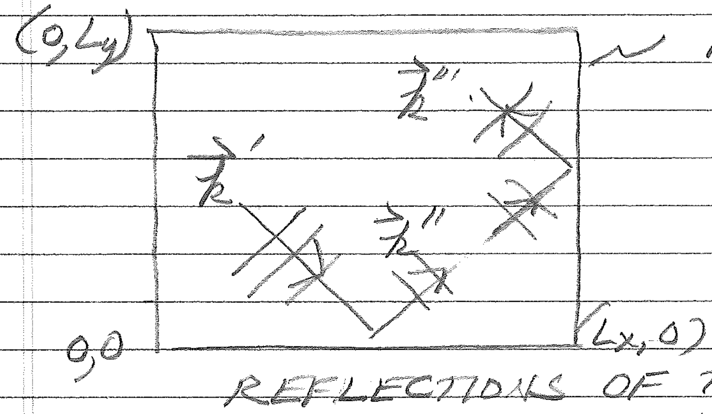
LECTURE 12

2 DIMENSIONAL SYSTEM - REVIEW OF LAST LECTURE



FIXED BOUNDARIES

$\psi(x,y,t)$ = HEIGHT OF THE VIBRATION FROM THE EQUILIBRIUM PLANE

REFLECTIONS OF THE PLANE WAVE SETS UP STANDING WAVES IN X, Y DIRECTION

SOLUTION TO WAVE EQUATION:

$$\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

IS

$$\psi = A \sin k_x x \sin k_y y \begin{Bmatrix} e^{-i\omega t} \\ \cos\omega t \\ \sin\omega t \end{Bmatrix}$$

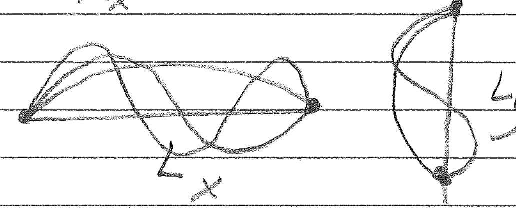
$$\vec{k} = k_x\vec{i} + k_y\vec{j}$$

BOUNDARY CONDITIONS $\sin k_x L_x = 0$

$\sin k_y L_y = 0$

$$k_x L_x = n\pi = \frac{2\pi}{\lambda_x} L_x \qquad n\lambda_x = 2L_x$$

$$k_y L_y = m\pi = \frac{2\pi}{\lambda_y} L_y$$



$$m\lambda_y = 2L_y$$

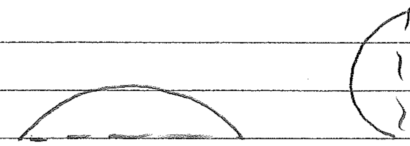
$$k^2 = k_x^2 + k_y^2 \qquad k = \sqrt{k_x^2 + k_y^2}$$

$$\omega = 2\pi f = vk \qquad v = \sqrt{\frac{T_s}{\frac{M}{A}}}$$

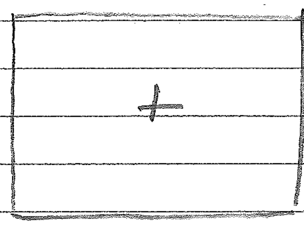
NODAL LINE PATTERNS

$n=1,\ m=1$

(n, m)



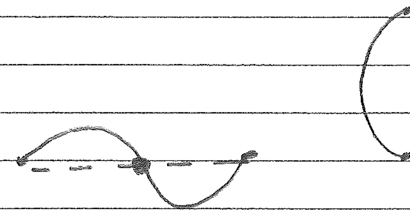
$n=1$ $m=1$

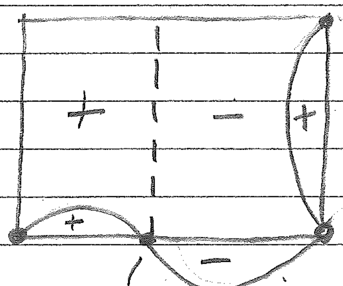


JUST A BULGE AT CENTER.

NOTE # OF NODAL LINE INSIDE RECTANGLE IS ONE UNIT LESS THAN n OR m.

$n=2\ m=1$



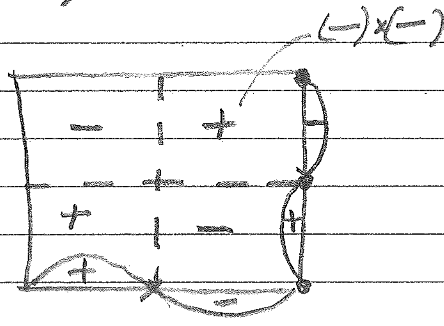


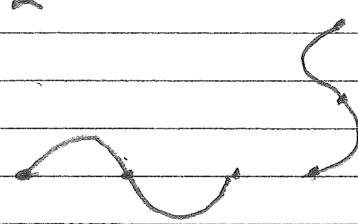
$\psi = \sin k_x x \sin k_y y\, e^{-i\omega t}$

$\sin k_x x = 0$ FOR ANY y.

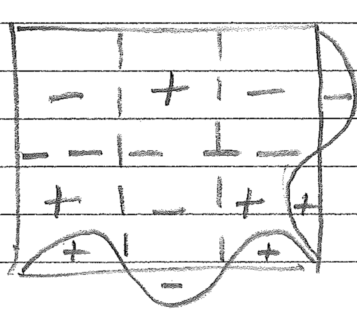
$k_x \frac{L_x}{2} = \pi \qquad \lambda_x = L_x$.

$(2, 2)$



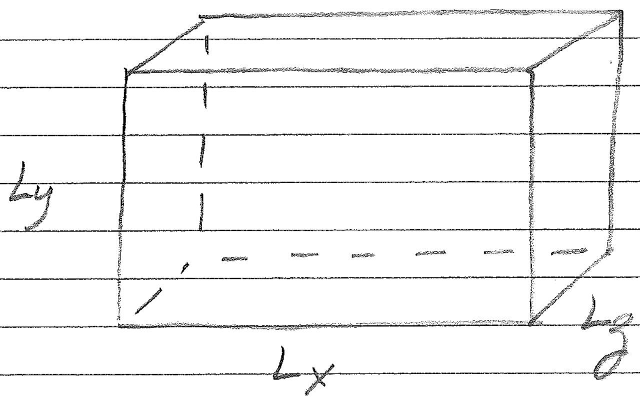


$(3, 2)$



BOXES – ROOMS – ARCHITECTURAL ACOUSTICS
3-D

$$k_x x + k_y y + k_z z = \vec{k}\cdot\vec{r}$$



$$n\lambda_x = 2L_x \quad m\lambda_y = 2L_y \quad p\lambda_z = 2L_z$$
$$p = 1, 2, 3, \ldots$$

$$\psi \sim \sin k_x x \sin k_y y \sin k_z z \, e^{-i\omega t}$$

FROM λ'S. FORM $\vec{k} = (k_x, k_y, k_z)$

$$= \left(\frac{2\pi}{\lambda_x}, \frac{2\pi}{\lambda_y}, \frac{2\pi}{\lambda_z}\right)$$

$$\vec{k} = \left(\frac{2\pi n}{2L_x}, \frac{2\pi m}{2L_y}, \frac{2\pi p}{2L_z}\right)$$

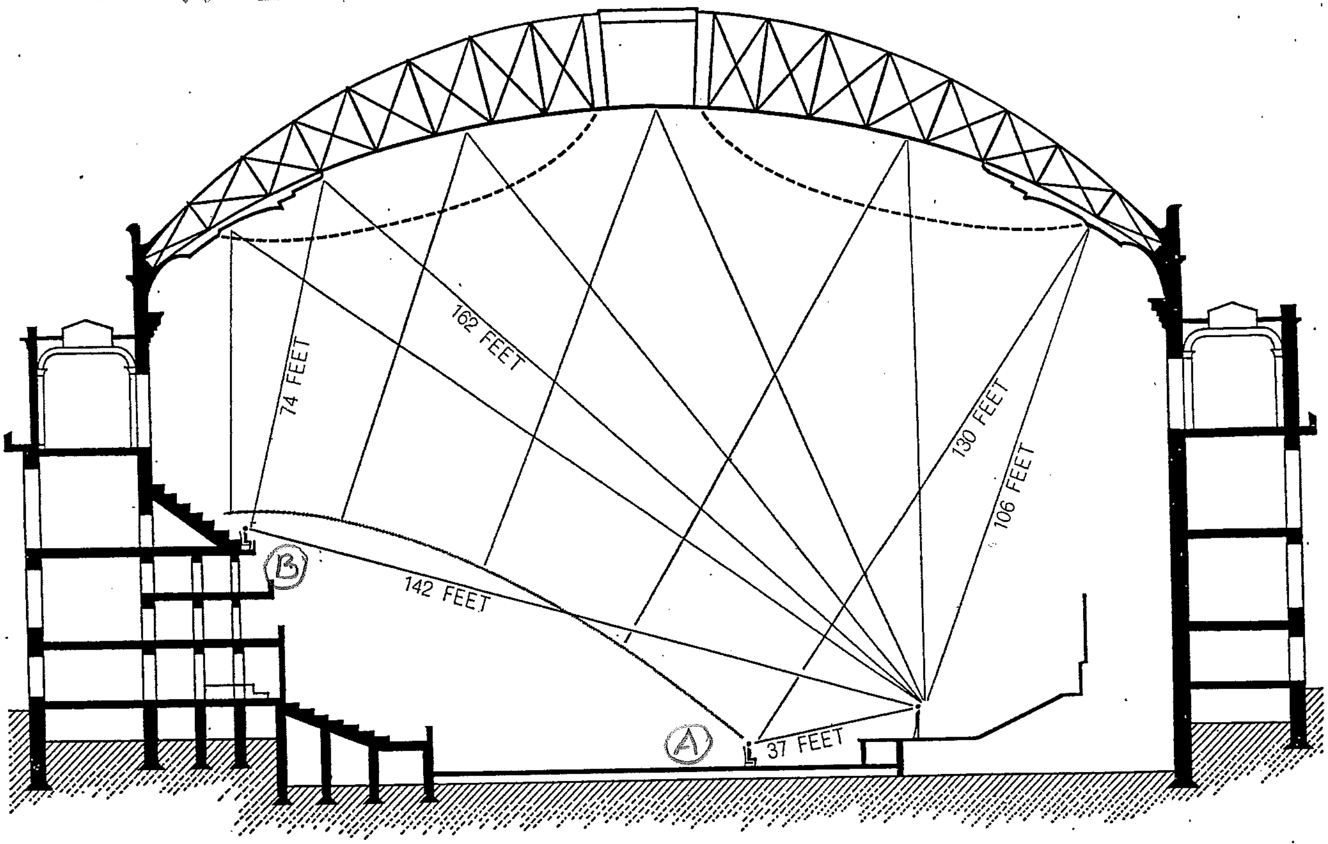
$$|\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\omega_{nmp} = v|\vec{k}| = v\sqrt{k_x^2 + k_y^2 + k_z^2}$$

v = SPEED OF SOUND IN AIR.

ω_{nmp} RESONANT ω'S FOR ROOM.
$= 2\pi f_{nmp}$. f_{nmp} RESONANT FREQUENCIES OF CAVITY WITH $\psi = 0$ ON BOUNDARIES

ARCHITECTURAL ACOUSTICS CONCERNED WITH ECHOES.



ROYAL ALBERT HALL, opened in 1871, was originally plagued by echoes reflected from the great dome. The colored lines show reflections of equal travel time. A listener in the front of the auditorium would hear an echo nearly a fifth of a second behind the direct sound. Echoes and reverberation were much reduced by installation of a velarium, or heavy fabric awning (*broken lines*).

FRONT LISTENER: $v_S \approx 340\ m/s \to 1115\ ft/s \sim 1000\ ft/s$.

$\frac{37}{1000} = .037$ DIRECT SOUND.

$\frac{236}{1000} \sim .236$ ECHO.

$-.037$ DIRECT

$\Delta t \sim .2$ SECS LATER.

(B) $\frac{142}{1000} = .142$ sec.

$\frac{162+74}{1000} = \frac{236}{1000} = .236S$ ECHO.

$-.142$ DIRECT

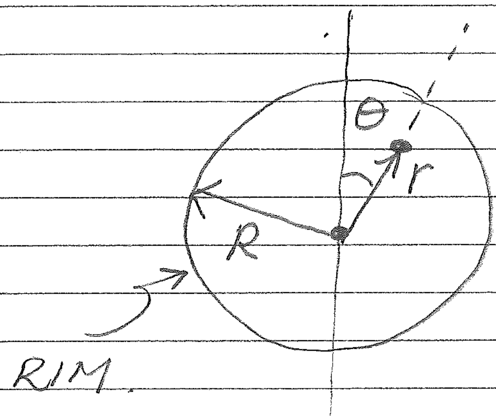
$\Delta t = .094 \sim .1$ sec.

2-D - CIRCULAR DRUMHEADS

$$\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}$$

SQUARE & RECTANGULAR
USE x, y COORDINATES

called $\nabla^2_{x,y}$



USE θ, r CYLINDRICAL (CIRCULAR) COORDINATES

$$\psi(x,y,t) \to \psi(r,\theta,t)$$

$$= R(r)\chi(\theta)\,e^{-i\omega t}$$

$$\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{v^2}(-i\omega)^2\psi = -\frac{\omega^2}{v^2}\psi = -k^2\psi$$

$$-k^2\psi = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}$$

$\psi = R(r)\chi(\theta)$ $\qquad \chi(\theta) = \sin n\theta$ or $\cos n\theta$

$$\frac{\partial^2\chi}{\partial\theta^2} = -n^2\begin{pmatrix}\sin n\theta\\ \cos n\theta\end{pmatrix} = -n^2\chi(\theta)$$

$$-k^2R(r)\cancel{\chi(\theta)} = \cancel{\chi(\theta)}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial R(r)}{\partial r} - \frac{1}{r^2}(n^2)R(r)\cancel{\chi}$$

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}r}\frac{\partial R(r)}{\partial r} - \frac{n^2}{r^2}R(r) + k^2R(r) = 0$$

$$\cancel{\frac{1}{r}}\cancel{r}\frac{\partial^2}{\partial r^2}R + \frac{1}{r}\frac{\partial R}{\partial r} - \frac{n^2}{r^2}R + k^2R = 0$$

$\times r^2$

$$r^2\frac{d^2}{dr^2}R(r) + r\frac{d}{dr}R + \left((kr)^2-n^2\right)R$$

$$= (kr)^2\frac{d^2R}{(d\,kr)^2} + (kr)\frac{dR}{d(kr)} + \left((kr)^2-n^2\right)R$$

$x = kr$ NEW VARIABLE

$$x^2 \frac{d^2}{dx^2}R(x) + x\frac{d}{dx}R(x) + (x^2 - n^2)R(x) = 0$$

$R(x) = J_n(x)$ BESSEL FUNCTION.

NOT TO WORRY $J_n(x)$ LOOKS LIKE $\cos x$

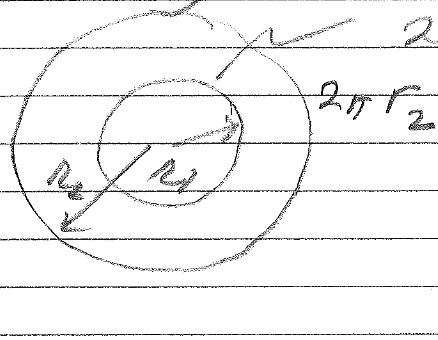
SEE COMPARISONS ON NEXT PAGE FROM MATHEMATICA.

$$J_n(x) \sim \frac{\sqrt{\frac{2}{\pi}}\cos\left[x - \frac{n\pi}{2} - \frac{\pi}{4}\right]}{\sqrt{x}}$$

IMPORTANT

FOR A SPHERICAL WAVE AMPLITUDE $\frac{1}{r}$ & INTENSITY $\sim \frac{1}{r^2}$ SO THAT TOTAL ENERGY CROSSING A SPHERE OF RADIUS r (AREA $= 4\pi r^2$) IS CONSTANT

ON A 2-D SURFACE WANT THE ENERGY GOING THROUGH CIRCLE

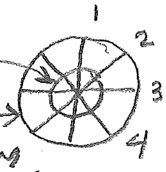


TO BE CONSTANT

AMPLITUDE MUST FALL AS $\frac{1}{\sqrt{r}}$, INTENSITY AS $\frac{1}{r}$. THE CIRCUMFERENCE $2\pi r \times$ INTENSITY $\frac{1}{r}$ = CONSTANT

2-D CIRCLES
P. 2A

INNER CIRCE



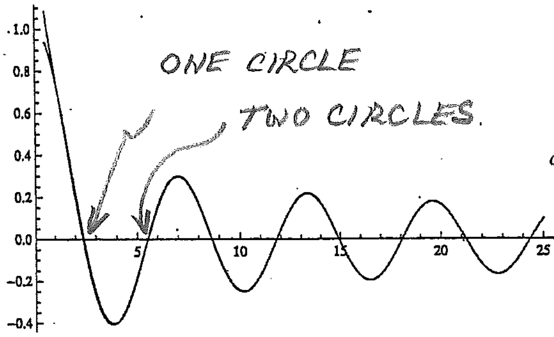
CIRCULAR NODE OR RIM

4 LINES
2 CIRCLES

(4,2)

In[30]:= Plot [{BesselJ[0, x], (2 / Pi) ^ .5 * Cos[x - Pi / 4] / x ^ .5}, {x, .5, 25}]

Out[30]=



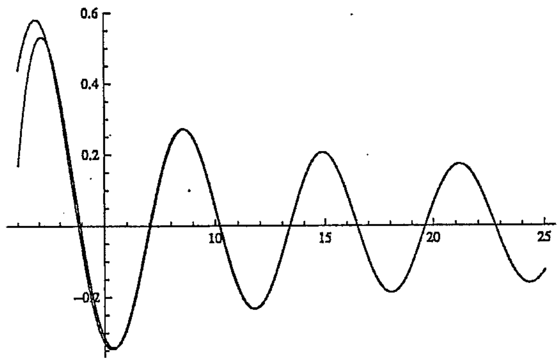
$$J_0(x) \sim \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{x}} \cos\left(x - \frac{\pi}{4}\right)$$

$J_0(x=0) = 1$ GIVES BULGE AT CENTER

J_0 FOR NO LINES

In[29]:= Plot [{BesselJ[1, x], (2 / Pi) ^ .5 * Cos[x - Pi / 2 - Pi / 4] / x ^ .5}, {x, 1, 25}]

Out[29]=



$$J_1(x) \sim \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{x}} \cos\left(x - \frac{\pi}{2} - \frac{\pi}{4}\right)$$

$J_1(x=0) = 0$

J_1 FOR ONE LINE

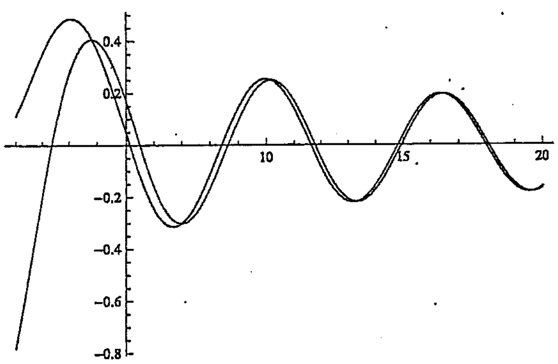
$$J_2(x) \sim \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{x}} \cos\left(x - 2\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$J_2(x=0) = 0$

J_2 FOR TWO LINES

Plot [{BesselJ[2, x], (2 / Pi) ^ .5 * Cos[x - 2 * Pi / 2 - Pi / 4] / x ^ .5}, {x, 1, 25}]

Out[21]=

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Table[N[BesselJZero[0, k]], {k, 1, 5}]
{2.40483, 5.52008, 8.65373, 11.7915, 14.9309}

Table[N[BesselJZero[1, k]], {k, 1, 5}]
{3.83171, 7.01559, 10.1735, 13.3237, 16.4706}

Table[N[BesselJZero[2, k]], {k, 1, 5}]
{5.13562, 8.41724, 11.6198, 14.796, 17.9598}

Table[N[BesselJZero[3, k]], {k, 1, 5}]
{6.38016, 9.76102, 13.0152, 16.2235, 19.4094}

Table[N[BesselJZero[4, k]], {k, 1, 5}]
{7.58834, 11.0647, 14.3725, 17.616, 20.8269}

Table[N[BesselJZero[5, k]], {k, 1, 5}]
{8.77148, 12.3386, 15.7002, 18.9801, 22.2178}

Table[N[BesselJZero[6, k]], {k, 1, 5}]
{9.93611, 13.5893, 17.0038, 20.3208, 23.5861}
```

$x = kr$