilar membrane nearest to the base of the cochlea; (C) medium-frequency waves affect the centre of the membrane; (D) and low-frequency waves preferentially stimulate the apex of the basilar membrane. (The locations of cochlear frequencies along the basilar membrane shown are a composite drawn from different sources.)

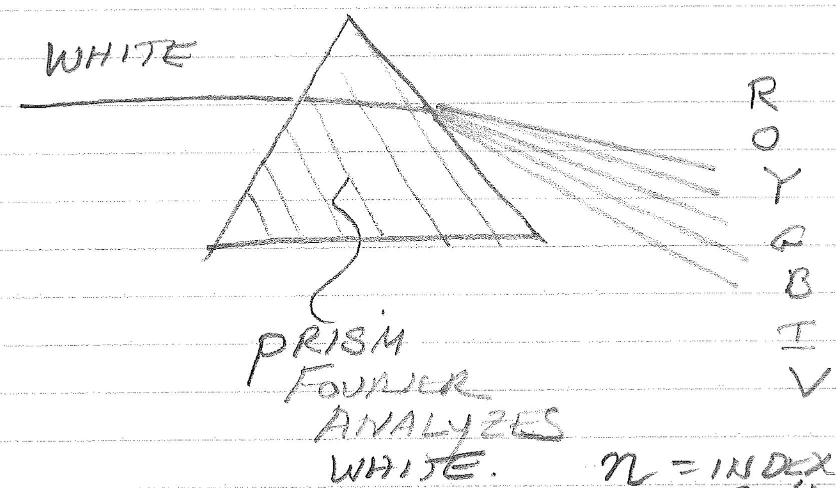
$f(t) = \sum a_n \sin 2\pi f_n t + b_n \cos 2\pi f_n t \qquad f_n = n f_1$

POWER IN MODE $n f_1$ IS $a_n^2 + b_n^2$.

a_n or b_n

BEST EXAMPLE OF A POWER SPECTRUM

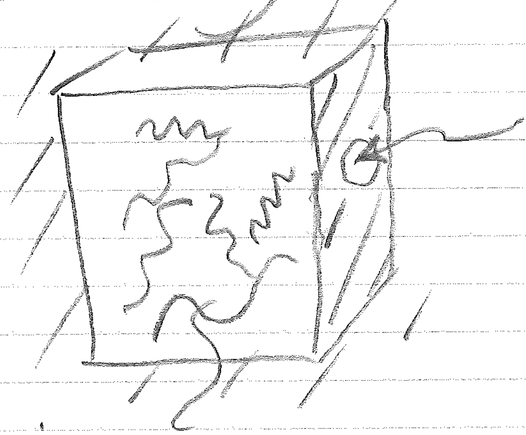
POWER SPECTRUM OR ENERGY SPECTRUM.



ASK HOW MUCH OF INTENSITY IS IN VARIOUS COLORS.

n = INDEX OF REFRACTION $= n(\lambda)$

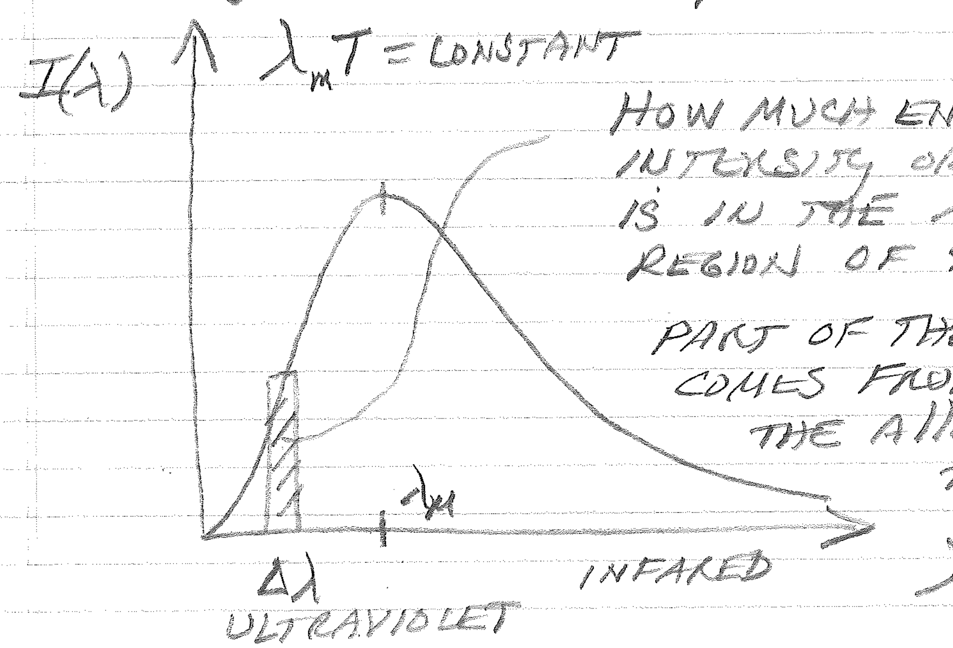
MOST FAMOUS INTENSITY SPECTRUM IS BLACK BODY RADIATION –



SPECTRUM OF E&M RADIATION

CAVITY HEATED TO TEMPERATURE T

$\lambda_m T$ = CONSTANT



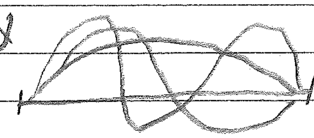
HOW MUCH ENERGY OR INTENSITY OR POWER IS IN THE λ TO $\lambda + \Delta\lambda$ REGION OF THE SPECTRUM.

PART OF THE ANSWER COMES FROM COUNTING THE ALLOWED λ'S IN THE VOLUME – SAME AS ARCHITECTURAL ACOUSTIC

$I(\lambda) \sim$ NUMBER OF STATES IN VOLUME WITH λ TO $\lambda + d\lambda$ $\times$ ENERGY ASSOCIATED WITH WAVELENGTH λ

$$\frac{hc/\lambda = hf}{e^{hc/\lambda k_B T} - 1}$$

DONE IN 3-DIMENS.



$\longleftarrow L \longrightarrow$

$V = L^3$; h = PLANCK'S CONSTANT

SEE P569 EQ. 25.2 ARCHITECTURAL ACOUSTICS.

NUMBER OF STATES (HARMONICS) FROM 0 TO f

$$N(f) = \frac{4\pi}{3} V \left(\frac{f}{v_{\text{SOUND OR LIGHT}}}\right)^3 + \text{SURFACE CORRECTIONS} + \text{CURVATURE CORRECTIONS}$$

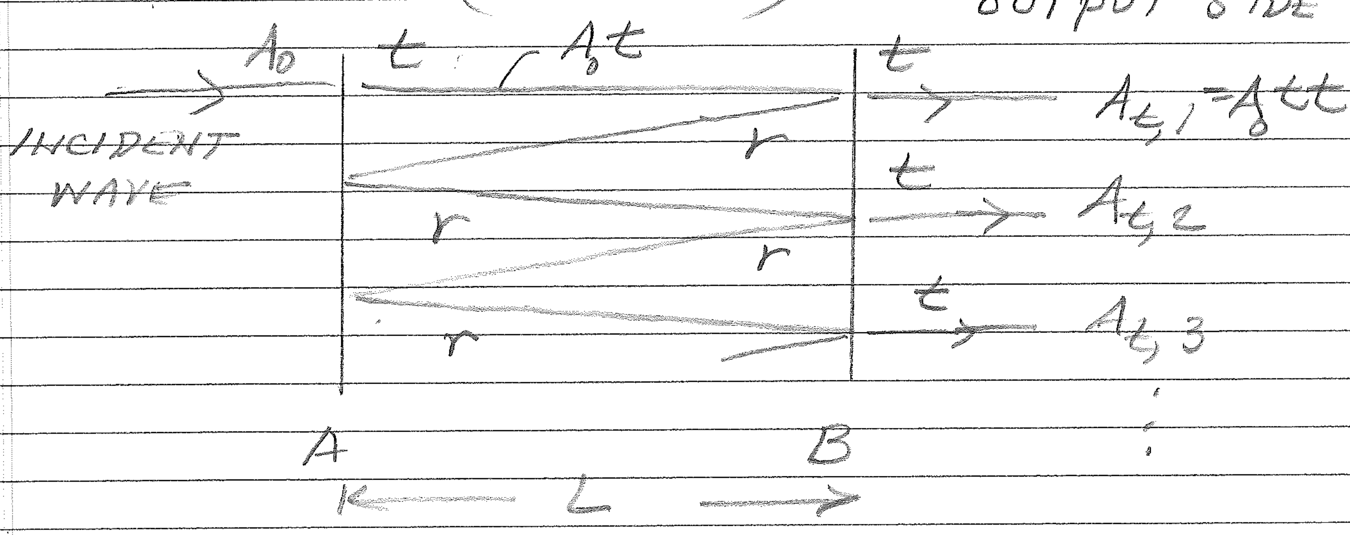
$f\lambda = v$

$\frac{f}{v} = \frac{1}{\lambda}$

$$N(\lambda) = \frac{4\pi}{3} V \left(\frac{1}{\lambda}\right)^3 ; \left|\frac{dN}{d\lambda}\right| = \frac{4\pi V}{\cancel{3}}\left(\frac{\cancel{3}}{\lambda^4}\right)$$

$N(\lambda)$ IMPORTANT ALSO IN STATISTICAL PHYSICS ASTROPHYSICS;.

LAST LECTURE (LECTURE 20)



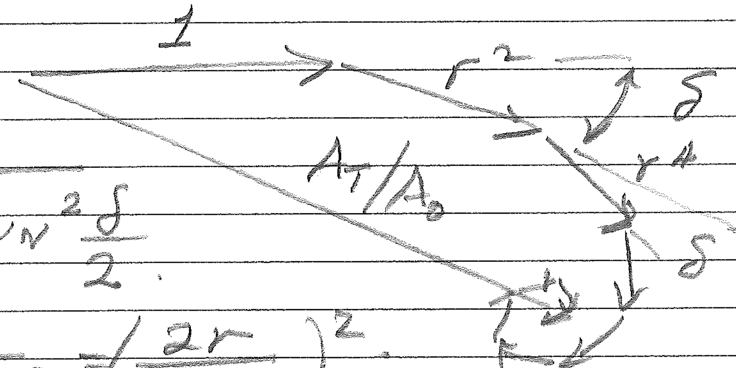
$A_{t,1} = A_0 tt$ TAKE PHASE AT B TO BE $= 0$

$A_{t,2} = A_0 tt r^2 e^{-i\delta}$ $\quad \delta = \omega t_0 \quad t_0 = \frac{2L}{v_s}$

$A_{t,3} = A_0 tt r^4 e^{-2i\delta}$

$\vdots$

LET $A_0 tt = 1$



$$\frac{I_T}{I_0} = \left|\frac{A_T}{A_0}\right|^2 = \frac{1}{1 + F\sin^2\frac{\delta}{2}}$$

$$F = \frac{4r^2}{(1-r^2)^2} = \left(\frac{2r}{1-r^2}\right)^2$$

WHEN $\sin^2\frac{\delta}{2} = 0$ $\quad \left|\frac{A_T}{A_0}\right|^2 = 1$

$\longrightarrow\longrightarrow\longrightarrow\longrightarrow$

$\frac{\delta}{2} = \pi, 2\pi, 3\pi$

$\delta = 2\pi, 4\pi, 6\pi, \ldots$

WHEN $\sin^2\frac{\delta}{2} = 1$

MINIMUM : $I_T/I_0 = \frac{1}{1+F}$

$\frac{\delta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\delta = \pi, 3\pi, 5\pi, \ldots$