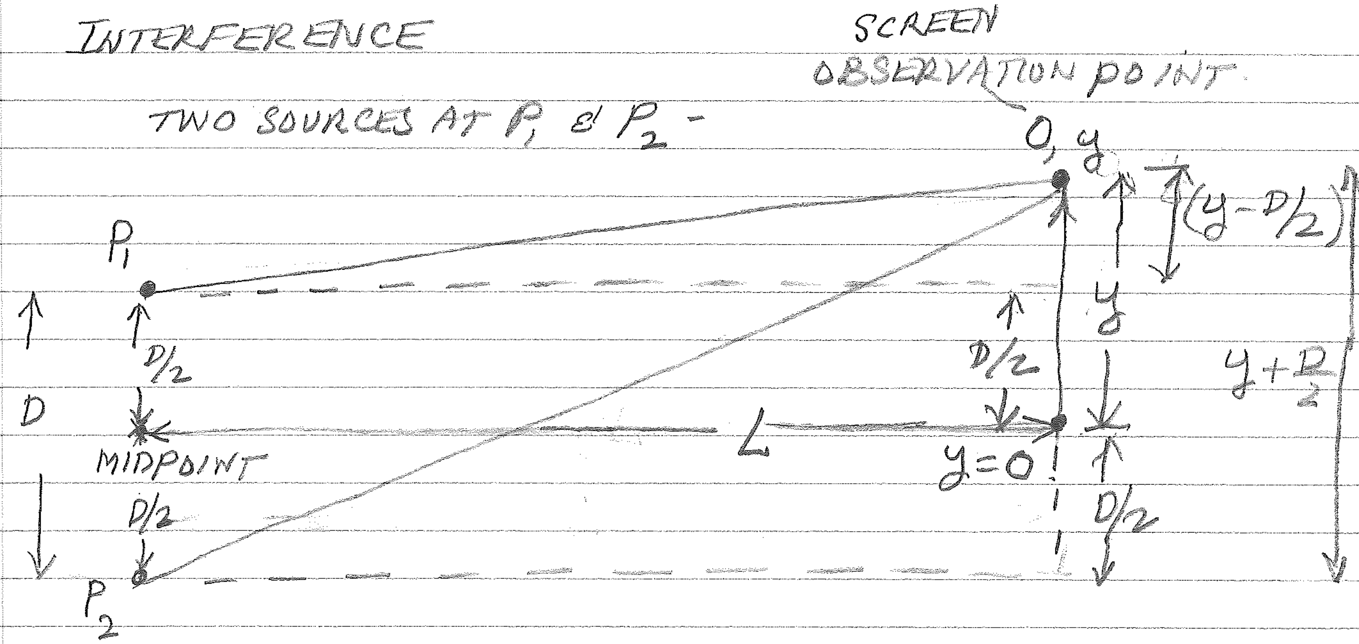
LECTURE 13

INTERFERENCE

TWO SOURCES AT P_1 & P_2 -



$$OP_2 = \sqrt{L^2 + \left(\frac{y}{2} + \frac{D}{2}\right)^2} \qquad OP_1 = \sqrt{L^2 + (y - D/2)^2}$$

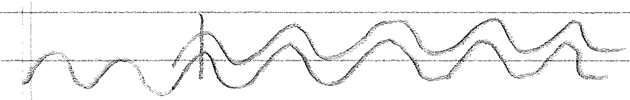
SOURCES AT P_1 & P_2 ARE IN PHASE

DISTANCES DIFFERENCE $OP_2 - OP_1$

IF $OP_2 - OP_1 = 0, \pm\lambda, \pm 2\lambda, \pm 3\lambda, \ldots$ CONSTRUCTIVE INTERFERENCE

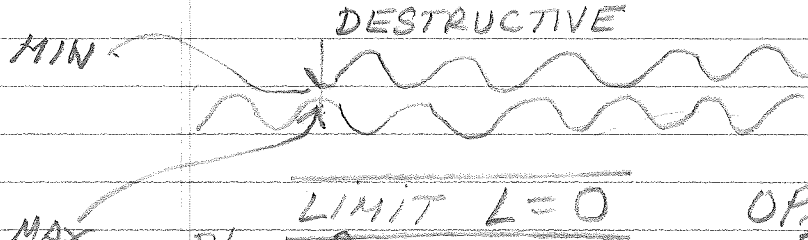
IF $OP_2 - OP_1 = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \ldots$ DESTRUCTIVE INTERFERENCE

CONSTRUCTIVE



WAVES ARE IN PHASE

DESTRUCTIVE



WAVE ARE OUT OF PHASE

LIMIT $L = 0$ $\quad OP_2 - OP_1 = \left(\frac{D}{2} - y\right) - \left(\frac{D}{2} + y\right) = -2y$

$D/2$ • P_1

$-\lambda, c$ $\quad -2y = 0, \pm\lambda, \pm 2\lambda, \ldots$ CONSTRUCTIVE

$-\lambda/2, c$ $\quad y = 0, \pm\frac{\lambda}{2}, \pm\lambda, \ldots$

$y = 0$

$-\lambda/2, c$

$-\lambda, c$ $\quad -2y = \pm\lambda/2, \pm\frac{3\lambda}{2}, \ldots$ DESTRUCTIVE

$-D/2$ • P_2 $\quad y = \pm\frac{\lambda}{4}, \pm\frac{3\lambda}{4}, \ldots$

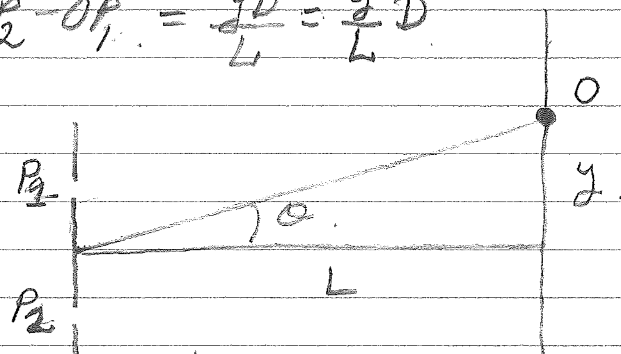
INTERFERENCE

LIMIT $L \gg y \pm D/2$ USE $\sqrt{1+x} \approx 1 + x/2$ FOR $x \ll 1$

$$\sqrt{L^2+(y\pm D/2)^2} = \sqrt{L^2\left(1+\frac{(y\pm D/2)^2}{L^2}\right)} = L\left(1+\frac{1}{2}\frac{(y\pm D/2)^2}{L^2}\right)$$

$$OP_2 - OP_1 = \frac{(y+D/2)^2}{2L} - \frac{(y-D/2)^2}{2L} = \frac{1}{2L}\left\{y^2+yD+\frac{D^2}{4}-\left(y^2-yD+\frac{D^2}{4}\right)\right\}$$

$$OP_2 - OP_1 = \frac{yD}{L} = \frac{y}{L}D$$

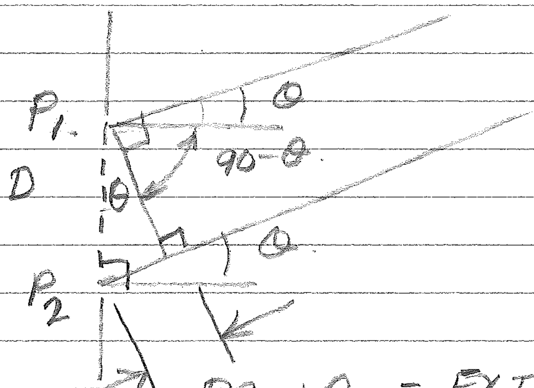


$\tan\theta = y/L$

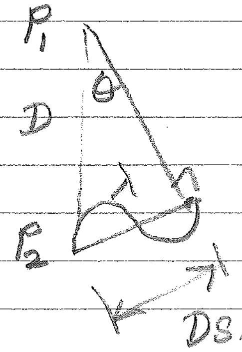
$\sin\theta = \dfrac{y}{\sqrt{L^2+y^2}} \approx \dfrac{y}{L}$

$y \ll L$

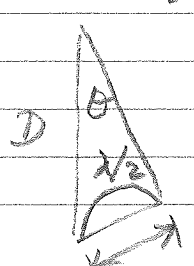
$OP_2 - OP_1 \cong D\sin\theta$



$D\sin\theta$ = EXTRA DISTANCE BETWEEN P_1 & P_2



$D\sin\theta = \lambda$. ALSO $0, 2\lambda, 3\lambda, ..$ CONSTRUCTIVE INTERFERENCE



$D\sin\theta = \frac{\lambda}{2}$, ALSO $\frac{3\lambda}{2}, \frac{5\lambda}{2}, ...$ DESTRUCTIVE INTERFERENCE

INTERFERENCE AS A VECTOR PROBLEM - COMPLEX PLANE

WAVE FROM P_1 $\quad e^{ikr_1}$ $\quad k=\frac{2\pi}{\lambda}$ $\quad r_1 = OP_1$

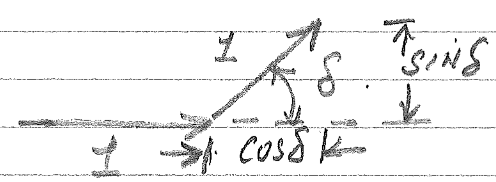
WAVE FROM P_2 $\quad e^{ikr_2}$ $\quad k=\frac{2\pi}{\lambda}$ $\quad r_2 = OP_2$

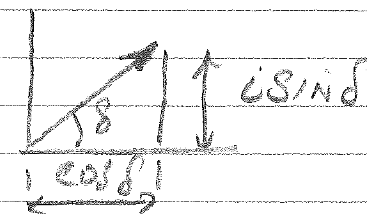
SUM $\quad e^{ikr_1}+e^{ikr_2} = e^{ikr_1}\left(1+e^{ik(r_2-r_1)}\right)$

$= e^{ikr_1}\left(1+e^{ik\Delta r}\right)$ $\qquad \boxed{k\Delta r = \delta \text{ PHASE DIFFERENCE}}$

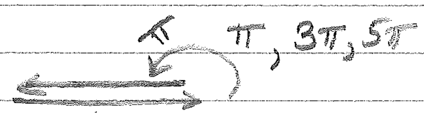
$= e^{ikr_1}\left(1+e^{i\delta}\right)$

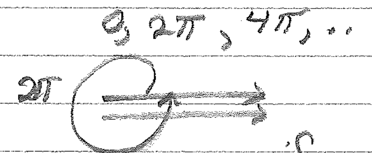
$e^{i\delta} = \cos\delta + i\sin\delta$





$\delta = \pi, 3\pi, 5\pi$ $\qquad \delta = 0, 2\pi, \ldots$





$1+e^{i\delta} = 0$ $\qquad 1+e^{i\delta} = 2$

$k\Delta r = \pi = \frac{2\pi}{\lambda}\Delta r$ $\qquad \delta = 2\pi = \frac{2\pi}{\lambda}\Delta r$

$\Delta r = \lambda/2$ ✓ $\qquad \Delta r = \lambda$ ✓

$\delta = 3\pi$ $\qquad \delta = 4\pi = \frac{2\pi}{\lambda}\Delta r$

$\Delta r = \frac{3\lambda}{2}$ ✓ $\qquad \Delta r = 2\lambda$ ✓

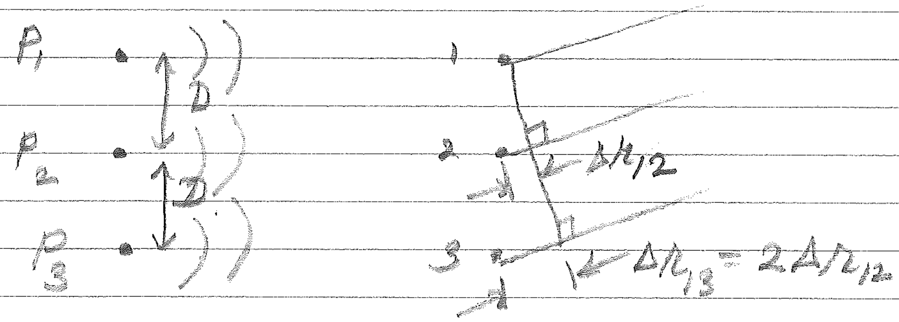
$I =$ INTENSITY $= |\text{AMPLIDE}|^2$

AMPLITUDE $Ae^{ikr_1}(1+e^{i\delta}) = Ae^{ikr_1+i\frac{\delta}{2}}\left(e^{-i\delta/2}+e^{i\delta/2}\right)$

$= Ae^{ikr_1+i\frac{\delta}{2}}\, 2\cos\delta/2$

$I = |A|^2\, 4\cos^2\delta/2$ $\qquad \delta = k\Delta r = kD\sin\theta = \frac{2\pi}{\lambda}D\sin\theta$

INTERFERENCE AS A VECTOR PROBLEM
USEFUL FOR MANY SOURCES - EACH SEPARATED
BY SAME DISTANCE & IN PHASE AT SOURCE.



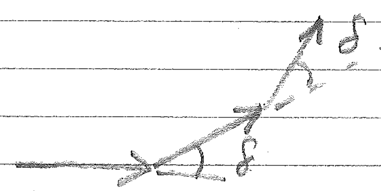
$\delta = k\Delta r = \frac{2\pi}{\lambda}\Delta r_{12}$

$\Delta r_{12} = D\sin\theta$

SCREEN POSITION
$\sin\theta \approx \tan\theta = y/L$

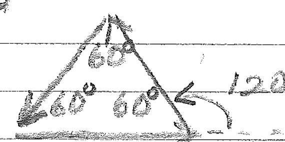
$e^{ikr_1}\left(1 + e^{ik\Delta r_{12}} + e^{2ik\Delta_{12}}\right)$

$e^{ikr_1}\left(1 + e^{i\delta} + e^{2i\delta}\right)$



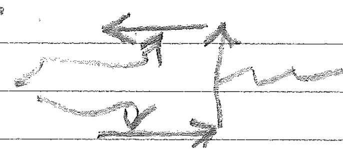
$\delta = 0$ $\xrightarrow{1}\xrightarrow{1}\xrightarrow{1}$ MAGNITUDE 3

$\delta = 120° = 2\pi \cdot \frac{120}{360} = \frac{2\pi}{3}$



$\left(1 + e^{i2\pi/3} + e^{2i\,2\pi/3}\right) = 0$

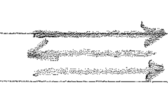
$\delta = 90° = \pi/2$



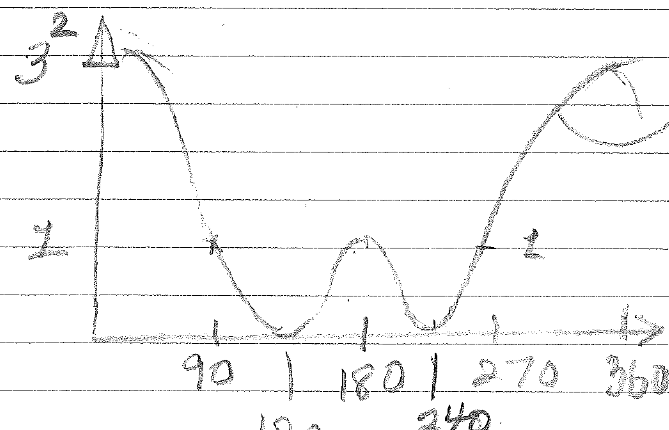
$(1 + i + (-1)) = i$ MAGNITUDE = 1

$\delta = 180 = \pi$

$(1 + (-1) + 1) = 1$

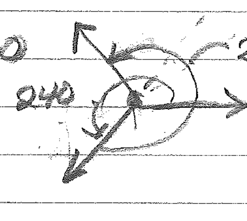


MAGNITUDE = 1.



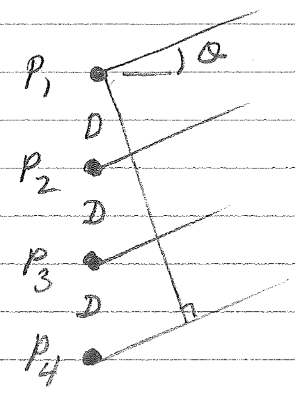
MAGNITUDE SQUARED
OF $\left|1 + e^{i\delta} + e^{2i\delta}\right|^2$

$\delta = 240$

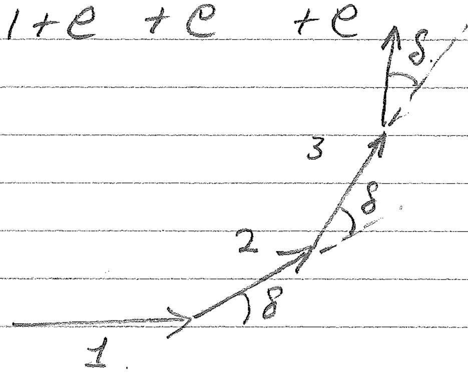


INTERFERENCE AS A VECTOR PROBLEM.

4 SOURCES.



$$\Sigma_4 \equiv 1+e^{i\delta}+e^{2i\delta}+e^{3i\delta}$$

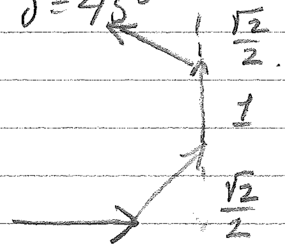


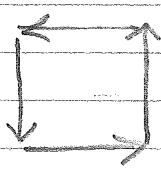
$$\delta = k\Delta r_{12} = \frac{2\pi}{\lambda} D\sin\theta.$$

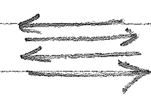
SPECIAL POINTS.

$\delta = 0, 2\pi, 3\pi, \ldots \quad \Sigma_4 = 1+e^{i\delta}+e^{2i\delta}+e^{3i\delta} = 4.$

$\delta = 45^\circ$ $\qquad$ $\delta = 90^\circ$ $\qquad$ $\delta = 180^\circ$





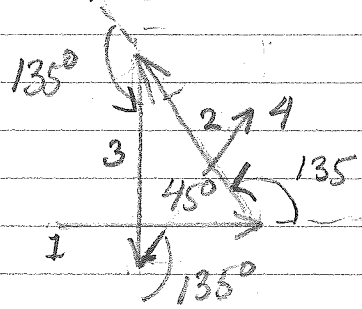


$\Sigma_4 = 1+(\sqrt{2}+1)i$ $\qquad$ $\Sigma_4 = 0$ $\qquad$ $\Sigma_4 = 0.$

$\delta = 135^\circ = 90+45^\circ \qquad 2\delta = 270^\circ \qquad 3\delta$

$\underline{3}$

$405 = 360+45$



$$1+e^{i\pi/2+i\pi/4}+e^{i\pi+i\pi/2}+e^{i3\pi/2+i\frac{3\pi}{4}}$$

$e^{i\pi/2} = i$

$e^{i\pi/4} = \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}.$

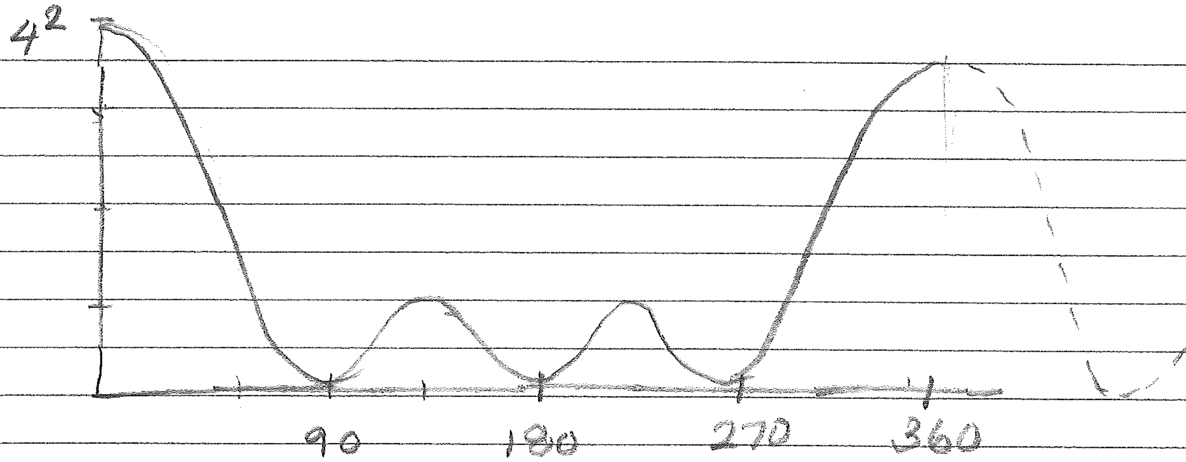
$e^{i\pi/2+i\pi/4} = e^{i\pi/2}e^{i\pi/4} = -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$

$e^{i\pi}e^{i\pi/2} = (-1)\,i = -i$

$e^{i3\pi/2}e^{i3\pi/4} = (-i)\,i\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$

$\Sigma_4 = 1+\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)-i+\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \underline{1}+i(\sqrt{2}-1)$

$1+.414i$



MAX ~ 135   MAX ~ 225

@ 135 $\Sigma_4 = 1 + .414i$

$|\Sigma_4| = 1 + (.414)$

SUM OF N TERMS.   N SOURCES IN PHASE.

$$\left(1 + e^{i\delta} + e^{2i\delta} + \dots + e^{(N-1)i\delta}\right)$$

LET $x = e^{i\delta}$

$$\Sigma_N = \left(1 + x^1 + x^2 + \dots + x^{N-1}\right) =$$

$$\left(1 + x + x^2 + \dots + x^{N-1}\right) + \left(x^N + x^{N+1} + \dots + x^\infty\right)$$
$$\underbrace{- \left(x^N + x^{N+1} + \dots + x^\infty\right)}_{\text{ADD \& SUBTRACT } x^N + x^{N+1} + \dots + x^\infty}$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^\infty$$

$$x^N + N^{N+1} + \dots + x^\infty = x^N\left(1 + x + x^2 + \dots x^\infty\right)$$
$$= \frac{x^N}{1-x}$$

$$\boxed{\Sigma_N = \frac{1}{1-x} - \frac{x^N}{1-x} = \frac{1}{1-x}\left(1 - x^N\right)}$$