A tentative theory of large distance physics

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The general nonlinear model (1979)

A renormalizable 2d field theory:

$$\int \mathrm{D}x \ \mathrm{e}^{-\int \mathrm{d}^2 z \ \frac{1}{2\pi} \ h_{\mu\nu}(x)\,\partial x^\mu\,\bar{\partial} x^\nu}$$

The fields are maps $x^\mu(z, \bar{z})$ from the plane to a target manifold (spacetime).

$h_{\mu\nu}(x)$ is a general riemannian metric, comprising infinitely many coupling constants, all naively dimensionless (renormalizable).

Renormalized at 2d distance μ^{-1},

$$\mu \frac{\partial}{\partial \mu} (h_{\mu\nu}(x)) = \beta_{\mu\nu} = 2R_{\mu\nu}(x) + \mathrm{O}(R^2)$$

$\beta = 0$ is Einstein's equation.

2d scale invariance is the equation of motion of General Relativity (without matter).

The coupling constants λ^i

Perturb around a solution of $\beta = 0$:

$$h_{\mu\nu}(x) = h^{(1)}_{\mu\nu}(x) + \sum_i \lambda^i \, h_{i,\mu\nu}(x)$$

The wave modes $h_{i,\mu\nu}(x)$ are the 2d fields

$$\phi_i(z,\bar{z}) = h_{i,\mu\nu}(x)\, \mu^{-2}\, \partial x^\mu \, \bar{\partial} x^\nu$$

The nearby 2d models are made by inserting

$$\mathrm{e}^{-\int \mathrm{d}^2 z\, \mu^2 \frac{1}{2\pi}\, \lambda^i \phi_i(z,\bar{z})}$$

Their anomalous dimensions $\gamma(i)$

$$\mathrm{dim}(\phi_i) = 2 + \gamma(i) \qquad \mathrm{dim}(\lambda^i) = -\gamma(i)$$

come from the linearized β-function

$$\left(-\nabla^\sigma \nabla_\sigma + \cdots\right)\; h_{i,\mu\nu}(x) = \gamma(i)\;\, h_{i,\mu\nu}(x)$$

Locally, the modes $h_{i,\mu\nu}(x)$ are plane waves with wave vectors $p(i)$ and

$$\gamma(i) \approx p(i)^2$$

At short 2d distances $\Lambda^{-1} \ll \mu^{-1}$

Running coupling constants:

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda_r^i = \beta^i(\lambda_r) = \gamma(i)\, \lambda_r^i + \cdots$$

Negative dimension λ_r^i are suppressed

$$\begin{aligned} \lambda^i &= (\Lambda/\mu)^{-\gamma(i)}\, \lambda_r^i \\ &= \mathrm{e}^{-L^2 \gamma(i)}\, \lambda_r^i \end{aligned}$$

where $L^2 = \ln(\Lambda/\mu)$ (a large number).

λ_r^i is *irrelevant* (λ^i *non-renormalizable*) if

$$\mathrm{e}^{-L^2 \gamma(i)} \approx 0 \qquad \text{e.g.} \quad \mathrm{e}^{-L^2 \gamma(i)} < \mathrm{e}^{-400}$$

Otherwise λ^i is *renormalizable*:

$$L^2 \gamma(i) < 400 \qquad p(i)^2 < 400/L^2$$

So L is a spacetime distance, acting as UV cut-off. The renormalizable λ^i are the spacetime wave modes at distances $> L$.

There are only a *finite* number (if spacetime is assumed compact).

The general nonlinear model in string theory

The string worldsurface in a curved spacetime is given by a general nonlinear model.

The 2d model is the background spacetime:

$$\mathrm{e}^{-\int \mathrm{d}^2 z\, \mu^2 \frac{1}{2\pi}\, \lambda^i \phi_i}$$

The λ^i are the wave modes of the spacetime metric plus other spacetime fields (including fermions).

The 2d field theory gives the string scattering amplitudes.

$\beta = 0$ is a consistency condition (*not* an equation of motion).

All $\gamma(i) \geq 0$ (tachyon-free backgrounds).

“Realistic” background spacetimes

Λ^{-1} is a nonzero worldsurface cutoff distance

$$\mu\Lambda^{-1} \approx 0 \qquad L^2 = \ln(\Lambda/\mu) \text{ is large}$$

String scattering is cut off at IR distance L.

The worldsurface is parametrized by the renormalizable λ_r^i, the wave modes at distances $> L$.

The background spacetime at large distance L is the 2d structure at short 2d distance Λ^{-1}.

String scattering takes place in a finite mechanical environment: a “realistic” version of string scattering.

The conventional background spacetimes are idealizations, at $\Lambda^{-1} = 0$, $L = \infty$. The renormalizable λ^i are then the exactly dimensionless zero modes, $\gamma(i) = 0$ (the moduli for $\beta = 0$).

The conventional string S-matrix is an idealization ($L = \infty$). It does not give a mechanical description of the experimental apparatus in which scattering takes place.

The λ-model

A mathematically natural, scale invariant 2d nonlinear model.

The λ_r^i are made into local sources $\lambda_r^i(z,\bar{z})$ which couple at distance Λ^{-1} to local 2d fields

$$\mathrm{e}^{-\int \mathrm{d}^2 z\, \Lambda^2 \frac{1}{2\pi}\, \lambda_r^i(z,\bar{z})\, \phi_{r,i}(z,\bar{z})}$$

The sources fluctuate at short 2d distances

$$\int \mathrm{D}\lambda_r \;\; \mathrm{e}^{-S(\lambda_r)} \;\; \mathrm{e}^{-\int \lambda_r^i\, \phi_{r,i}}$$

$$S(\lambda_r) = \int \mathrm{d}^2 z\, \frac{1}{2\pi} T^{-1} g_{ij}(\lambda_r)\, \partial\lambda_r^i\, \bar{\partial}\lambda_r^j$$

The target manifold is the manifold of background spacetimes.

The couplings are given by the natural metric

$$T^{-1} g_{ij}(\lambda_r) = Z \left\langle\, \phi_{r,i}\, \phi_{r,j} \,\right\rangle \quad \text{(on the plane)}$$

$$T^{-1} = Z \left\langle 1 \right\rangle = g_s^{-2}\, V$$

The *a priori* measure

$$\left(\prod_z \int \mathrm{d}\rho_r(\Lambda, \lambda_r(z,\bar{z}))\right) \quad \mathrm{e}^{-S(\lambda_r)} \quad \mathrm{e}^{-\int \lambda_r^i\, \phi_{r,i}}$$

The *a priori* measure $\mathrm{d}\rho_r(\Lambda, \lambda_r)$ summarizes the λ-fluctuations at 2d distances $< \Lambda^{-1}$.

As Λ^{-1} increases, more λ-fluctuations are included. At the same time, the λ_r^i are driven towards $\beta = 0$ by the RG.

The measure obeys a driven diffusion equation

$$-\Lambda \frac{\partial}{\partial \Lambda} \mathrm{d}\rho_r = \nabla_i \left[T\, g^{ij}(\lambda_r) \nabla_j + \beta^i(\lambda_r)\right] \mathrm{d}\rho_r$$

The gradient property of the RG flow

$$\beta^i(\lambda_r) = T\, g^{ij}\, \partial_j \left(T^{-1} a(\lambda_r)\right)$$

implies a unique equilibrium given by

$$0 = \left[T\, g^{ij} \nabla_j + \beta^i\right] \mathrm{d}\rho_r$$

$$\mathrm{d}\rho_r = \mathrm{e}^{-T^{-1} a(\lambda_r)}\, \mathrm{dvol}(\lambda_r)$$

The *a priori* measure

$$\mathrm{d}\rho_r = \mathrm{e}^{-T^{-1}a(\lambda_r)}\,\mathrm{dvol}(\lambda_r)$$

is a QFT (a functional integral on the space-time wave modes λ^i at distances $> L$).

It is a specific QFT in a specific state.

Its equation of motion is $\beta = 0$, expressed by the equilibrium equation

$$0 = \left[T\,g^{ij}\nabla_j + \beta^i\right]\,\mathrm{d}\rho_r$$

The *a priori* measure is the *quantum* background spacetime.

The diffusion process forgets its initial condition, so the λ-model is background independent, dynamically.

The λ-model acts at 2d distances out to Λ^{-1}.

At longer 2d distances, $> \Lambda^{-1}$, there is an effective worldsurface produced by the λ-model.

At each large distance L, the λ-model produces two complementary descriptions of physics:

- a QFT at distances $> L$,
- a string S-matrix at distances $< L$.

The λ-model is formulated so that these stay always compatible, for all large L.

The λ-model was needed at short distance in the worldsurface in order to make a consistent "realistic" background spacetime, over all large spacetime distances L. (A diffusion process can be reversed only from the equilibrium state.)

The λ-model works entirely at large distance.

The λ-model builds the QFT from the largest spacetime distances *down* to L (the diffusion process assuring locality).

QFT is not derived from microscopic physics.

QFT, as produced by the λ-model, describes spacetime physics *only* at large distances L.

$\ln(\Lambda/\mu) = L^2$ must be effectively a divergence, so that the renormalization of the general non-linear model will be accurate.

Presumably, L can still be taken smaller than the smallest observable distance, since

$$(100\text{TeV})^{-1} = 10^{14}$$

in units of the Planck length.

So, if this theory is right, the λ-model will give all observable physics.

It will be a comprehensive, self-contained theory of physics.

The task now is to figure out what QFT (or QFTs) the λ-model produces.

The λ-model is merely a somewhat elaborate 2d nonlinear model. As such, it is a well-defined nonperturbative theory.

It is mathematically natural, nothing is adjustable in its formulation.

Among many questions: are there nonperturbative 2d phenomena in the λ-model (e.g., 2d instantons or defects) that have interesting effects on the QFT (by making quantum corrections to $\beta^i(\lambda)$)?

These would appear as novel physical phenomena at large distances in spacetime.

Crucial question:

Do nonperturbative 2d effects in the λ-model make corrections to β^i that break the degeneracies and produce small masses?