1. (6.42 Finding the vector potential)

\[ \nabla \times \mathbf{A} = \mathbf{B}_0 \hat{k} \Rightarrow \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} = \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \]

and \[ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \mathbf{B}_0 \]

The easiest is to make all the derivatives zero except for \( \frac{\partial A_y}{\partial x} \)

\[ A_x = A_z = 0 \]

\[ A_y = B_0 \times \quad (0, B_0, 0) \]

Another choice that is often made for a uniform field is more symmetrical in the coordinates perpendicular to the field.

\[ A_z = 0, \quad A_y = \frac{B_0}{2} x, \quad A_x = -\frac{B_0}{2} y \]

\( \left(-\frac{B_0}{2} y, \frac{B_0}{2} x, 0\right) \)
2. (6.43 Vector potential inside a wire)

\[ J = \frac{I}{\pi r_0^2} \]
\[ 2\pi r B(r) = \frac{\mu_0 I r^2}{r_0^2} \]
\[ \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r_0^2} r \hat{\theta} \]

Since the problem gives \( \vec{A} \) in Cartesian coordinates, let's first do the curl in Cartesian coordinates

\[ \nabla \times [A_0 \hat{z}(x^2+y^2)] = \frac{\partial}{\partial y} A_0(x^2+y^2) \hat{x} - \frac{\partial}{\partial x} A_0(x^2+y^2) \hat{y} \]
\[ = A_0 \cdot 2y \hat{x} - A_0 \cdot 2x \hat{y} \]

compare with \( \vec{B} \) above in Cartesian coordinates

\[ \vec{B}(r) = \frac{\mu_0 I}{2\pi r_0^2} r \left( -\frac{y}{r} \hat{x} + \frac{x}{r} \hat{y} \right) \]

\[ \Rightarrow A_0 = -\frac{\mu_0 I}{4\pi r_0^2} \]

matches everywhere inside the wire

More natural to compute the curl of \( A_0 r^2 \hat{z} \) in cylindrical coordinates

\[ \nabla \times \vec{A} = -\frac{\partial}{\partial r} (A_0 r^2) \hat{\theta} = -2A_0 r \hat{\theta} \]

\[ \Rightarrow A_0 = -\frac{\mu_0 I}{4\pi r_0^2} \]
3. (6.46 * Field from a wire frame)

First consider the field contribution from one segment using Biot-Savart law.

\[ \mathbf{B}_E \propto \frac{1}{r^2} (\hat{j} + \hat{k}) \]
\[ \mathbf{B}_D \propto \frac{1}{r^2} (\hat{l} + \hat{j}) \]
\[ \mathbf{B}_A \propto \frac{1}{r^2} (\hat{j} - \hat{k}) \]
\[ \mathbf{B}_B \propto \frac{1}{r^2} (-\hat{l} + \hat{j}) \]

Total \( \mathbf{B} \propto \hat{j} \)

Simplify by constructing a wire arrangement with current \( I \) in each segment. Cancel in pairs so field at \( P \) is zero.

Superposition \( \rightarrow \) field is same as single loop.

Total \( \mathbf{B} \propto \hat{j} \)
First, use Biot-Savart to find contributions from straight segments.

\[ \int_{-\infty}^{\infty} dB = \int_{-\infty}^{0} \frac{\mu_0 I (dx) \, z}{4\pi \left( x^2 + z^2 \right)^{3/2}} = \frac{\mu_0 I \, z}{4\pi} \frac{x}{\left( x^2 + z^2 \right)^{1/2}} \]

\[ \hat{B} = \frac{\mu_0 I}{4\pi \, z} \text{ out of page} \]

This makes sense since the fields from the two segments that make up an infinite wire have to add to \( \frac{\mu_0 I}{2\pi \, z} \text{ out of page} \).

The circular arc is \( \frac{\mu_0 I}{4\pi} \int \frac{r \, d\theta}{r^2} = \frac{\mu_0 I}{4\pi} \text{ out of page} \).

So total field at \( \hat{B} \) is \( 2 \, \frac{\mu_0 I}{4\pi \, 4r} + \frac{\mu_0 I}{4\pi \, 4r} = \mu_0 I \left( \frac{2 + 1}{4r} \right) \).
5. (6.54 Force between a wire and a loop)

Square is distance $l$ below the wire. $dF = I dl \times B$

drawing on right shows that force on segment $C$
cancels force on segment $A$.

The vertical components of $F$ cancel

The horizontal component of each $F$ is

\[
\mu_0 I_1 I_2 l \left( \frac{l}{2} \right) = \frac{\mu_0 I_1 I_2 l^2}{2\pi \left( r^2 + \left( \frac{l}{2} \right)^2 \right)}
\]

\[
= \frac{\mu_0 I_1 I_2 l^2}{4\pi \left( r^2 + \left( \frac{l}{2} \right)^2 \right)}
\]

Add the two contributions:

\[
F = \frac{\mu_0 I_1 I_2 l^2}{2\pi r^2 + \left( \frac{l}{2} \right)^2}
\]

$R = \sqrt{r^2 + \left( \frac{l}{2} \right)^2}$

to the left.
6. (6.56 Field at the tip of a cone)

\[
\text{slice into rings of width } dl \\
\text{total charge on ring is } \sigma \cdot 2\pi l \sin \theta \, dl \\
\text{current} = \frac{\text{charge}}{\text{time}} = \frac{\sigma 2\pi l \sin \theta \, dl}{(2\pi/w)}
\]

Recall that \( B \) on axis of ring of radius \( b \) at height \( z \) is

\[
\frac{\mu_0 I b^2}{2(b^2 + z^2)^{3/2}}
\]

from each ring in cone

\[
\frac{dB}{dz} = \frac{\mu_0}{2} \sigma w l \sin \theta \, dl \frac{(l \sin \theta)^2}{2 \left( l^2 \sin^2 \theta + l^2 \cos^2 \theta \right)^{3/2}}
\]

\[
= \frac{\mu_0}{4} \sigma w \sin^3 \theta \, dl
\]

\[
B = \frac{\mu_0 \sigma w \sin^3 \theta L}{4} \text{ up along axis}
\]
7. (6.57 A rotating cylinder)

another ring problem

Find field at \( z = 0 \) - same for all \( z \)

\[ \text{current} = \sigma \cdot 2\pi R dz = \sigma \omega R dz \]

\[ \text{current} = \frac{2\pi R dz}{(2\pi/\omega)} \]

\[ \text{field of ring} = \frac{\mu_0 R^2 \sigma \omega R dz}{2 (R^2 + z^2)^{3/2}} \]

\[ B = \frac{\mu_0 \sigma \omega R^3}{2} \int_{-\infty}^{\infty} \frac{dz}{(R^2 + z^2)^{3/2}} = \frac{\mu_0 \sigma \omega R^3}{2} \frac{\pi}{R^2 (R^2 + z^2)^{1/2}} \bigg|_{z=0}^{z=\infty} \]

\[ B = \mu_0 \sigma \omega R \]

8. (6.58 Rotating cylinders)

\[ \frac{Q_1}{\ell} \quad \text{this is a capacitor} \]

\[ \text{transfer charge/length to get voltage difference} \]

\[ r = \frac{Q}{2\pi r_1 L} \quad \text{The field inside is the same everywhere} \]

\[ \text{Field on axis from 6.57 is} \]

\[ \frac{\mu_0 Q}{2\pi r_1 L} = \frac{\mu_0}{2\pi} \frac{W}{L} \]

\[ \text{go back to chapter 3 to get capacitance/length of coaxial cylinders} \]

\[ E \text{ field from } r_1 \text{ to } r_2 = \frac{Q}{\varepsilon_0 2\pi r L} \]

\[ \text{potential difference} = \frac{Q}{2\pi \varepsilon_0 L} \int \frac{dr}{r} = \frac{Q}{2\pi \varepsilon_0 L} \ln \left( \frac{r_2}{r_1} \right) \]

\[ \text{so} \quad \frac{Q}{L} = \frac{V \cdot 2\pi \varepsilon_0}{\ln \left( \frac{r_2}{r_1} \right)} \]

\[ B = \frac{\mu_0 W}{2\pi} \frac{2\pi \varepsilon_0 V}{\ln \left( \frac{r_2}{r_1} \right)} \]

\[ = 1.1 \times 10^{-6} \text{ gauss} \]
9. (6.59 Scaled down solenoid)

\[
\begin{align*}
B' &= \mu_0 I n \\
I &= \frac{B}{\mu_0 n} = \frac{0.1}{4\pi \times 10^{-7} \times 100} \\
&= 79.6 \text{ A}
\end{align*}
\]

length of wire = \(200 \cdot \pi \cdot 1 \text{ m} = (200 \pi) \text{ m}\), area of wire = \(\frac{\pi}{4} \text{ m}^2\)

in the scaled down solenoid

\[
\begin{align*}
I' &= \frac{B'}{\mu_0 n} = \frac{0.1}{4\pi \times 10^{-7} \times 1000} \\
&= 79.6 \text{ A}
\end{align*}
\]

length of wire = \(200 \cdot \pi \cdot 0.1 \text{ m} = 20 \pi \text{ m}\), area of wire = \(\frac{\pi}{400} \text{ m}^2\)

a) \(V' = I' R' = (0.1) I \cdot (0.1) R = IR = 120 \text{ V}\)

(note that the wire cannot be made of copper, since the numbers given here determine \(p = \frac{RA}{L} = 1.88 \times 10^{-4} \text{ m}^{-1}\)

which is about \(10^4 \times \) the resistivity of copper (\(1.72 \times 10^{-8} \text{ m}^{-1}\))

I should have told you to cross the word "copper" out of the statement of the problem)
b) \[ P = IV, \quad P' = I'V' = (0.1I)V = 0.1P \]

The smaller coil dissipates 10 times less power than the large coil.

However, the surface area is 100 times smaller so it is dissipating 10 times more power per area and will be harder to cool.
(a) Think of the field on the axis as being the sum of the fields of the rings of current that make up the solenoid.

So the field at $P_2$ is equal to the field at $P_3$.

Also, for a given $I$ and $n$ (# of turns/length), $P_1$, $P_2$ slowly increases with the length $L$ of the solenoid (more rings contribute to field).

$$B_{p_1}(2L) = B_{p_2}(L) + B_{p_3}(L) = 2B_{p_2}(L) \geq B_{p_1}(L)$$

So $B_{p_2}(L)$ is slightly more than $\frac{B_{p_1}(L)}{2}$.
11. (6.63c Solenoids and superposition)

\[ \text{semi-infinite} \]

The flux through the coil cross-section a large distance back into interior is the same as if the coil were infinite \( \Phi_j \).

Put 2 semi-infinite coils together to get an infinite coil. Flux adds, so
\[ 2 \Phi_{si} = \Phi_j \]

and
\[ \Phi_{si} = \frac{1}{2} \Phi_j = \frac{1}{2} \text{ the flux through the coil at a large distance back} \]