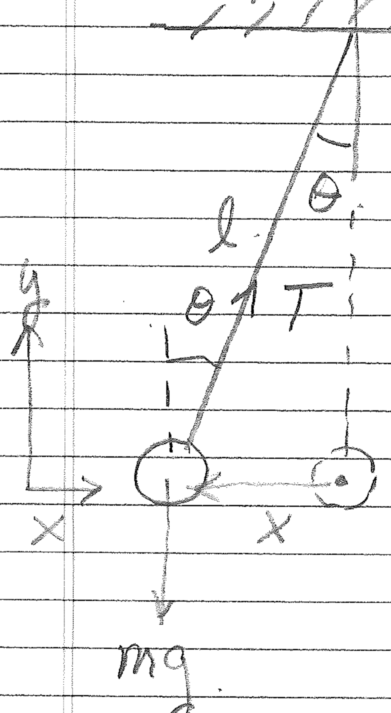
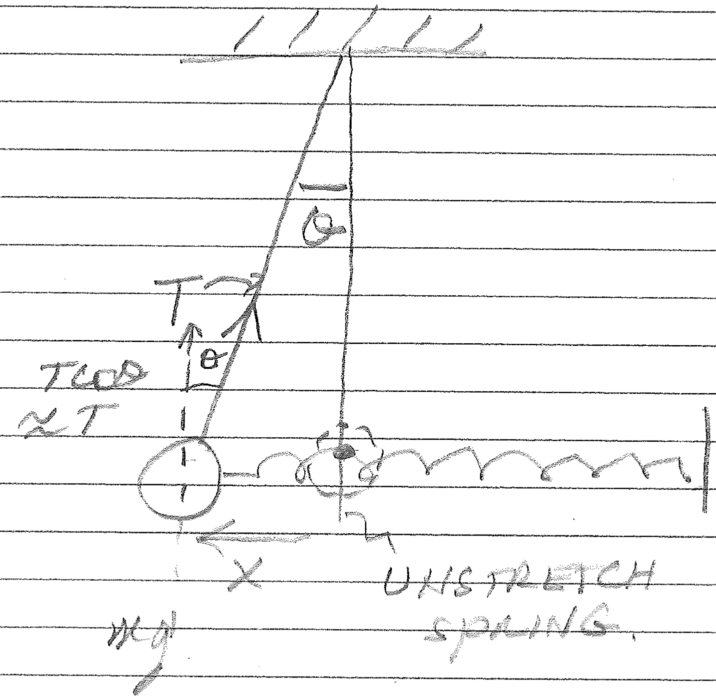
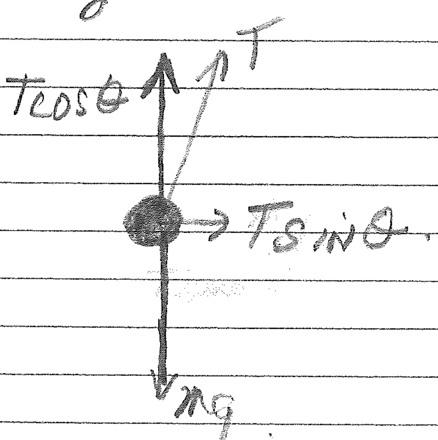
EXAMPLES OF SHM.







$\theta \approx 0 \quad \cos\theta \cong 1.$

$T\cos\theta - mg = 0 \quad T \approx mg$

$T\sin\theta = m a_x$

$\sin\theta = x/l$

SIGN $\xleftarrow{-x}$

$$-mg\frac{x}{l} = m\frac{d^2x}{dt^2}$$

$k = mg/l \quad k/m = g/l.$

$T \approx mg.$

ΣF_x

$T\sin\theta + F_s =$

$$mg\frac{x}{l} + kx$$

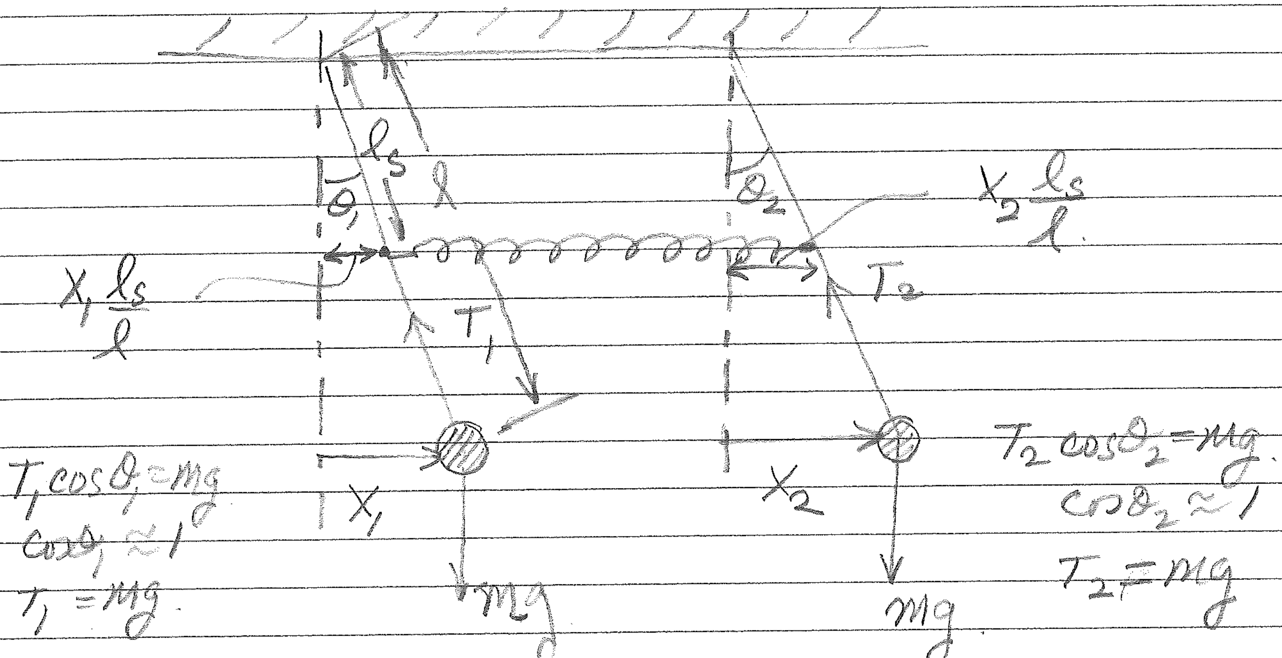
$$-\left(\frac{mg}{l} + k\right)x = m\frac{d^2x}{dt^2}$$

$$-\left(\frac{g}{l} + \frac{k}{m}\right)x = \underbrace{\cancel{m}\frac{d^2x}{dt^2}}_{\text{MISTAKE}}$$

$x = A\cos\omega_0 t$

$$\rightarrow \quad \omega_0^2 = \frac{g}{l} + \frac{k}{m}.$$

COUPLED SYSTEMS (P. 1)



$$m\,\frac{d^2X_1}{dt^2} = -T_1 \sin\theta_1 - k(X_1 - X_2)\frac{l_s}{l} \qquad \sin\theta_1 = \frac{X_1}{l}$$

$$m\,\frac{d^2X_1}{dt^2} = -mg\frac{X_1}{l} - k(X_1 - X_2)\frac{l_s}{l}$$

① $$\frac{d^2X_1}{dt^2} = -\frac{g}{l}X_1 - \left(\frac{k\,l_s/l}{m}\right)(X_1 - X_2)$$

② $$\frac{d^2X_2}{dt^2} = -\frac{g}{l}X_2 - \left(\frac{k\,l_s/l}{m}\right)(X_2 - X_1)$$

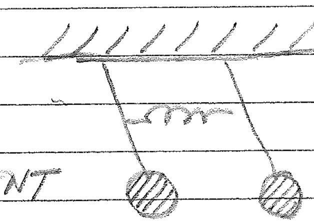
HERE $X_2 - X_1$ IF X_1 ✓ $\frac{k\,l_s}{m\,l}X_2$

TO SOLVE — USE FOLLOWING PROCEDURE

ADD ① & ②

③ $$\frac{d^2}{dt^2}(X_1 + X_2) = -\frac{g}{l}(X_1 + X_2) + 0$$

LOOKS LIKE A SYSTEM THAT
OSCILLATES AS SHOWN
NO SPRING COUPLING IS PRESENT



$$\underbrace{\omega_\Sigma = \sqrt{g/l}}_{\text{FOR SUM}} \qquad X_\Sigma \equiv X_1 + X_2 \qquad \frac{d^2X_\Sigma}{dt^2} = -\omega_\Sigma^2 X_\Sigma$$

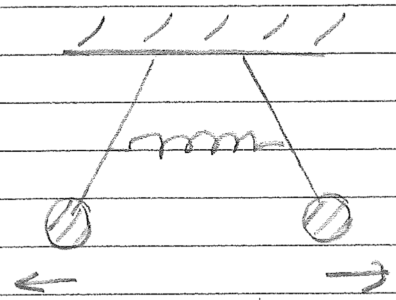
COUPLED SYSTEMS (P. 2)

SUBTRACT ① & ②

$$④ \quad \frac{d^2(x_1-x_2)}{dt^2} = -\frac{g}{l}(x_1-x_2) - \frac{2k\,l_s/l}{m}(x_1-x_2)$$

$$= -\left(\frac{g}{l} + \frac{2k\,l_s/l}{m}\right)(x_1-x_2)$$

LOOKS LIKE AN EQUATION THAT IS SIMPLE FOR JUST x_1-x_2 ONLY.



$$\omega_\Delta = \sqrt{\frac{g}{l} + \frac{2k(l_s/l)}{m}}$$

FOR DIFFERENCE

SPRING STRETCHES $2(x_1-x_2)$

$$X_\Delta \equiv x_1 - x_2$$

$$\frac{d^2 X_\Delta}{dt^2} = -\omega_\Delta^2 X_\Delta$$

NOTE $X_\Sigma \equiv x_1 + x_2$

$X_\Delta \equiv x_1 - x_2$

$$x_1 = \frac{X_\Sigma + X_\Delta}{2} \qquad x_2 = \frac{X_\Sigma - X_\Delta}{2}$$

$$X_\Sigma = A_\Sigma \cos\omega_\Sigma t + B_\Sigma \sin\omega_\Sigma t$$ SOLUTION TO ③

$$X_\Delta = A_\Delta \cos\omega_\Delta t + B_\Delta \sin\omega_\Delta t$$ SOLUTION TO ④

INITIAL CONDITION DETERMINES $A_\Sigma, B_\Sigma, A_\Delta, B_\Delta$.

USE $\cos\omega_\Sigma t$, $\cos\omega_\Delta t$ WHEN NO INITIAL SPEEDS JUST INITIAL DISPLACEMENT

EX $x_1 = A$ AT $t=0$ & $x_2 = 0$ NO INITIAL v's

THE WRITE

$$x_1 = A_\Sigma \cos\omega_\Sigma t + A_\Delta \cos\omega_\Delta t$$

$$x_2 = A_\Sigma \cos\omega_\Sigma t - A_\Delta \cos\omega_\Delta t$$

COUPLED SYSTEMS (P.3)

At $t=0$ $\quad X_1 = A = A_\Sigma + A_\Delta$

$$X_2 = 0 = A_\Sigma - A_\Delta \qquad A_\Sigma = A_\Delta$$

$$\Rightarrow A = 2A_\Sigma \quad A_\Sigma = A/2 = A_\Delta$$

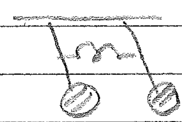
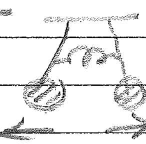
$$X_1 = \frac{A}{2}\left\{\cos\omega_\Sigma t + \cos\omega_\Delta t\right\}$$

$$X_2 = \frac{A}{2}\left\{\cos\omega_\Sigma t - \cos\omega_\Delta t\right\}$$

USE TRIG IDENTITIES:

$$X_1 = \frac{A}{2}\, 2\cos\left(\frac{\omega_\Sigma+\omega_\Delta}{2}t\right)\cos\left(\frac{\omega_\Sigma-\omega_\Delta}{2}t\right)$$

$$X_2 = \frac{A}{2}\, 2\cos\left(\frac{\omega_\Sigma+\omega_\Delta}{2}t\right)\sin\left(\frac{\omega_\Sigma-\omega_\Delta}{2}t\right)$$

$\omega_\Sigma = \sqrt{g/l}$  ; $\omega_\Delta = \sqrt{\frac{g}{l} + \frac{2k}{m}\left(\frac{l_s}{l}\right)}$ 

WHEN X_1 TERM $\cos\frac{\omega_\Sigma-\omega_\Delta}{2}t = \pm 1$, $\sin\frac{\omega_\Sigma-\omega_\Delta}{2}t = 0$

$\cos\frac{\omega_\Sigma-\omega_\Delta}{2}t = \pm 1$, $\frac{\omega_\Sigma-\omega_\Delta}{2}t = 0, \pi, 2\pi$

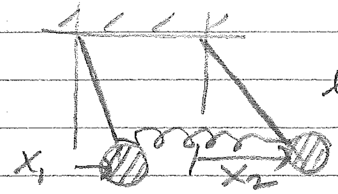
X_1, X_2 OSCILLATE OUT OF PHASE

TIME FOR X_2 TO REACH 1^{ST} MAXIMUM IN OSCILLATIONS

$\sin\frac{\omega_\Sigma-\omega_\Delta}{2}t = \pm 1$, $\left|\frac{\omega_\Sigma-\omega_\Delta}{2}\right|t = \pi/2$

THEN $X_1 \sim \cos\frac{\omega_\Sigma-\omega_\Delta}{2}t = 0$

WILBERFORCE PENDULUM – COUPLED TORSIONAL TO SPRING VIBRATION



PENDULUMS COUPLED WITH A SPRING. X_1 & X_2 COUPLED

BEFORE

① $m\dfrac{d^2X_1}{dt^2} = -mg\dfrac{X_1}{l} - k(X_1 - X_2)$

$= -mg\dfrac{X_1}{l} - kX_1 + kX_2$

COUPLES X_1 to X_2

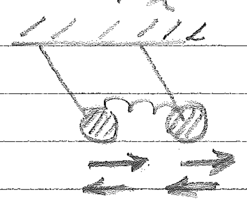
② $m\dfrac{d^2X_2}{dt^2} = -mg\dfrac{X_2}{l} - k(X_2 - X_1)$

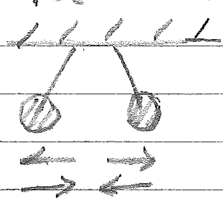
$= -mg\dfrac{X_2}{l} - kX_2 + kX_1$

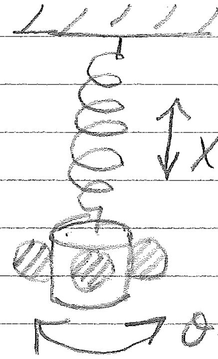
COUPLES X_2 to X_1

SOLUTIONS

$\omega_1 = \sqrt{\dfrac{g}{l}}$ $\qquad \omega_2 = \sqrt{\dfrac{g}{l} + \dfrac{2k}{m}}$







COUPLES SPRING VIBRATIONS TO TORSIONAL ROTATIONS X & θ (NO g)

SIMILAR. WRITE

① $m\dfrac{d^2X}{dt^2} = -kX - \beta_c\theta$

② $I\dfrac{d^2\theta}{dt^2} = -K_S Y_S \theta - \beta_c X$

K_S = SPRING PROPERTIES

β_c IS A COUPLING COEFFICIENT

$\omega_S = \sqrt{\dfrac{k}{m}}$ NO COUPLING WITH ROTATION

$\omega_R = \sqrt{\dfrac{K_S Y_S}{I}}$ NO COUPLING WITH SPRING

GOAL IS TO FIND CORRESPONDING ω_1 AND ω_2. ANY MOTION CAN BE WRITTEN IN TERMS OF ω_1 & ω_2

WILBERFORCE PENDULUM – SOLUTION

① $\dfrac{d^2X}{dt^2} = -\omega_S^2 X - \dfrac{\beta_c}{m}\theta$ $\qquad$ ② $\dfrac{d^2\theta}{dt^2} = -\omega_R^2\theta - \dfrac{\beta_c}{I}X$

STEP 2 SOLVE ② FOR X

③ $X = -\dfrac{I}{\beta_c}\left(\dfrac{d^2\theta}{dt^2} + \omega_R^2\theta\right)$

NEED ALSO $\dfrac{d^2X}{dt^2} = \dfrac{d^2}{dt^2}\left(-\dfrac{I}{\beta_c}\left(\dfrac{d^2\theta}{dt^2} + \omega_R^2\theta\right)\right)$

④ $\dfrac{d^2X}{dt^2} = -\dfrac{I}{\beta_c}\left(\dfrac{d^4\theta}{dt^4} + \omega_R^2\dfrac{d^2\theta}{dt^2}\right)$

SUBSTITUTE ③ & ④ INTO ①