~~LECTURE 17~~

WAVES & QUANTUM MECHANICS (QM)
PARALLELS WITH SOUND WAVES.

PARTICLES ON THE ATOMIC LEVEL HAVE WAVE ASPECTS.

NON-RELATIVISTIC

PARTICLE ASPECT : MOMENTUM $\vec{p} = m\vec{v}$ -

NON-RELATIVISTIC ENERGY

$E = \frac{1}{2}mv^2$ - KINETIC ENERGY

$= \frac{1}{2}\frac{(mv)^2}{m} = \frac{1}{2}\frac{p^2}{m}$.

WAVE ASPECT $\lambda = h/p$ deBROGLIE CONNECTION

h = PLANCK'S CONSTANT

$= 6.626 \times 10^{-34}$ JOULE·SEC

$= 4.135 \times 10^{-15}$ eV·SEC

$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J.S.

γ- PHOTON $E = pc$ FROM $E^2 = (pc)^2 + (mc^2)^2$ $m = 0$

$$p_\gamma = h/\lambda \Rightarrow E_\gamma = p_\gamma c = \frac{hc}{\lambda} = hf$$

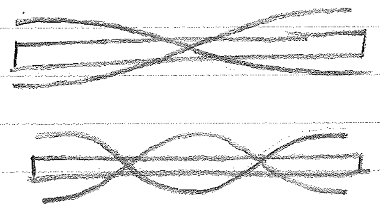
WAVE ASPECTS FOR PARTICLES SHOW UP

1) BOHR QUANTIZATION OF ENERGY LEVELS
2) DOUBLE SLIT INTERFERENCE
3) DIFFRACTION
4) LIKE SOUND WAVES A WAVE EQUATION EXISTS FOR PARTICLE WAVES - SCHROEDINGER EQUATION

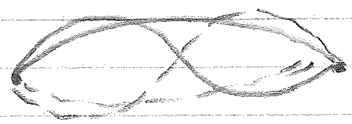
Parallels WITH SOUND

WAVES & QUANTUM MECHANICS.

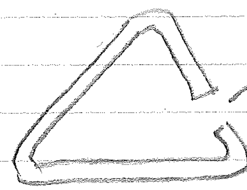
Allowed λ's



BAR - FREE ENDS.
LONGITUDINAL
$n\lambda = 2L$.



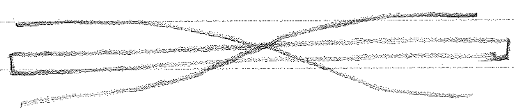
STRING.
$n\lambda = 2L$.



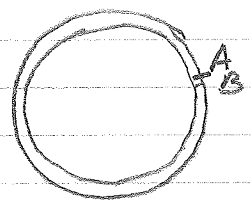
~ CUT

TRIANGLE IN MUSIC.

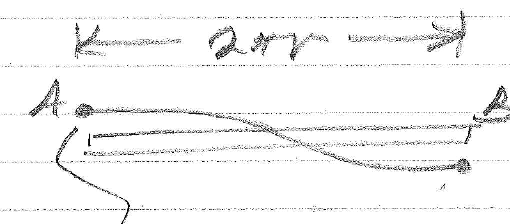
OPEN IT UP - SAME AS STRAIGHT BAR.



RING WITH NO CUT

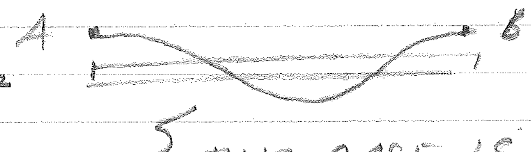


IF YOU OPEN IT UP

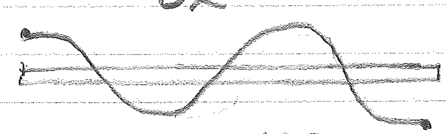


WAVE ON THIS END DOES NOT MATCH UP WITH WAVE ON B

$2\pi r = 1\lambda$ →

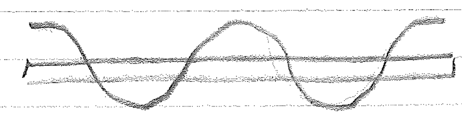


THIS CASE IS OK



THIS CASE DOES NOT MATCH

$2\pi r = 2\lambda$

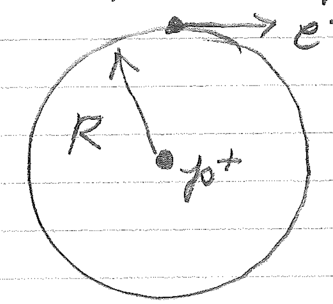


OK

WAVES & QUANTUM MECHANICS

BOHR'S CONDITION

ELECTRON MOVED IN CIRCULAR ORBITS
AROUND PROTON p



THE ELECTRONS HAVE A WAVE LENGTH $\lambda = h/p$

$p = mv$ = MOMENTUM OF ELECTRON

CONDITION ON λ $\quad n\lambda = 2\pi R = n\frac{h}{p}$

BECAUSE OF THIS CONDITION
THE RADII ARE QUANTIZED, i.e., ONLY CERTAIN
RADII ARE ALLOWED. CORRESPONDINGLY
ONLY CERTAIN ENERGIES ARE ALLOWED

NOTE ON A STRING. ONLY CERTAIN λ'S
$n\lambda = 2L$. AND CORRESPONDINGLY
ω, f ARE "QUANTIZED"

$$\lambda f = v \qquad f = \frac{v}{\lambda} \qquad f_n = n\frac{v}{2L} \quad n = 1, 2, 3, ..$$

FOR PARTICLES THAT ARE NOT SUBJECT
TO A POTENTIAL - KINETIC ENERGY ONLY

$$E = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 2m} \qquad k = \frac{2\pi}{\lambda}, \ \frac{1}{\lambda} = \frac{k}{2\pi}$$

$$E = \left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} k^2 = \frac{\hbar^2 k^2}{2m}$$

SOUND $\quad \omega = vk \quad (f = v/\lambda)$

WAVES & QUANTUM MECHANICS

BOHR'S ATOM.

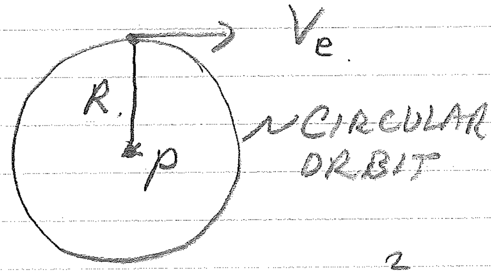
$$n\lambda = 2\pi R = n\frac{h}{p}$$

$$Rp = n\frac{h}{2\pi} = n\hbar \qquad \vec{L} = \vec{r}\times\vec{p}$$

$$r \perp p$$

$$L = n\hbar \qquad |\vec{L}| = rp$$

ANGULAR MOMENTUM IS QUANTIZED.



acceleration – TOWARD p^+ CENTER!

$$a = \frac{V_e^2}{R} \sim \text{CONSTANT SPEED.}$$

$$F=ma: \; F_{ep^+} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e a = m_e\frac{V_e^2}{R}$$

SQUARE

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R} = m_e V_e^2 \qquad V_e^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e R}$$

P.E. = POTENTIAL ENERGY $\quad -\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}$

K.E. = KINETIC ENERGY $\quad \frac{1}{2}m_e V_e^2$

T.E. = TOTAL ENERGY $= K.E. + P.E. = \frac{1}{2}m_e V_e^2 - \frac{1}{4\pi\epsilon_0}\frac{e^2}{R}$

$$= -\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}\right) = \frac{P.E.}{2} = -\frac{|P.E.|}{2}$$

QUANTIZATION OF RADII

$$Rp = R m_e V_e = n\hbar \qquad R^2 m_e^2 V_e^2 = n^2\hbar^2$$

SUBSITUTE F=ma $\quad R^2 m_e^2 \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e R} = n^2\hbar^2$

$$R = n^2\hbar^2\frac{4\pi\epsilon_0}{e^2 m_e}$$

$$\equiv n^2 a_0 \leftarrow \text{BOHR RADIUS}$$

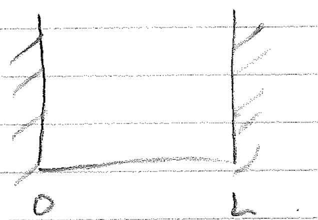
WAVES & QUANTUM MECHANICS.

$$T.E. = -\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}\right) = E \qquad E_n = -\frac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{n^2 a_0}\right)$$

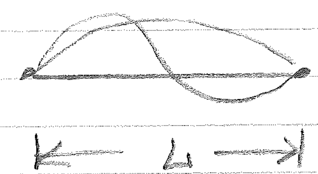
$$E_n = -\frac{13.6\,eV}{n^2}$$

OTHER IMPORTANT PARALLELS.

PARTICLE FREE TO MOVE ONLY FROM $X=0$ TO $X=L$.



SAME AS STRING WITH FIXED ENDS.



$$E = \frac{\hbar^2}{2m}k^2 \qquad \omega = vk$$

HERE QUADRATIC E VERSUS k. HERE LINEAR ω VERSUS k.

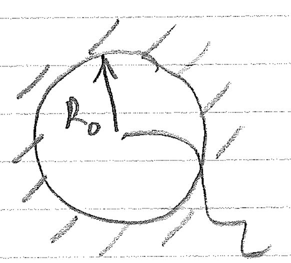
$$n\lambda = 2L \qquad n\lambda = 2L$$

$$\frac{1}{\lambda} = \frac{n}{2L} \quad \frac{2\pi}{\lambda} = k = \frac{2\pi n}{2L}$$

$$E = \frac{(\hbar 2\pi)^2}{2m}\frac{n^2}{4L^2} \qquad \omega = vk$$

$$= n^2\frac{h^2}{8mL^2} \qquad = v\cdot\frac{2\pi n}{2L}$$

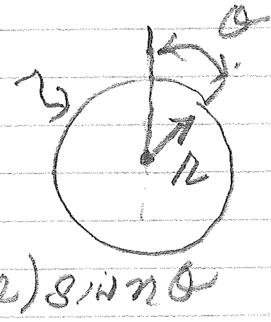
$$\cancel{2\pi} f = n\frac{v}{2L}\cancel{2\pi}$$



ELECTRON FREE INSIDE CIRCLE

CIRCULAR DRUM.

FIXED END

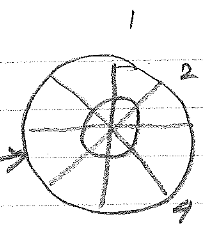


$$J_n(kr)\sin n\theta$$

WAVES & QUANTUM MECHANICS.

$$E_{nm} = \frac{\hbar^2 k_{nm}^2}{2m_e}$$

FIXED RIM



DRUM
n LINES = 4
m = 2 CIRCLES

$$k_{nm} = \frac{X_{nm}}{R_0}$$

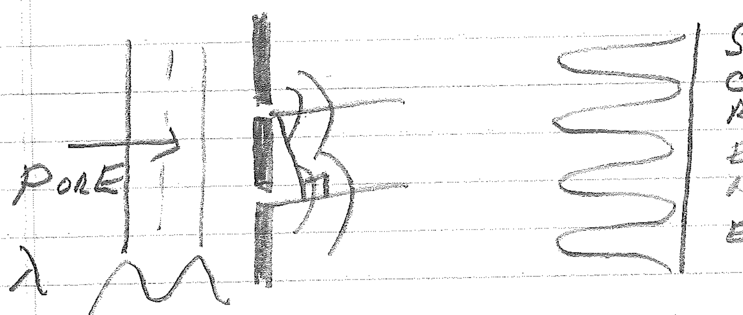
X_{nm} IS THE m'th ZERO OF $J_n(X_{nm}) = 0$

THE m'th ZERO IS AT R_0, THE RADIUS OF THE DRUM OR THE CIRCULAR WELL.

THE n IS SIMILAR TO THE n THAT APPEARS IN THE BOHR CONDITION

$$n\lambda = 2\pi R_0$$

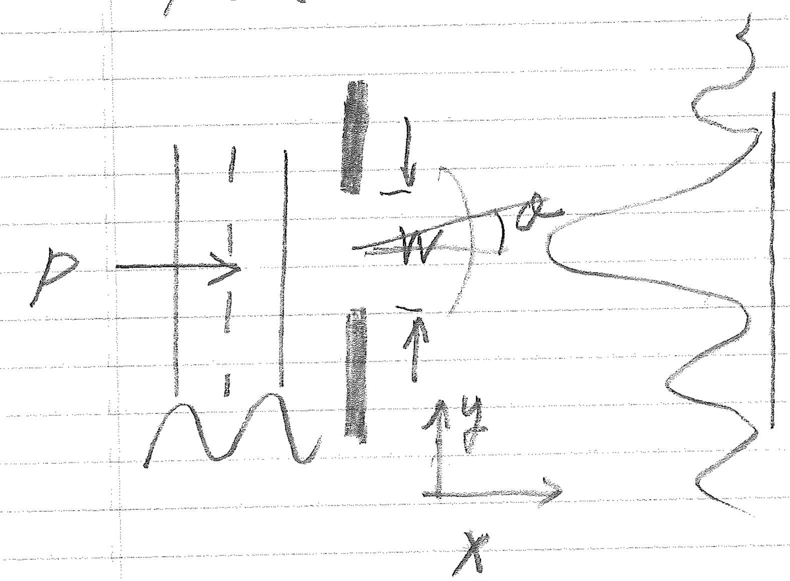
INTERFERENCE - ELECTRONS - USE $\lambda = h/p$



$p = mv$

$E = \frac{p^2}{2m}$

$p = \sqrt{2mE}$



USE $\lambda = h/p$ AS ABOVE

OR.

USE UNCERTAINTY RELATION

$\Delta p_y \Delta y \gtrsim \hbar/2$

$\Delta y = W$

$\Delta p_y \gtrsim \frac{1}{W}\frac{\hbar}{2}$