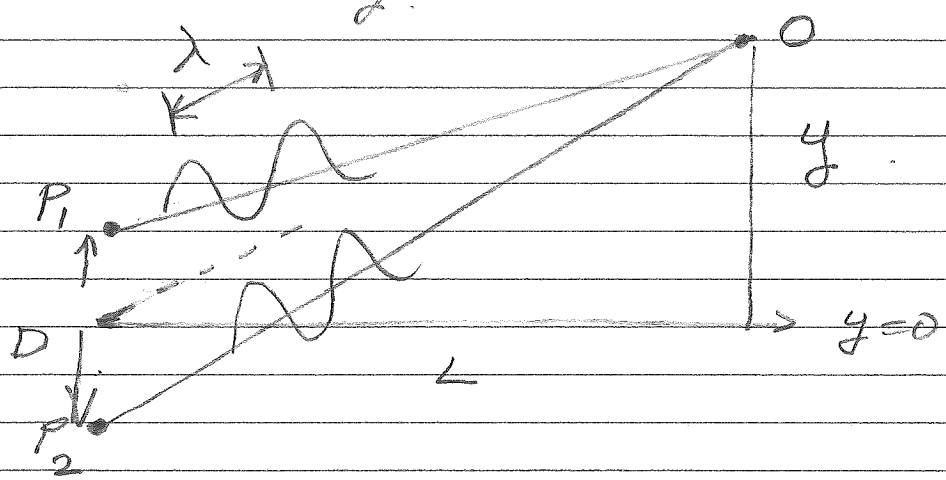
LECTURE 14

INTERFERENCE DIFFRACTION CONTINUED.

SUMMARY - INTERFERENCE

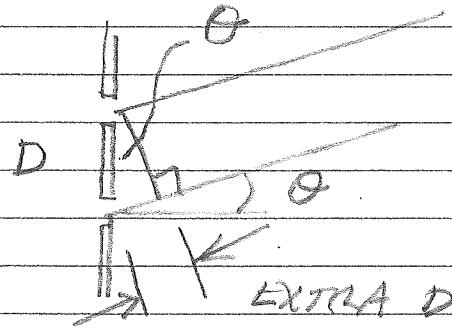


$$OP_2 - OP_1 = \sqrt{L^2+\left(y+\frac{D}{2}\right)^2} - \sqrt{L^2+\left(y-\frac{D}{2}\right)^2}$$

$= n\lambda$ CONSTRUCTIVE $n = 0, 1, 2, \ldots$

$\left(m+\frac{1}{2}\right)\lambda$ DESTRUCTIVE $m = 0, 1, 2, \ldots$

FRAUNHOFER LIMIT $L \gg \left(y \pm D/2\right)$

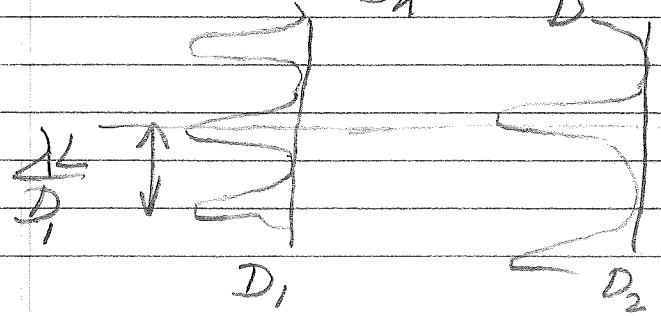


EXTRA DISTANCE $= D\sin\theta = D\frac{y}{L}$

CONSTRUCTIVE $D\frac{y}{L} = n\lambda \quad y = y_n$

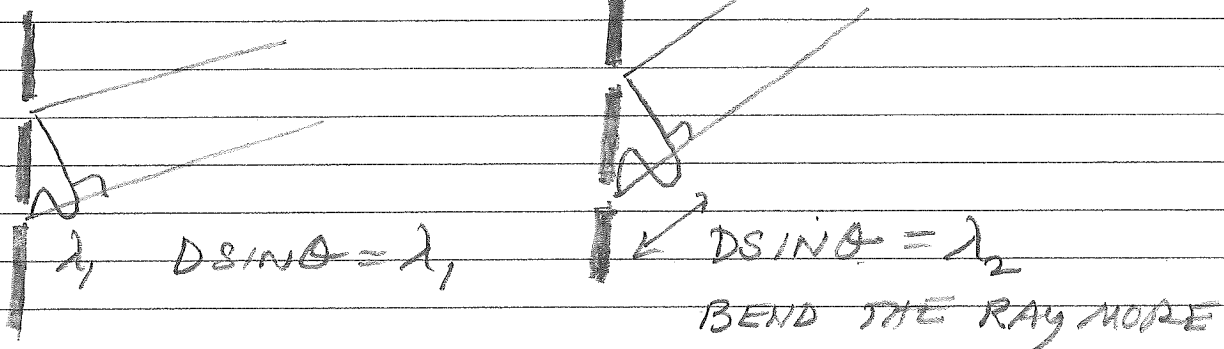
$$D\frac{y_n}{L} = n\lambda$$

$$y_n = \frac{n\lambda L}{D} \sim \frac{1}{D}$$ SMALLER D LARGER y_n.



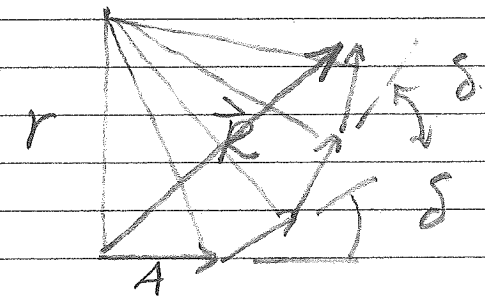
INTERFERENCE / DIFFRACTION SUMMARY

BIGGER λ BIGGER ψ



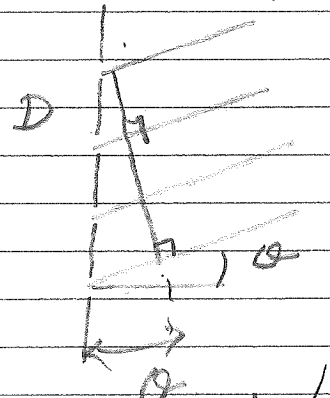
BEND THE RAY MORE TO ACCOMODATE LARGER λ.

N - SOURCES.



$$\delta = k\Delta r = \frac{2\pi}{\lambda}\Delta r$$

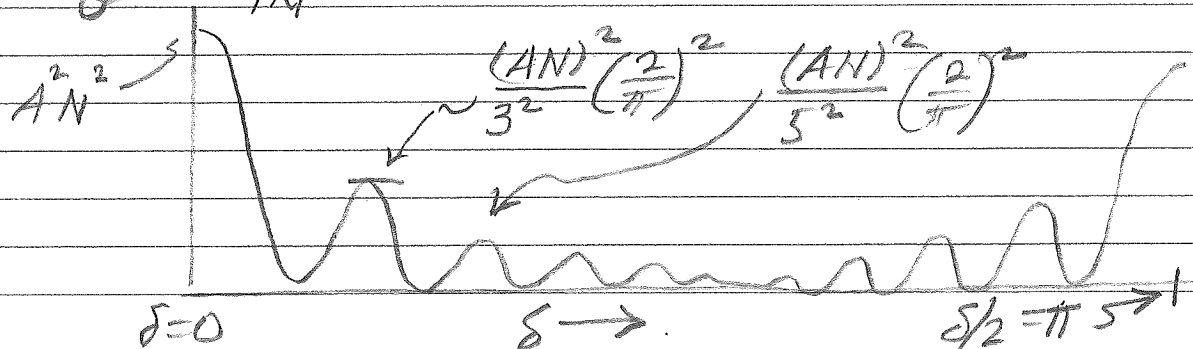
$$\Delta r = D\sin\theta$$



$$1 + e^{i\delta} + e^{2i\delta} + \ldots + e^{i(N-1)\delta}$$

$$r = \frac{A/2}{\sin\delta/2}$$

$$|R| = A\left|\frac{\sin N\delta/2}{\sin\delta/2}\right|$$



INTERFERENCE

ALGEBRAIC METHOD N-SLITS

$$1 + e^{i\delta} + e^{2i\delta} + \dots + e^{(N-1)i\delta}$$

$$x = e^{i\delta}$$

$$1 + x + x^2 + \dots + x^{N-1}$$

NOTE $1 + x + x^2 + \dots x^{\infty} = \frac{1}{1-x}$ Multiply by $(1-x)$

$$\begin{array}{l} 1 + x + x^2 + \dots x^{\infty} \\ \qquad \otimes \quad (1-x) \\ \hline 1 + x + x^2 + \dots x^{\infty} \\ \quad -x - x^2 - \dots -x^{\infty} \\ \hline 1 + 0 + 0 \qquad\qquad 0 \end{array}$$

WRITE $(1 + x + \dots x^{N-1}) = (1 + x + \dots x^{\infty}) - (x^N + x^{N+1} + \dots x^{\infty})$

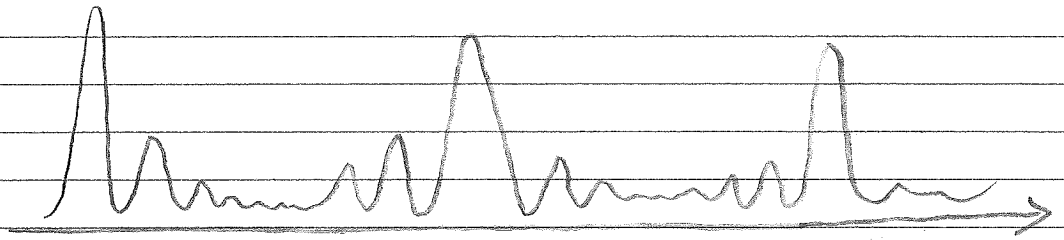
$$\frac{1}{1-x} - x^N(1 + x + \dots + x^{\infty}) = \frac{1}{1-x} - \frac{x^N}{1-x}$$

$$= \frac{1-x^N}{1-x} = \frac{1 - e^{iN\delta}}{1 - e^{i\delta}} =$$

$$\frac{e^{+iN\delta/2}\left(e^{-iN\delta/2} - e^{iN\delta/2}\right)}{e^{i\delta/2}\left(e^{-i\delta/2} - e^{i\delta/2}\right)} \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

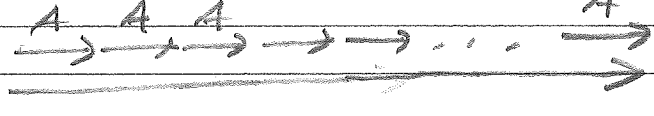
$$= e^{i(N-1)\delta/2} \; \frac{\sin\frac{N\delta}{2}}{\sin(\delta/2)}$$

$$\text{MAGNITUDE} = \left| \frac{\sin N\delta/2}{\sin \delta/2} \right|$$

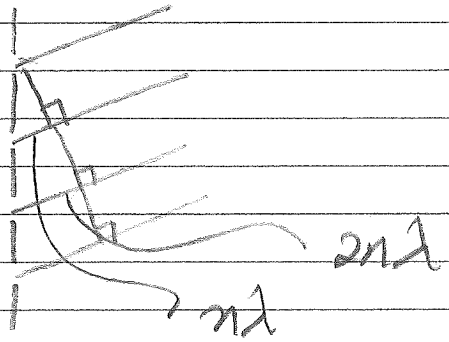


$\delta = 0$ $\qquad \frac{\delta}{2} = \pi$ $\qquad \frac{\delta}{2} = 2\pi$

THE LARGE PEAKS OCCUR WHEN THE DENOMINATOR IN R WHICH IS $\sin \delta/2$ IS ZERO THE $R = 0/0$ BUT THE ANSWER IS $R = AN$ SINCE THE VECTORS LINE UP



AT $\delta = 2\pi, 4\pi, \ldots$



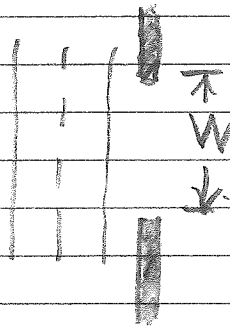
$\delta = kd$

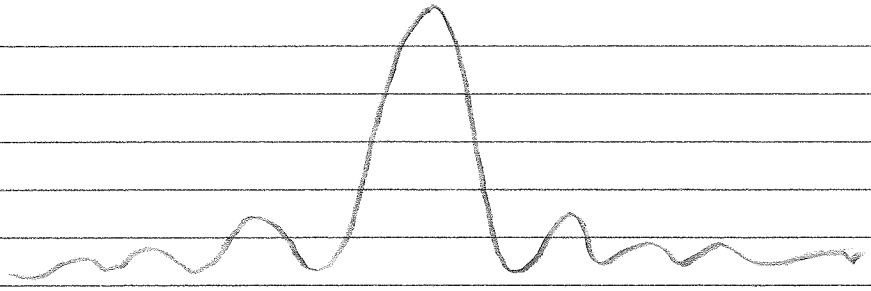
$= \frac{2\pi}{\lambda} D \sin\theta = 2n\pi$

$$\boxed{D \sin\theta_n = n\lambda}$$

POSITION OF PRINCIPLE MAXIMA

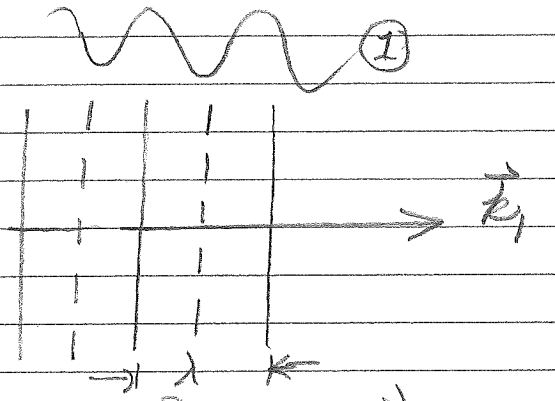
IN A DIFFRACTION CASE OF AN OPENING THE INITIAL BEHAVIOR IS SIMILAR BUT THERE IS ONLY ONE MAIN PEAK.

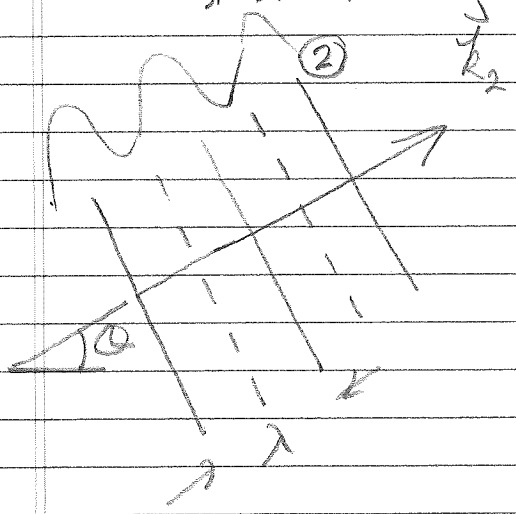


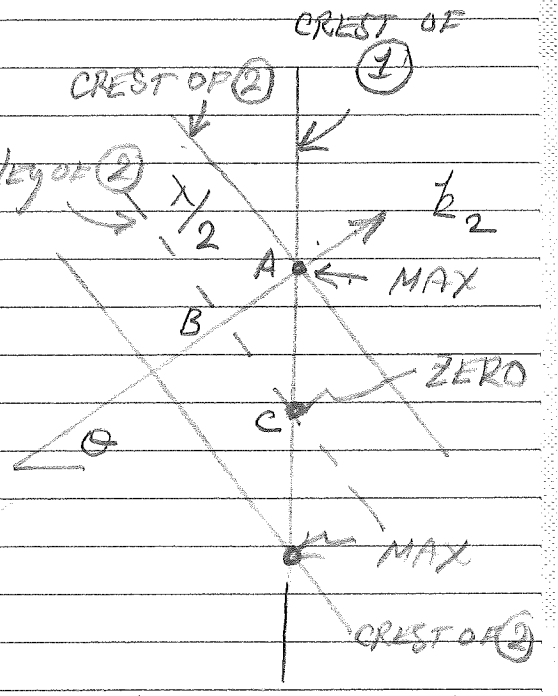


THIS BEHAVIOR DOES NOT REPEAT

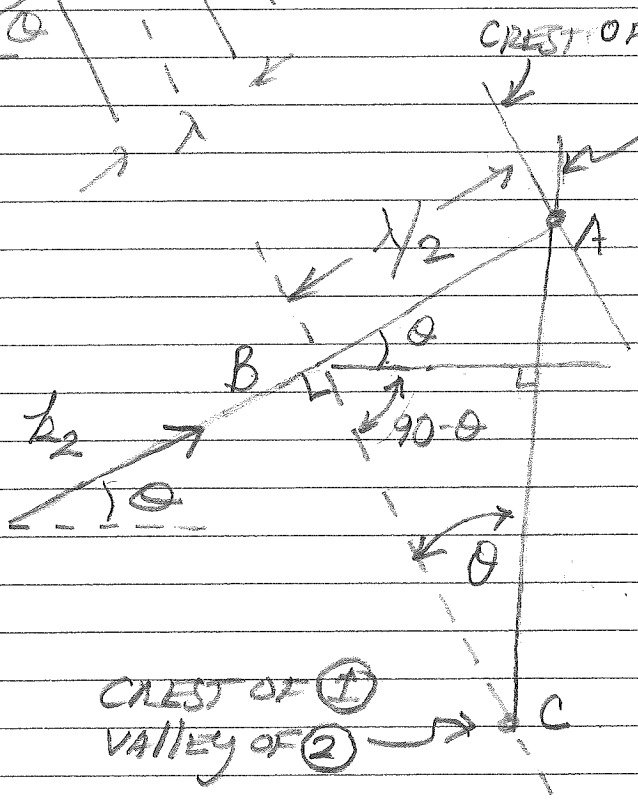
INTERFERENCE – 2 PLANE WAVES MOVING AT AN ∠ θ WITH RESPECT TO EACH OTHER.







INTERFERENCE FRINGES



$$\sin\theta = \frac{\lambda/2}{AC}$$

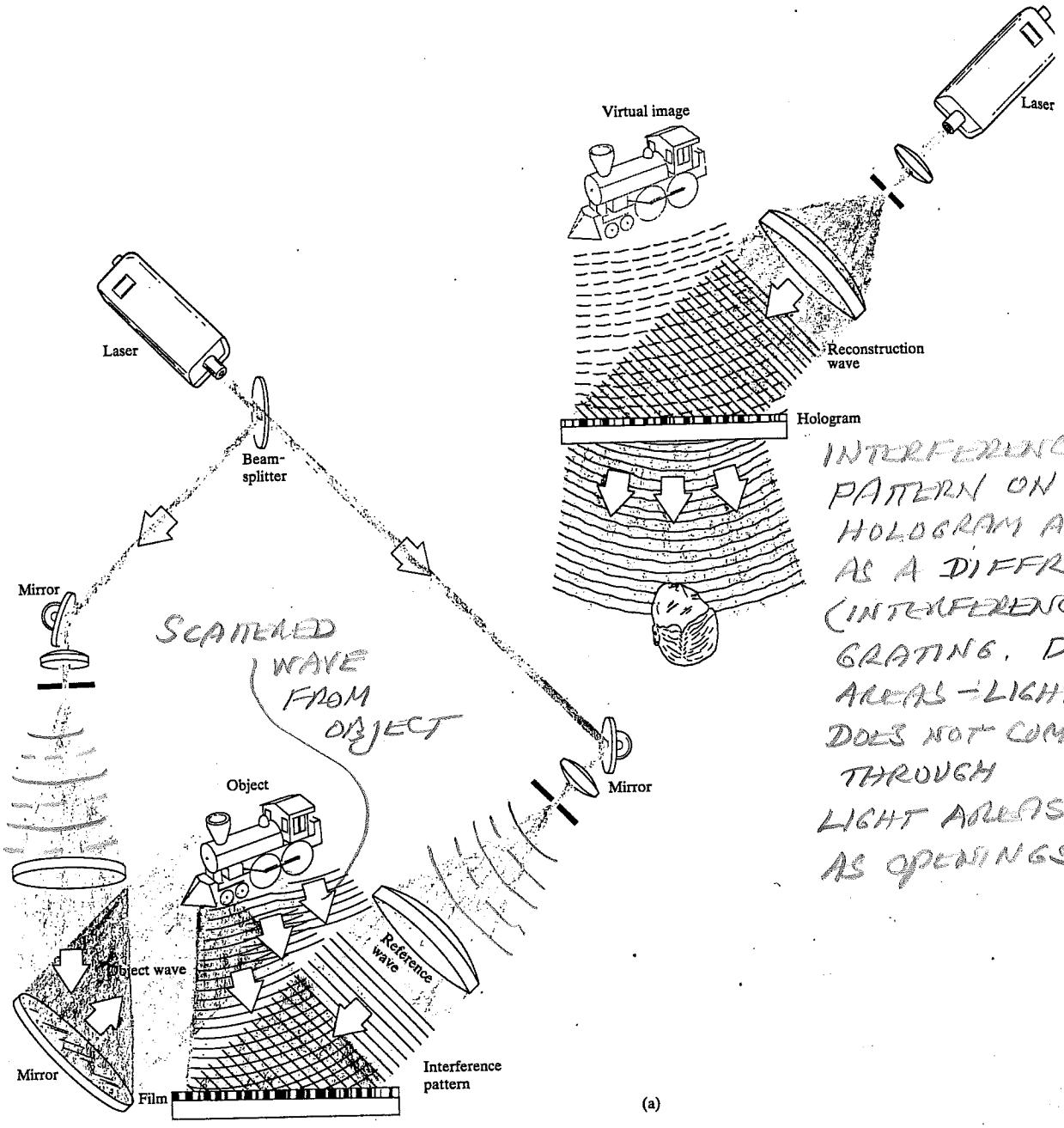
$$AC = \frac{\lambda/2}{\sin\theta}$$

DISTANCE BETWEEN MAX & ZERO

GIVEN AC, λ θ IS DETERMINED

INTERFERENCE PATTERN OF ① & ② IS A SERIES OF BRIGHT AND DARK LINES.

USED IN HOLOGRAPHY < OBJECT WAVE ①
REFERENCE WAVE ②



SCATTERED WAVE FROM OBJECT

INTERFERENCE PATTERN ON HOLOGRAM ACTS AS A DIFFRACTION (INTERFERENCE) GRATING. DARK AREAS = LIGHT DOES NOT COME THROUGH LIGHT AREAS ACT AS OPENINGS

Figure 13.50 (a) The creation of a transmission hologram of a toy locomotive. (b) Replay of a transmission hologram.

SCATTERED WAVE FROM OBJECT PLUS REFERENCE WAVE MAKE AN INTERFERENCE PATTERN ON A FILM. INTERFERENCE PATTERN CONTAINS COMPLETE INFORMATION ABOUT OBJECT WAVE — AMPLITUDE OF WAVE INCLUDING ITS PHASE.