
QED (Including 2-photon) Corrections and Observables for Exclusive Processes

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Hampton University/JLAB

**Electron-Ion Collider Workshop: Electron-Nucleon
Exclusive Reactions**

Rutgers University, March 14, 2010



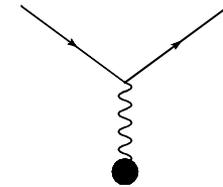
Plan of talk

- **Elastic electron-proton scattering beyond the leading order in QED**
- **Models for two-photon exchange**
- **Single-spin asymmetries**
 - **Diffractive mechanism in ep-scattering via two-photon exchange**
 - **Novel features of a single-spin asymmetry**
 - **Comparison with experiment**
 - **Possible new insights from EIC**
- **Summary**



Elastic Nucleon Form Factors

- Based on one-photon exchange approximation



$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = \frac{-ie^2}{q^2} \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(q^2) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m_N} F_2(q^2)) u_p$$

- Two techniques to measure

$$\sigma = \sigma_0 (G_M^2 \tau + \varepsilon \cdot G_E^2) \quad : \text{Rosenbluth technique}$$

$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau} \sqrt{2\varepsilon(1-\varepsilon)}}{G_M \tau \sqrt{1-\varepsilon^2}} \quad : \text{Polarization technique}$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

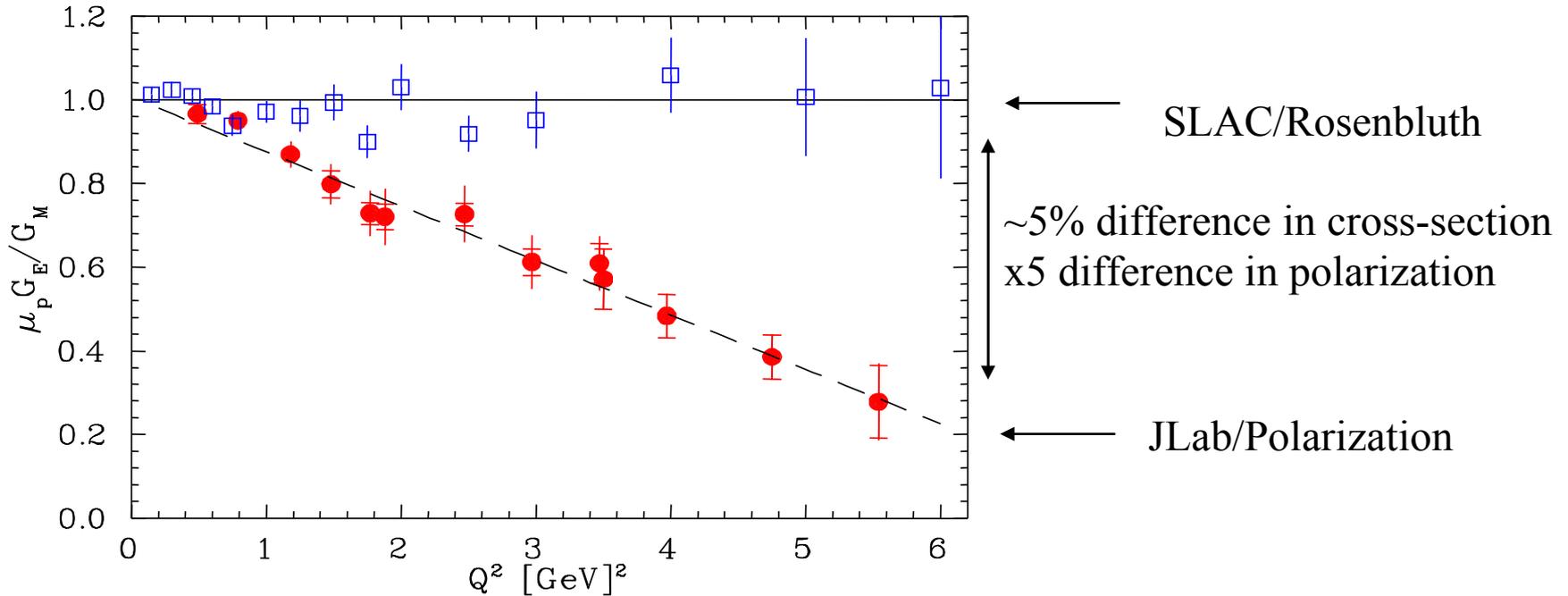
$$\tau = \frac{-q^2}{4m_N^2}, \quad \varepsilon = (1 - 2 \frac{q_{lab}^2}{q^2} \tan^2 \frac{\theta_e}{2})^{-1}$$

$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalov; Arnold, Carlson, Gross



Do the techniques agree?

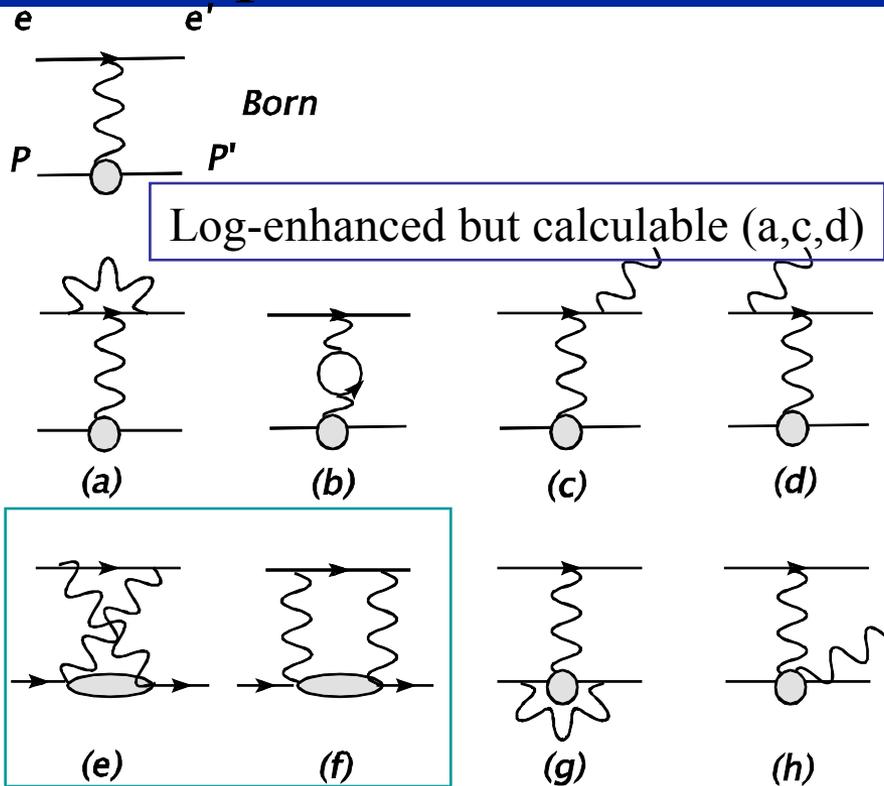


- Both early SLAC and Recent JLab experiments on (super)Rosenbluth separations followed $G_E/G_M \sim \text{const}$, see I.A. Quattan et al., Phys.Rev.Lett. 94:142301,2005
- JLab measurements using polarization transfer technique give different results (Jones'00, Gayou'02)

Radiative corrections, in particular, a short-range part of 2-photon exchange is a likely origin of the discrepancy



Complete radiative correction in $O(\alpha_{em})$



Radiative Corrections:

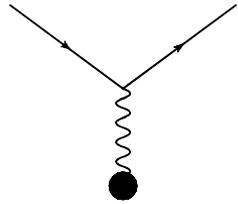
- Electron vertex correction (a)
 - Vacuum polarization (b)
 - Electron bremsstrahlung (c,d)
 - Two-photon exchange (e,f)
 - Proton vertex and VCS (g,h)
 - Corrections (e-h) depend on the nucleon structure
- Guichon&Vanderhaeghen'03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

Main issue: Corrections dependent on nucleon structure

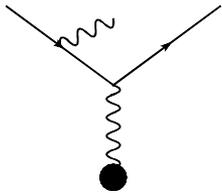
Model calculations:

- Blunden, Melnitchouk, Tjon, Phys.Rev.Lett.**91**:142304,2003
- Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

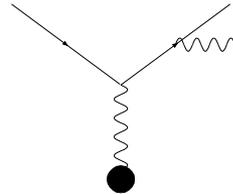
Basics of QED radiative corrections



(First) Born approximation

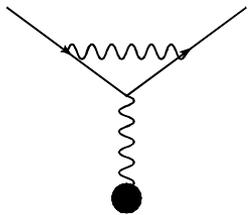


Initial-state radiation



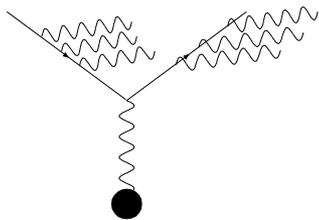
Final-state radiation

Cross section $\sim d\omega/\omega \Rightarrow$ integral diverges logarithmically: **IR catastrophe**



Vertex correction \Rightarrow cancels divergent terms; Schwinger (1949)

$$\sigma_{\text{exp}} = (1 + \delta)\sigma_{\text{Born}}, \quad \delta = \frac{-2\alpha}{\pi} \left\{ \left(\ln \frac{E}{\Delta E} - \frac{13}{12} \right) \left(\ln \frac{Q^2}{m_e^2} - 1 \right) + \frac{17}{36} + \frac{1}{2} f(\theta) \right\}$$



Multiple soft-photon emission: solved by exponentiation, Yennie-Frautschi-Suura (YFS), 1961

$$(1 + \delta) \rightarrow e^\delta$$

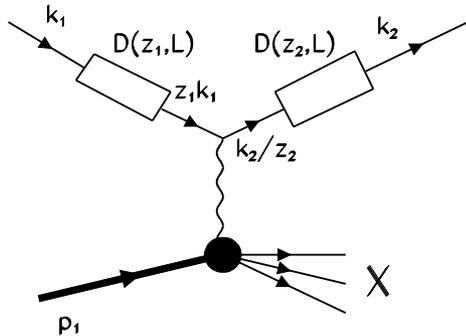
Basic Approaches to QED Corrections

- **L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).**
 - **Considered both elastic and inelastic inclusive cases. No polarization.**
- **D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).**
 - **Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.**
- **E.A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 7333 (1985); E.A. Kuraev, N.P.Merenkov, V.S. Fadin, Yad. Fiz. 47, 1593 (1988).**
 - **Developed a method of electron structure functions based on Drell-Yan representation; currently widely used at e^+e^- colliders.**



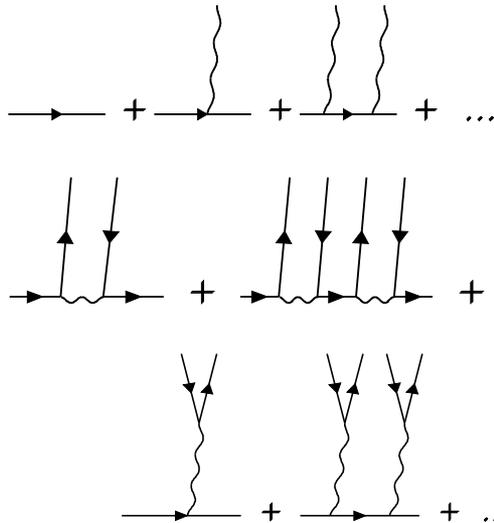
Electron Structure Functions

- For polarized $ep \rightarrow e'X$ scattering, AA et al, JETP 98, 403 (2004); elastic ep: AA et al. PRD 64, 113009 (2001).



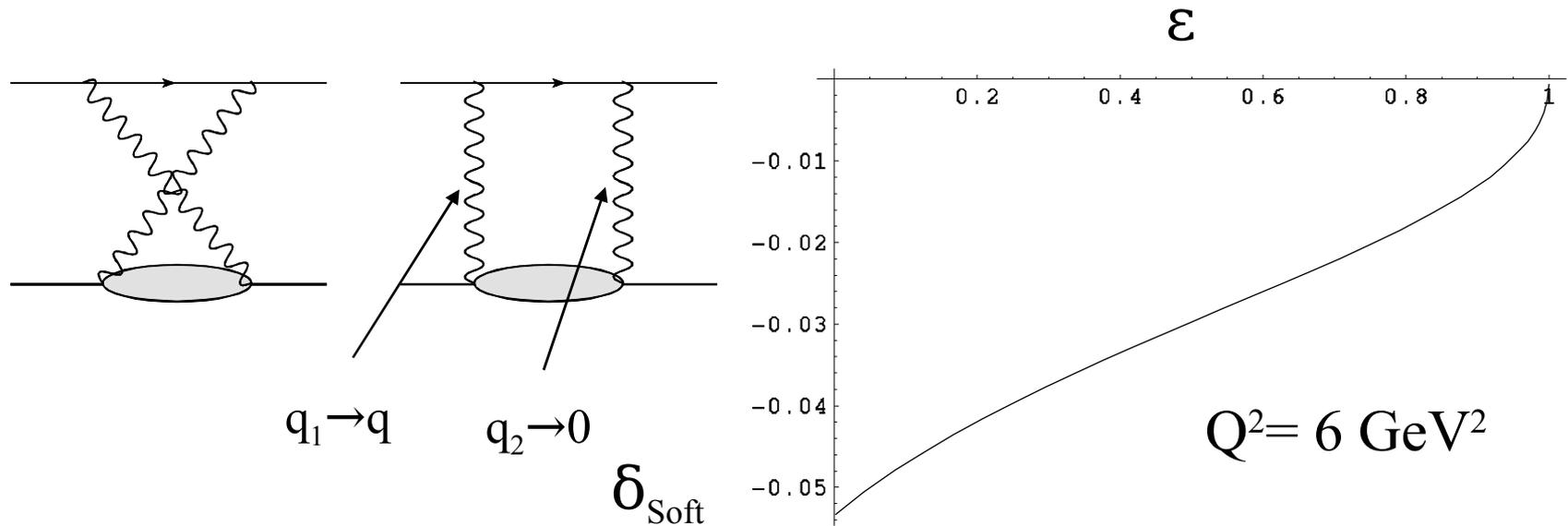
- Resummation technique for collinear photons (=peaking approx.)
- Difference <0.5% from previous calculation including hard brems

- Bystritskiy, Kuraev, Tomasi-Gustafson (2007) claimed this approach resolves Rosenbluth vs polarization discrepancy... but used incorrect energy cutoff $\Delta E/E$ of 3% (instead of e.g. 1.5%) => miscalculated rad.correction by ~5% (absolute)



Separating soft 2-photon exchange

- Tsai; Maximon & Tjon ($k \rightarrow 0$); similar to Coulomb corrections at low Q^2
- Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- Already included in experimental data analysis
- **NB:** Corresponding effect to polarization transfer and/or asymmetry is zero



A similar approach can be applied for any exclusive reaction

What is missing in the calculation?

- 2-photon exchange contributions for non-soft intermediate photons
 - Can estimate based on a text-book example from *Berestetsky, Lifshitz, Pitaevsky: Quantum Electrodynamics*
 - Double-log asymptotics of electron-quark backward scattering

$$\delta = -\frac{e_q e}{8\pi^3} \log^2 \frac{s}{m_q^2}$$

- Negative sign for backward ep-scattering; zero for forward scattering → Can (at least partially) mimic the electric form factor contribution to the Rosenbluth cross section
- Numerically ~3-4% (for GeV electrons and $m_q \sim 300$ MeV, backward angles); zero at forward angles
- Motivates a more detailed calculation of 2-photon exchange at quark level



Lorentz Structure of ep-scattering amplitude

Three generalized form factors ($m_e \rightarrow 0$ case) are functions of two Mandelstam invariants. Specific dependence is determined by nucleon structure

$$M_{fi} = M_{fi}^{1\gamma} + M_{fi}^{2\gamma}$$

$$M_{fi}^{1\gamma} = \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

$$M_{fi}^{2\gamma} = V_e \otimes V_p + A_e \otimes A_p$$

$$V_e = \bar{u}_e \gamma_\mu u_e, V_p = \bar{u}_p (F_1'(s, u) \gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2'(s, u)) u_p$$

$$A_e = \bar{u}_e \gamma_\mu \gamma_5 u_e, A_p = \bar{u}_p G_A(s, u) \gamma_\mu \gamma_5 u_p$$

Observables in terms of generalized form factors

$$\sigma_R = |G_M'|^2 + \frac{\varepsilon}{\tau} |G_E'| + \sqrt{\frac{(1+\tau)(1-\varepsilon^2)}{\tau}} G_M \text{Re}(G_A') + O(\alpha^2)$$

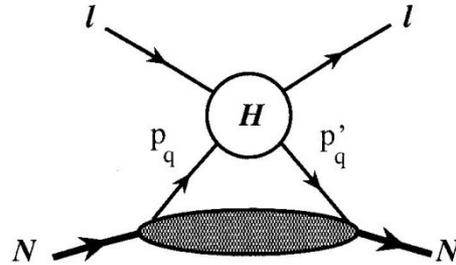
$$P_n \sigma_R = A_n \sigma_R = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \left[\text{Im}(G_E' * G_M') + \sqrt{\frac{(1+\tau)(1-\varepsilon)}{\tau(1+\varepsilon)}} G_E \text{Im}(G_A') + O(\alpha^2) \right]$$

$$P_s \sigma_R = A_s \sigma_R = -P_e \sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}} \left[\text{Re}(G_E' * G_M') + \sqrt{\frac{(1+\tau)(1+\varepsilon)}{\tau(1-\varepsilon)}} G_E \text{Re}(G_A') + O(\alpha^2) \right]$$

$$P_l \sigma_R = A_l \sigma_R = P_e \left[\sqrt{1-\varepsilon^2} |G_M'|^2 + 2 \sqrt{\frac{1+\tau}{\tau}} G_M \text{Re}(G_A') + O(\alpha^2) \right]$$

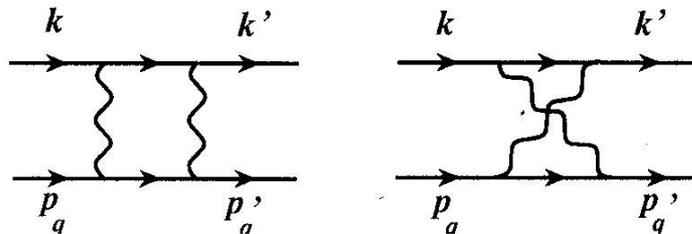


Calculations using Generalized Parton Distributions



Model schematics:

- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
 - Use Grammer-Yennie prescription



Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Phys.Rev.Lett.**93**:122301,2004; Phys.Rev.D**72**:013008,2005

Short-range effects; on-mass-shell quark

Two-photon probe directly interacts with a (massless) quark (cf *Khriplovich*, 1973; *Brown et al*, 1973); Emission/reabsorption of the quark is described by GPDs

$$A_{eq \rightarrow eq}^{2\gamma} = \frac{e_q^2}{t} \frac{\alpha_{em}}{2\pi} (V_\mu^e \otimes V_\mu^q \times f_V + A_\mu^e \otimes A_\mu^q \times f_A),$$

$$V_\mu^{e,q} = \bar{u}_{e,q} \gamma_\mu u_{e,q}, \quad A_\mu^{e,q} = \bar{u}_{e,q} \gamma_\mu \gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi] \log(-\frac{t}{\lambda^2}) - \frac{t}{2} [\frac{1}{s} (\log(\frac{u}{t}) + i\pi) - \frac{1}{u} \log(-\frac{s}{t})] +$$

$$+ \frac{(u^2 - s^2)}{4} [\frac{1}{s^2} (\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2} \log(-\frac{s}{t}) (\log(-\frac{s}{t}) + i2\pi)] + i\pi \frac{u^2 - s^2}{2su}$$

$$f_A = -\frac{t}{2} [\frac{1}{s} (\log(\frac{u}{t}) + i\pi) + \frac{1}{u} \log(-\frac{s}{t})] +$$

$$+ \frac{(u^2 - s^2)}{4} [\frac{1}{s^2} (\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2} \log(-\frac{s}{t}) (\log(-\frac{s}{t}) + i2\pi)] + i\pi \frac{t^2}{2su}$$

Note the additional effective (axial-vector)² interaction; absence of mass terms
Dimensional counting: at the quark level 2g amplitude has the same asymptotics as
Born amplitude



'Hard' contributions to generalized form factors

GPD integrals

$$A \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q + E^q),$$

$$B \equiv \int_{-1}^1 \frac{dx}{x} \frac{[(\hat{s} - \hat{u}) \tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3]}{(s-u)} \sum_q e_q^2 (H^q - \tau E^q)$$

$$C \equiv \int_{-1}^1 \frac{dx}{x} \tilde{f}_1^{hard} \text{sgn}(x) \sum_q e_q^2 \tilde{H}^q$$

Two-photon-exchange form factors from GPDs

$$\delta \tilde{G}_M^{hard} = C$$

$$\delta \tilde{G}_E^{hard} = -\left(\frac{1+\varepsilon}{2\varepsilon}\right) (A - C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}} B$$

$$\tilde{F}_3 = \frac{M^2}{\nu} \left(\frac{1+\varepsilon}{2\varepsilon}\right) (A - C)$$



Two-Photon Contributions (cont.)

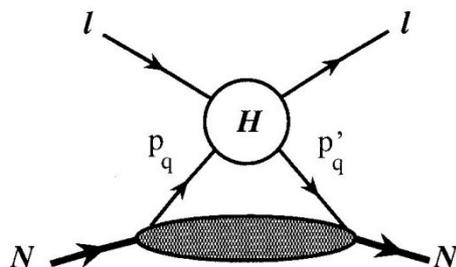
Blunden et al. have calculated elastic contribution of TPE

Afanasev, Brodsky, Carlson et al,
PRD 72:013008 (2005); PRL

93:122301 (2004)

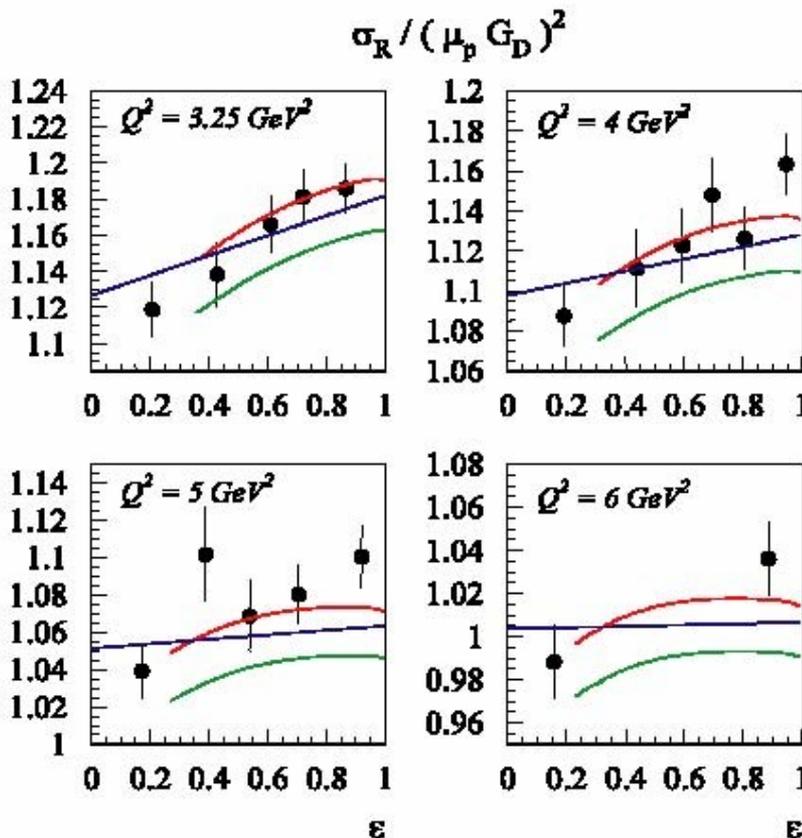
Model schematics:

- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
- Assume factorization



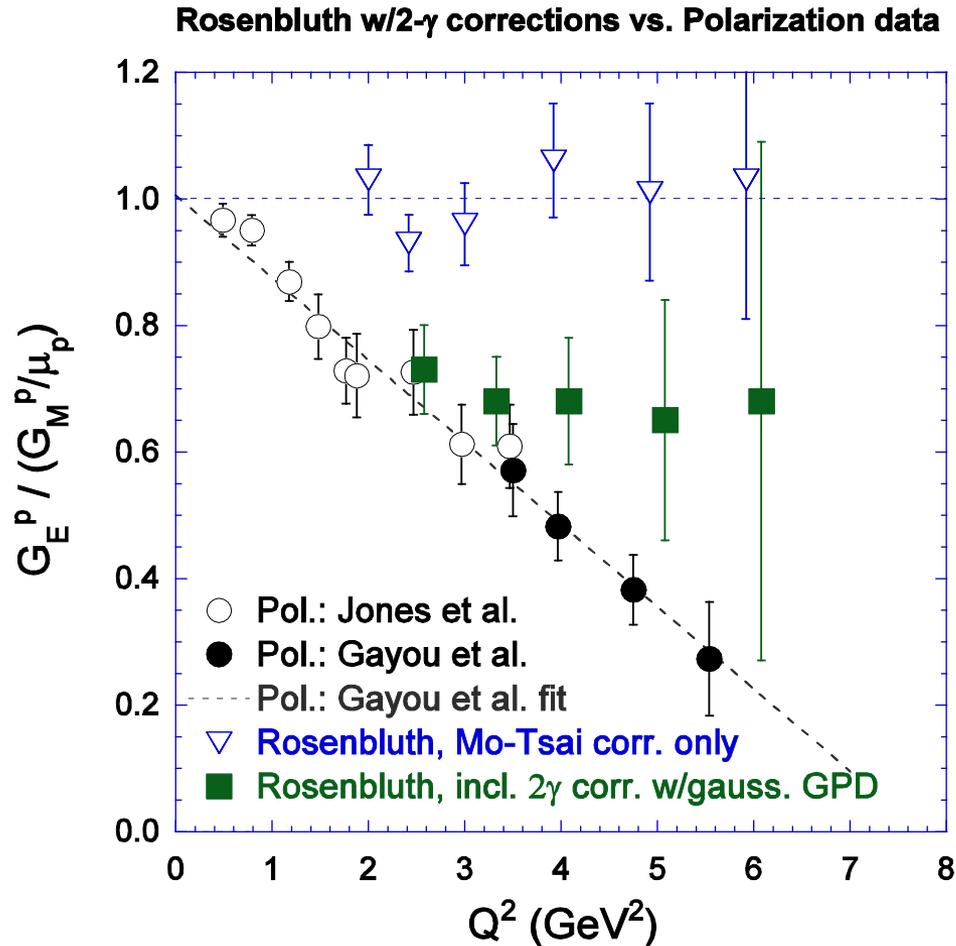
Resolves ~50% of discrepancy

Polarization transfer
 $1\gamma + 2\gamma$ (hard)
 $1\gamma + 2\gamma$ (hard+soft)



Updated Ge/Gm plot

AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Phys.Rev.Lett.93:122301, 2004; Phys.Rev.D72:013008, 2005



Full Calculation of Bethe-Heitler Contribution

Additional work by AA et al., using MASCARAD (*Phys.Rev.D64:113009,2001*)
 Full calculation including soft and hard bremsstrahlung

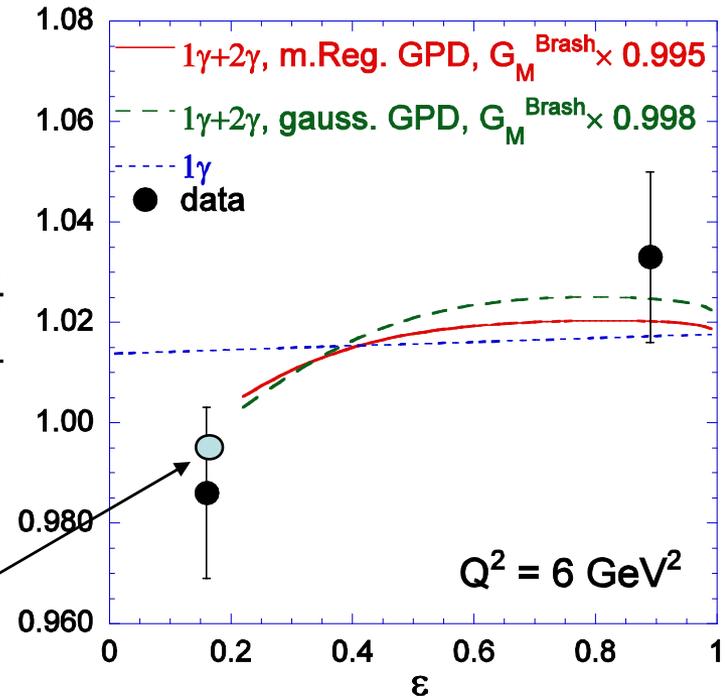
Radiative leptonic tensor in full form
 AA et al, *PLB 514, 269 (2001)*

$$L_{\mu\nu}^r = -\frac{1}{2} \text{Tr}(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1 + \gamma_5 \hat{\xi}_e)(\hat{k}_1 + m)\bar{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\mu - \frac{\gamma_\mu \hat{k} \gamma_\alpha}{2k \cdot k_1} - \frac{\gamma_\alpha \hat{k} \gamma_\mu}{2k \cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k \cdot k_1} - \frac{k_{2\alpha}}{k \cdot k_2} \right) \gamma_\nu - \frac{\gamma_\alpha \hat{k} \gamma_\nu}{2k \cdot k_1} - \frac{\gamma_\nu \hat{k} \gamma_\alpha}{2k \cdot k_2}$$

Cross section for ep elastic scattering

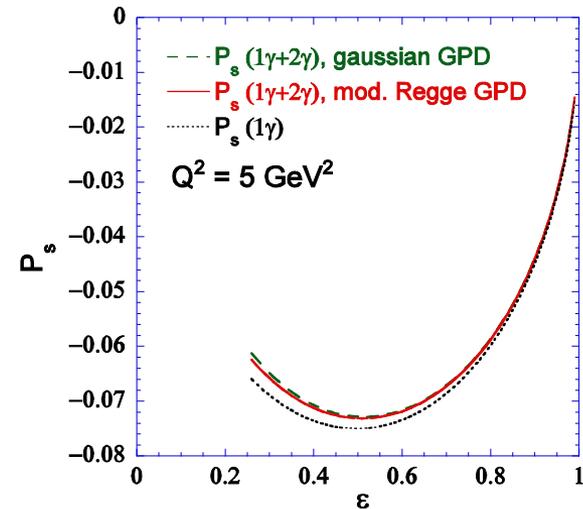
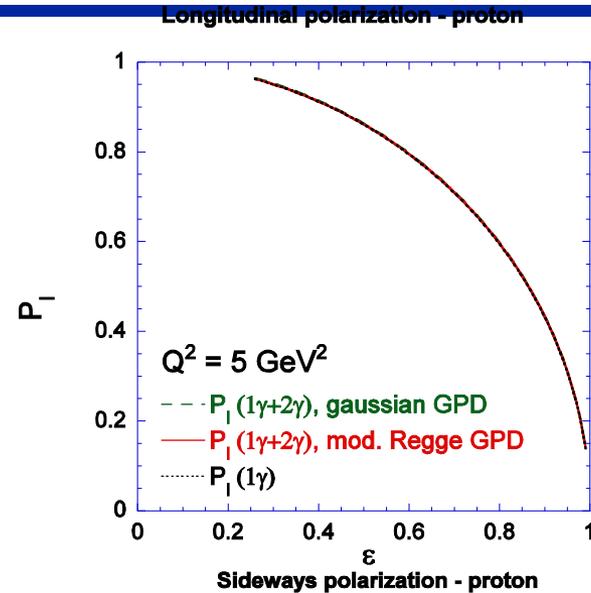
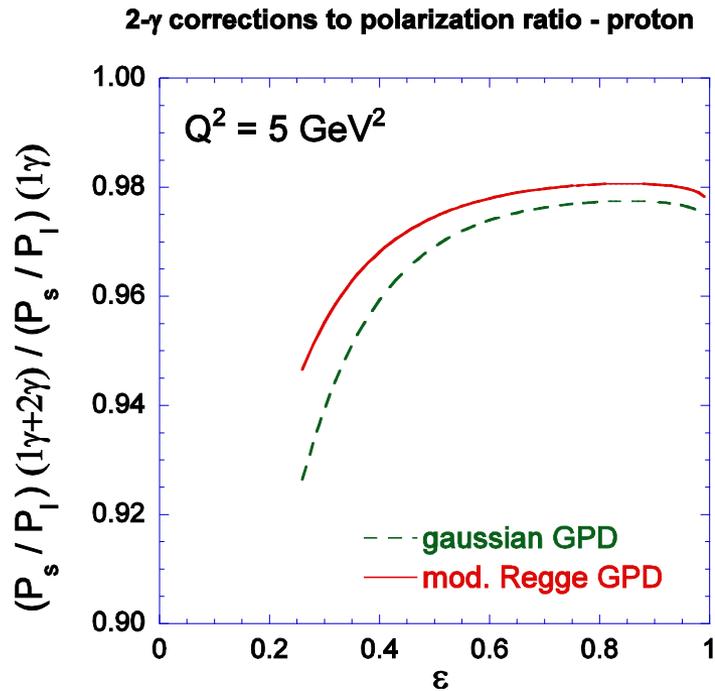


Additional effect of full soft+hard brem \rightarrow +1.2% correction to ϵ -slope
Resolves additional ~25% of Rosenbluth/polarization discrepancy!



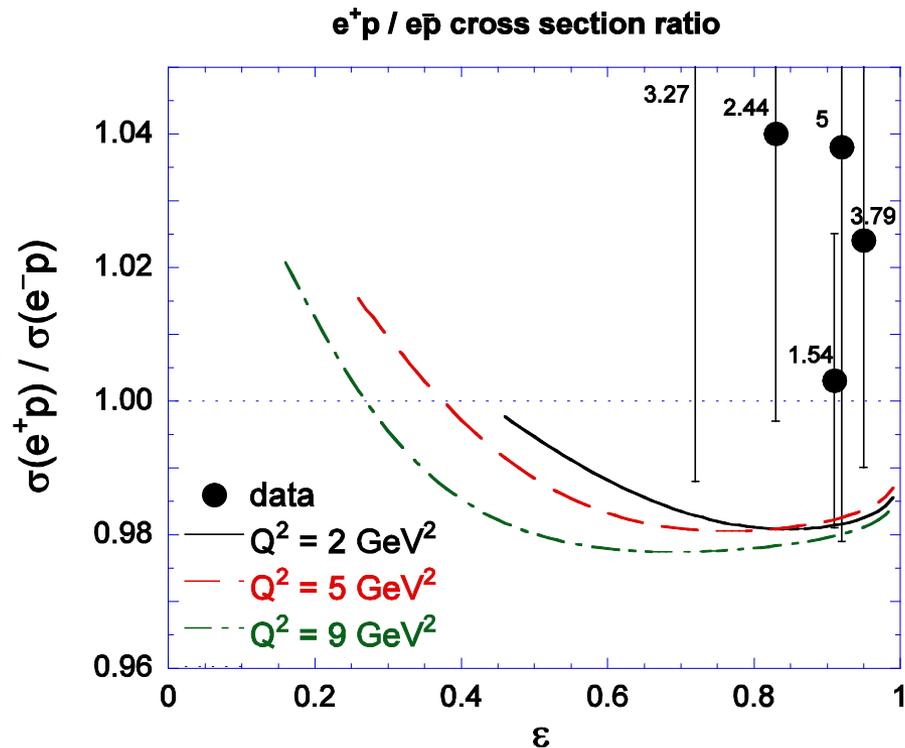
Polarization transfer

- Also corrected by two-photon exchange, but with little impact on G_{ep}/G_{mp} extracted ratio



Charge asymmetry

- **Cross sections of electron-proton scattering and positron-proton scattering are equal in one-photon exchange approximation**
 - **Different for two- or more photon exchange**



To be measured in JLab Experiment 04-116,
Spokepersons AA, W. Brooks, L. Weinstein, et al.
Also at DESY (Olympus) and Novosibirsk



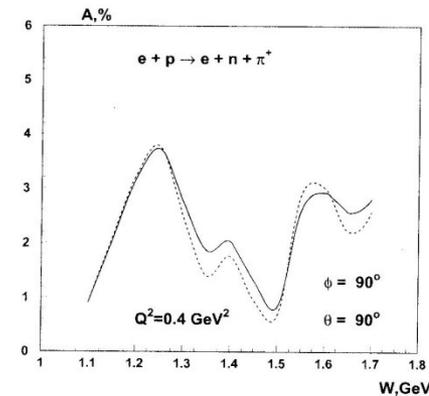
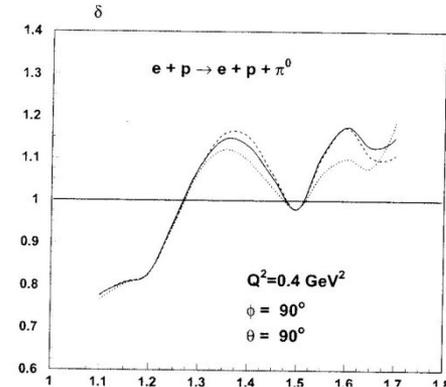
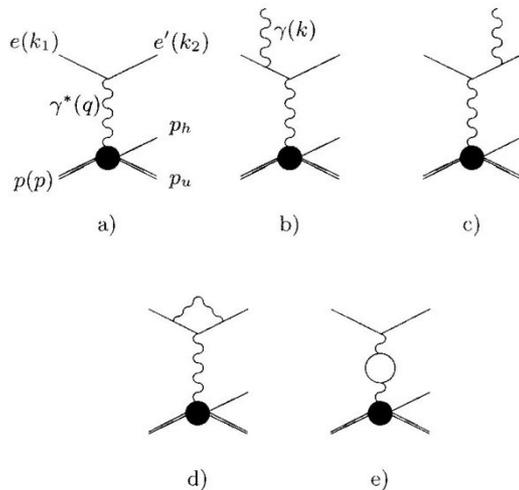
Radiative Corrections for Exclusive Processes

- Photon emission is a part of any electron scattering process: accelerated charges radiate
- Exclusive electron scattering processes such as $p(e, e' h_1) h_2$ are in fact inclusive $p(e, e' h_1) h_2 n\gamma$,
where we can produce an infinite number of low-energy photons
- But low-energy photons do not affect polarization observables, thanks to Low theorem



RC for Electroproduction of Pions

- AA, Akushevich, Burkert, Joo, Phys.Rev.D66, 074004 (2002)
 - Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation; extension to DVMP is straightforward

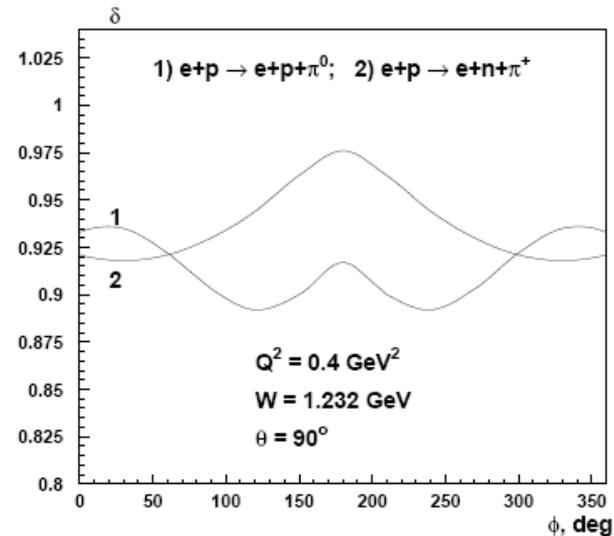
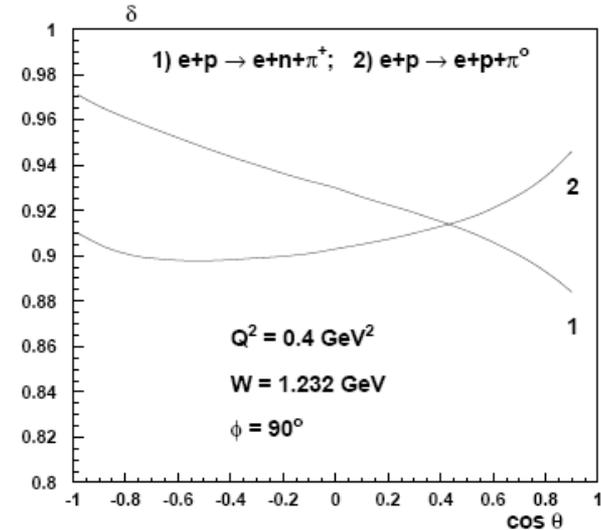


See <http://www.jlab.org/RC> for other codes
 Used in data analysis at JLab
 (and MIT, HERMES, MAMI,...)



Angular Dependence of Rad. Corrections

- **Rad. Corrections** introduce additional angular dependence on the experimentally observed cross section of electroproduction processes, both exclusive and semi-inclusive



Rad. Corrections to e^+e^- pair production

- Usual corrections+charge asymmetric corrections

PHYSICAL REVIEW

VOLUME 173, NUMBER 4

20 SEPTEMBER 1968

Second Born Corrections to Wide-Angle High-Energy Electron Pair Production and Bremsstrahlung*

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AND

JOHN R. GILLESPIE

Centre de Physique Théorique, Ecole Polytechnique,† Paris, France and Stanford Linear Accelerator Center,
Stanford University, Stanford, California 94305

(Received 15 April 1968)

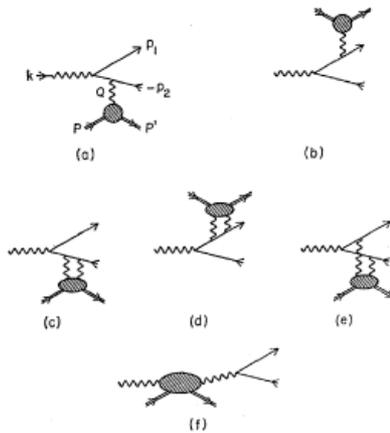


FIG. 1. Feynman diagrams for electron pair production. (a)–(e) give the Bethe-Heitler amplitude through second order in the electromagnetic interaction with the nucleus. Diagram (f) represents the virtual Compton contribution to pair production and includes contributions from the nuclear-pole terms, nucleon and nuclear excitations, and neutral vector-meson production.

*Need to be re-visited in view
of time-like DVCS measurements
at JLAB*

Single-Spin Asymmetries in Elastic Scattering

Parity-conserving

- Observed spin-momentum correlation of the type:

$$s \cdot k_1 \times k_2$$

where $k_{1,2}$ are initial and final electron momenta, s is a polarization vector of a target OR beam

- For elastic scattering asymmetries are due to *absorptive part* of 2-photon exchange amplitude

Parity-Violating

$$s \cdot k_1$$



Normal Beam Asymmetry in Moller Scattering

- Pure QED process, $e^+e^- \rightarrow e^+e^-$
 - Barut, Fronsdal, Phys.Rev.120:1871 (1960): Calculated the asymmetry in first non-vanishing order in QED $O(\alpha)$
 - Dixon, Schreiber, Phys.Rev.D69:113001,2004, Erratum-ibid.D71:059903,2005: Calculated $O(\alpha)$ correction to the asymmetry



$$A_n \propto \frac{2M_\gamma \text{Im}(M_{2\gamma})}{M_\gamma^2} \xrightarrow{\sqrt{s} \gg m_e} \alpha \frac{m_e}{\sqrt{s}} f(\theta)$$

SLAC E158 Results (K. Kumar, private communication):

$A_n(\text{exp}) = 7.04 \pm 0.25(\text{stat})$ ppm

$A_n(\text{theory}) = 6.91 \pm 0.04$ ppm



Single-Spin Target Asymmetry $s_T \cdot k_1 \times k_2$

De Rujula, Kaplan, De Rafael, Nucl.Phys. B53, 545 (1973):

Transverse polarization effect is due to the absorptive part of the non-forward Compton amplitude for off-shell photons scattering from nucleons

See also AA, Akushevich, Merenkov, hep-ph/0208260

$$A_{l,p}^{el,in} = \frac{8\alpha}{\pi^2} \frac{Q^2}{D(Q^2)} \int dW^2 \frac{S + M^2 - W^2}{S + M^2} \frac{dQ_1^2}{Q_1^2} \frac{dQ_2^2}{Q_2^2} \frac{1}{\sqrt{K}} B_{l,p}^{el,in}$$

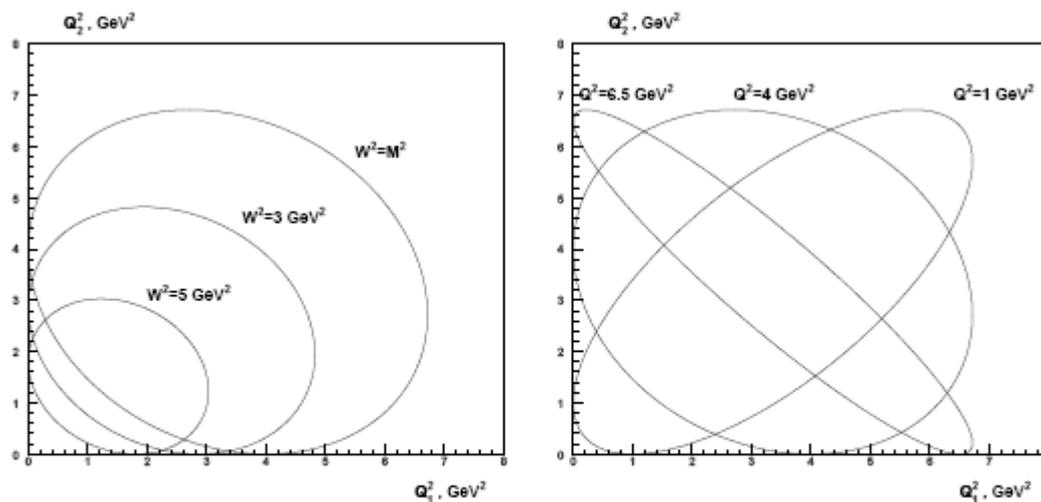


Figure 2. Integration region over Q_1^2 and Q_2^2 in Eq.(2) for elastic ($W^2 = M^2$) and inelastic contributions. The latter (left) is given for $Q^2=4 \text{ GeV}^2$ and two values of W^2 , which is an integration variable in this case. The elastic case is shown on the right as a function of external Q^2 . The electron beam energy is $E_b = 5 \text{ GeV}$.



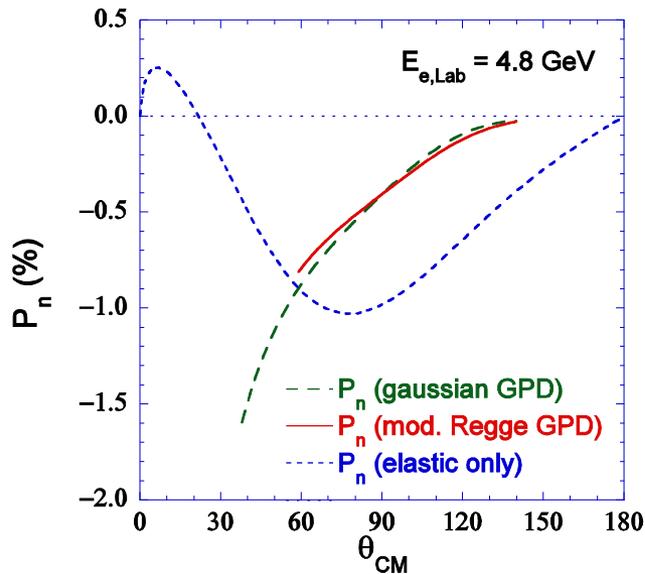
Quark+Nucleon Contributions to Target Asymmetry

- Single-spin asymmetry or polarization normal to the scattering plane
- Handbag mechanism prediction for single-spin asymmetry of elastic eN-scattering on a polarized nucleon target (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

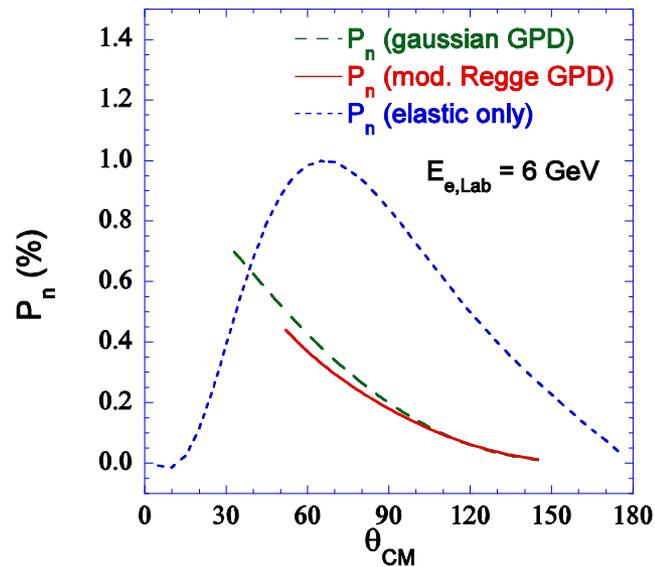
$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left[G_E \operatorname{Im}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}} G_M \operatorname{Im}(B) \right] \quad \textit{Only minor role of quark mass}$$

No dependence on GPD \tilde{H}

Normal Polarization or Analyzing Power - Neutron



Normal Polarization or Analyzing Power - Proton



Data coming from JLAB E05-015

(Inclusive scattering on normally polarized ^3He in Hall A)



Beam Single-Spin Asymmetry: Early Calculations

- ***Spin-orbit interaction of electron moving in a Coulomb field***
N.F. Mott, Proc. Roy. Soc. London, Set. A 135, 429 (1932);
- ***Interference of one-photon and two-photon exchange Feynman diagrams in electron-muon scattering: Barut, Fronsdal, Phys.Rev.120, 1871 (1960)***
- ***Extended to quark-quark scattering SSA in pQCD: Kane, Pumplin, Repko, Phys.Rev.Lett. 41, 1689 (1978)***

$$\Delta(\vartheta) = \frac{v}{1-v^2} 2Z\alpha \frac{v\sqrt{1-v^2}}{\sin^2(\vartheta/2)} \frac{\sin^3(\vartheta/2)}{\cos(\vartheta/2)} \ln \frac{1}{\sin(\vartheta/2)}.$$

$$A_n \propto \frac{\alpha \cdot m_e \cdot \theta^3}{E}, \text{ for } \theta \ll 1$$

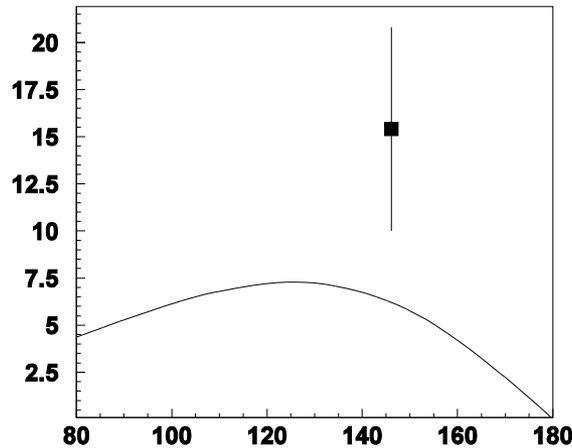
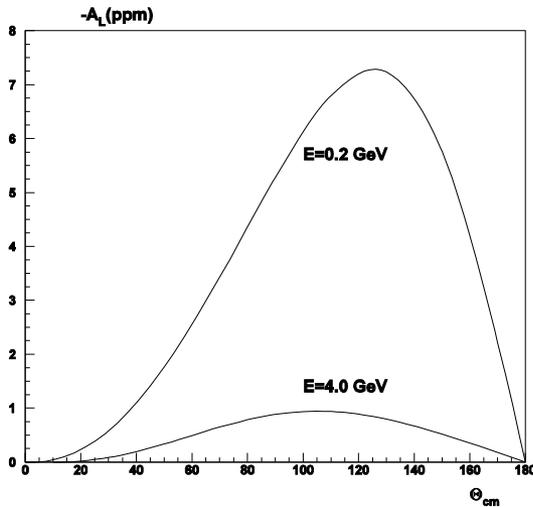
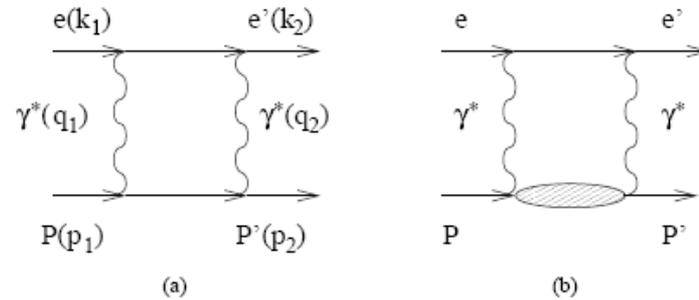
(small – angle scattering)



Proton Mott Asymmetry at Higher Energies

AA, Akushevich, Merenkov,
 hep-ph/0208260

Transverse beam SSA,
 units are parts per million



- Asymmetry due to absorptive part of two-photon exchange amplitude; shown is elastic intermediate state contribution
- Nonzero effect first observed by SAMPLE Collaboration (S.Wells et al., PRC63:064001,2001) for 200 MeV electrons

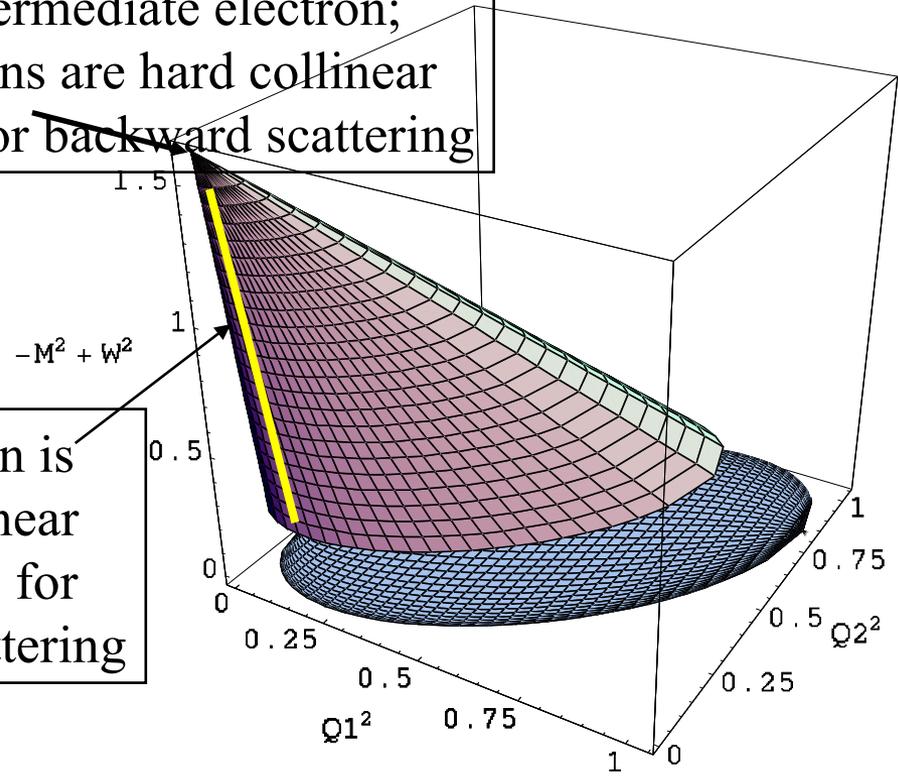


Phase Space Contributing to the absorptive part of 2γ -exchange amplitude

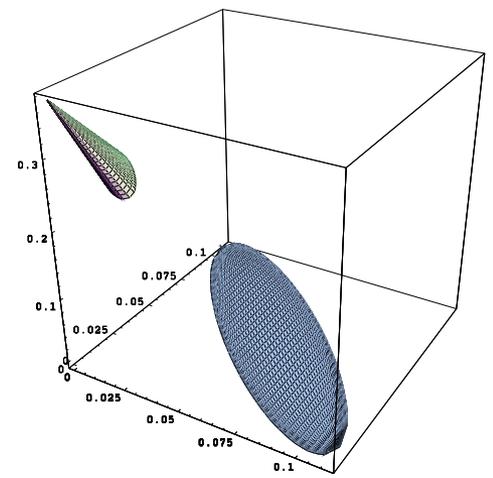
- 2-dimensional integration (Q_1^2, Q_2^2) for the elastic intermediate state
- 3-dimensional integration (Q_1^2, Q_2^2, W^2) for inelastic excitations

'Soft' intermediate electron;
Both photons are hard collinear
Dominates for backward scattering

One photon is hard collinear
Dominates for forward scattering



Examples: MAMI A4
E= 855 MeV
 $\Theta_{cm}= 57$ deg;
SAMPLE, E=200 MeV



Special property of Mott asymmetry

- Mott asymmetry above the nucleon resonance region
 - (a) does not decrease with beam energy
 - (b) is enhanced by large logs
- (AA, Merenkov, PL B599 (2004)48; hep-ph/0407167v2 (erratum))
- Reason for the unexpected behavior: exchange of hard collinear quasi-real photons and diffractive mechanism of nucleon Compton scattering
 - For $s \gg -t$ and above the resonance region, the asymmetry is given by:

$$A_n^e(\text{diffractive}) = \sigma_p \frac{(-m_e)\sqrt{Q^2}}{8\pi^2} \cdot \frac{F_1 - \tau F_2}{F_1^2 + \tau F_2^2} (\log(\frac{Q^2}{m_e^2}) - 2) \cdot \text{Exp}(-bQ^2)$$

Compare with asymmetry caused by Coulomb distortion at small $\theta \Rightarrow$
may differ by orders of magnitude depending on scattering kinematics

$$A_n^e(\text{Coulomb}) \propto \alpha \frac{m_e}{\sqrt{s}} \theta^3 \rightarrow A_n^e(\text{Diffractive}) \propto \alpha m_e (\sqrt{s}) \theta \cdot R_{\text{int}}^2$$

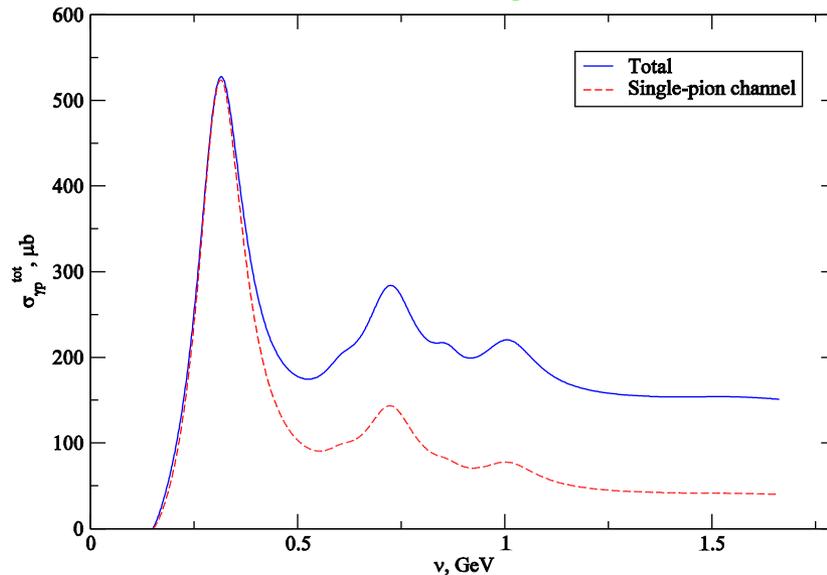


Input parameters

For small-angle ($-t/s \ll 1$) scattering of electrons with energies E_e , normal beam asymmetry is given by the energy-weighted integral

$$A_n \propto \frac{1}{E_e^2} \int_{v_{th}}^{E_e} dv \cdot v \sigma_{\gamma p}^{tot}(v; q_{1,2}^2 \approx 0)$$

Total photoabsorption cross section
Proton target



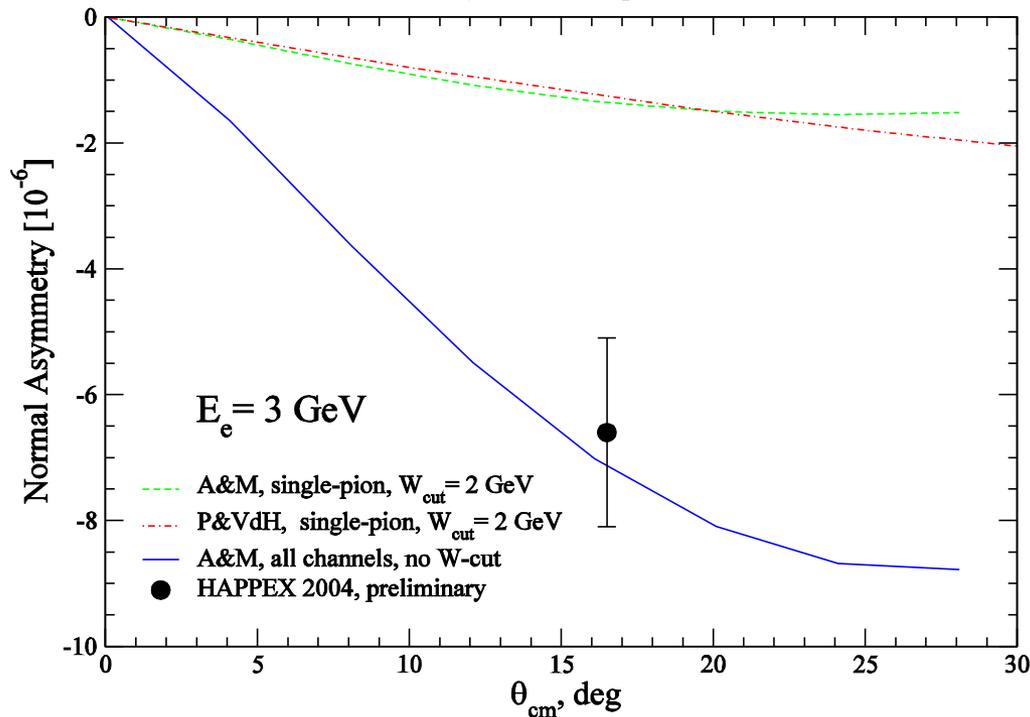
$\tau_{\gamma p}$ from N. Bianchi et al.,
Phys.Rev.C54 (1996)1688
(resonance region) and
Rock&Halzen,
Phys.Rev. D70 (2004) 091901



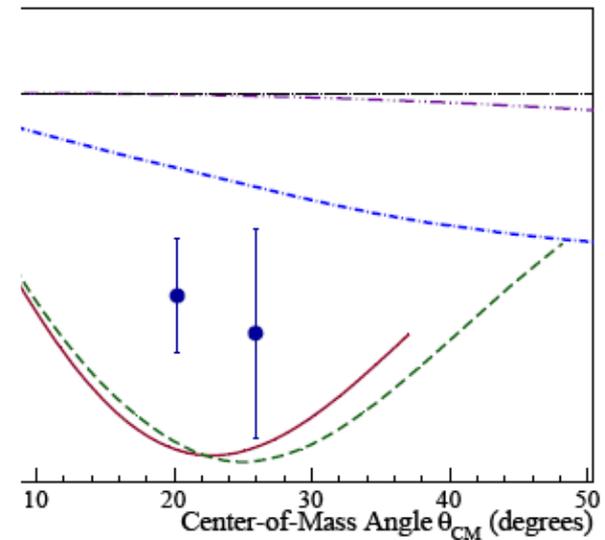
Predictions vs experiment for Mott asymmetry

Use fit to experimental data on $\sigma_{\gamma p}$ (dotted lines include only one-pion+nucleon intermediate states)

Normal beam asymmetry for elastic ep-scattering
Unitarity-based model predictions



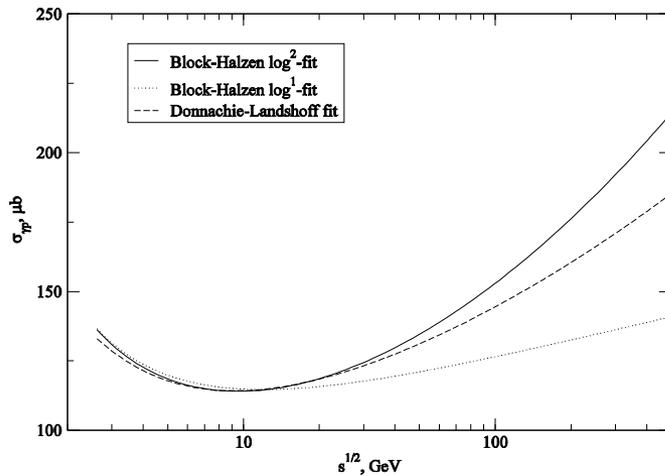
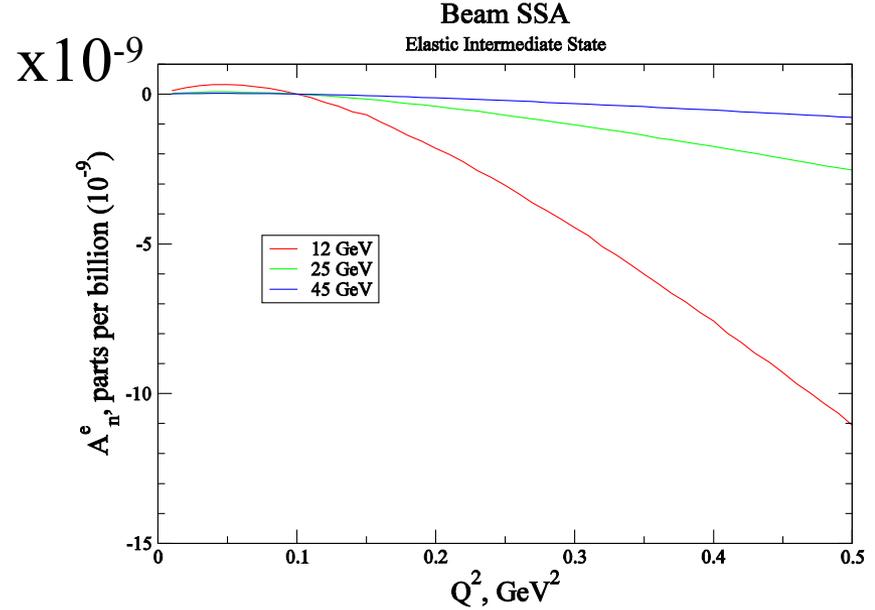
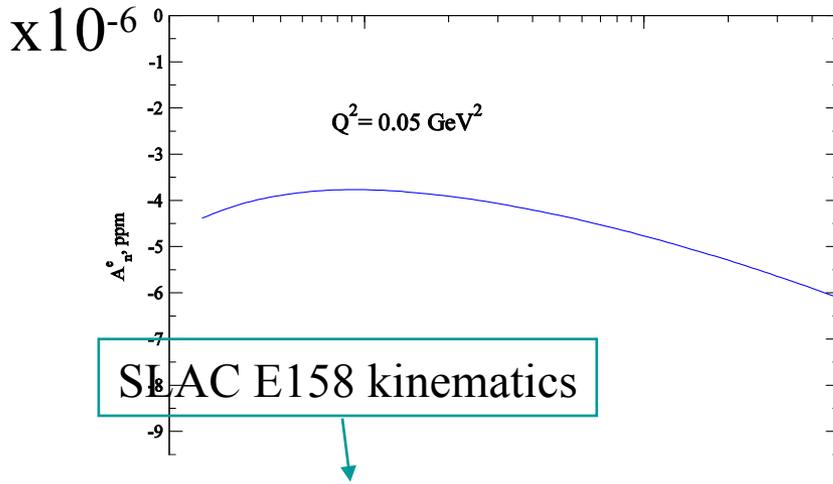
0 arXiv 0705.1525[nucl-ex]



Estimated normal beam asymmetry
for Qweak: **-5ppm**



Predict no suppression for Mott asymmetry with energy at fixed Q^2

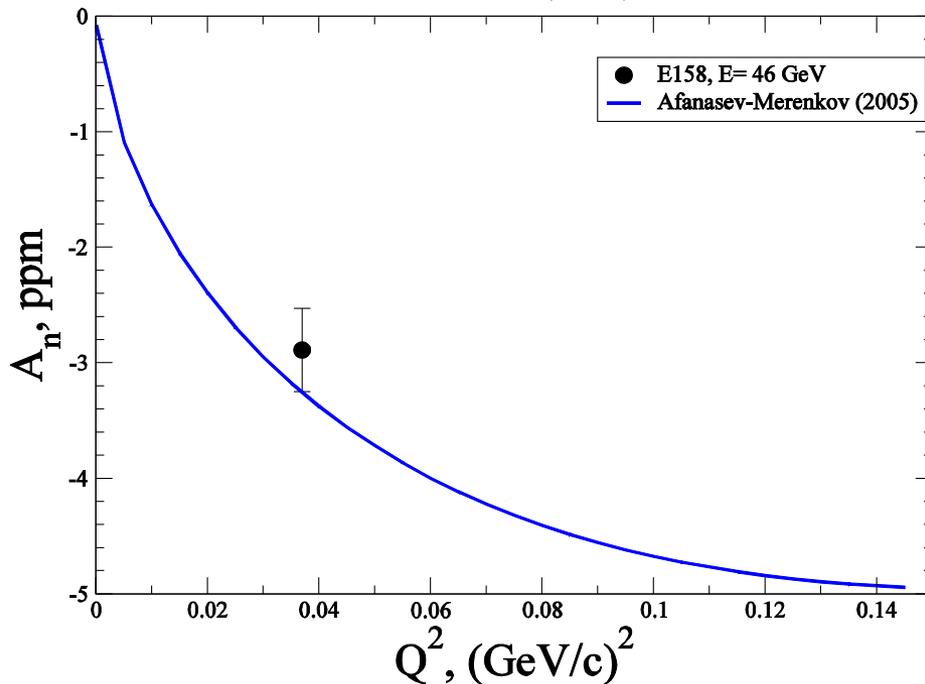


- At 45 GeV predict beam asymmetry parts-per-million (diffraction) vs. parts-per billion (Coulomb distortion)



Comparison with E158 data

Elastic e+p scattering
Normal beam asymmetry

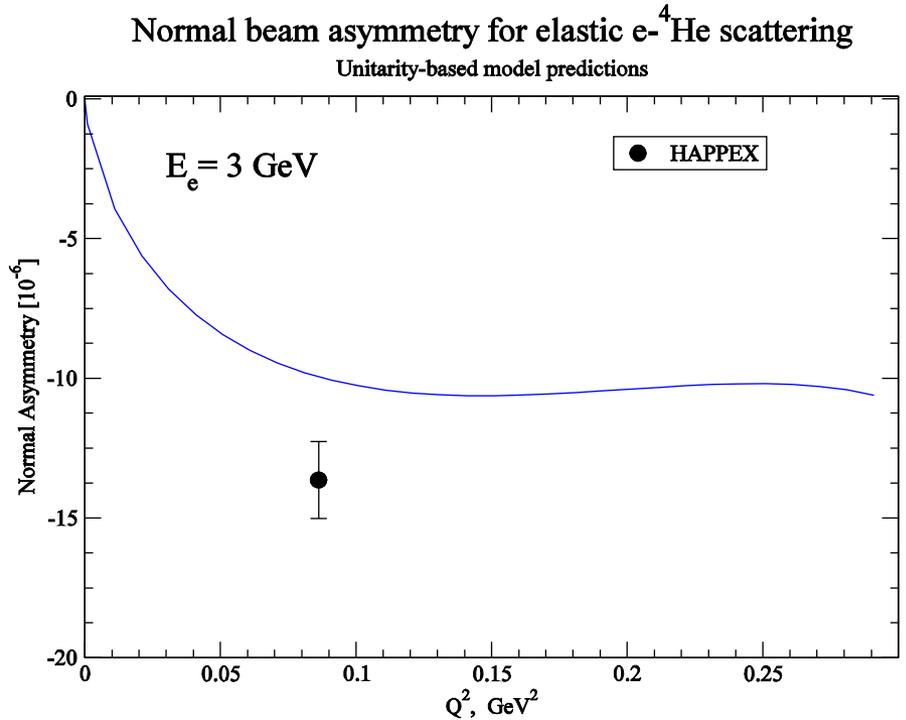
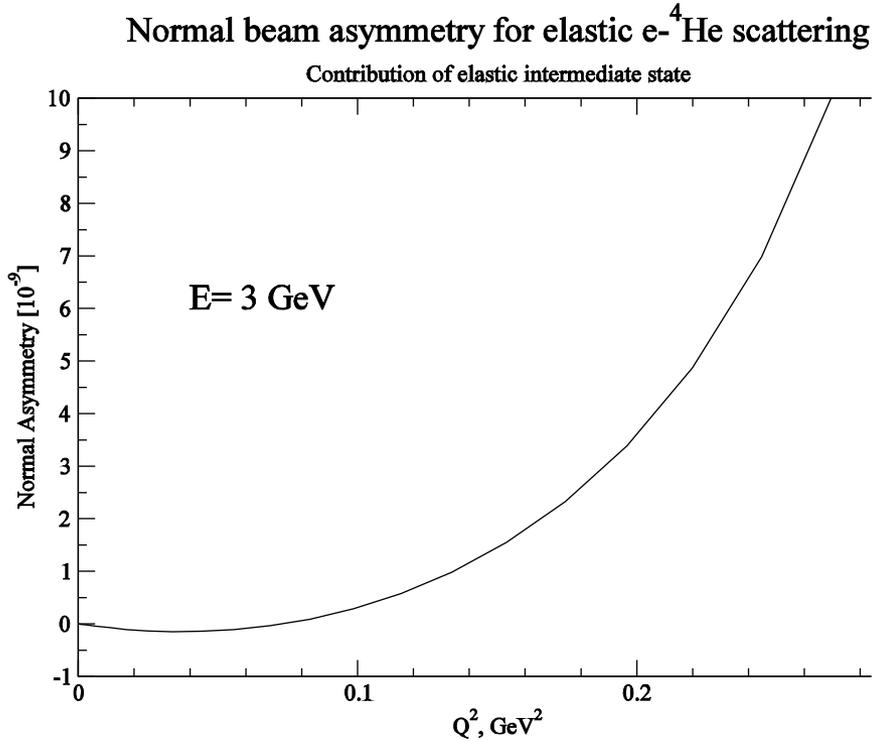


- SLAC E158: 46 GeV beam on fixed-target protons
 $A_n = -2.89 \pm 0.36(\text{stat}) \pm 0.17(\text{syst})$ ppm
(K. Kumar, private communication)
 - Theory (AA, Merenkov):
 $A_n = -3.2$ ppm
- Need to include QED radiative correction**
- Good agreement justifies application of this approach to the real part of two-boson exchange (Gorchtein's talk on γZ box)



Mott Asymmetry on Nuclei

- Important systematic correction for parity-violation experiments (~ -10 ppm for HAPPEX on ^4He , ~ -5 ppm for PREX on Pb), see AA *arXiv:0711.3065 [hep-ph]*; also Gorchtein, Horowitz, Phys.Rev.C77:044606,2008
- Coulomb distortion: only 10^{-10} effect (Conner&Horowitz, Phys.Rev.C72:034602,2005)



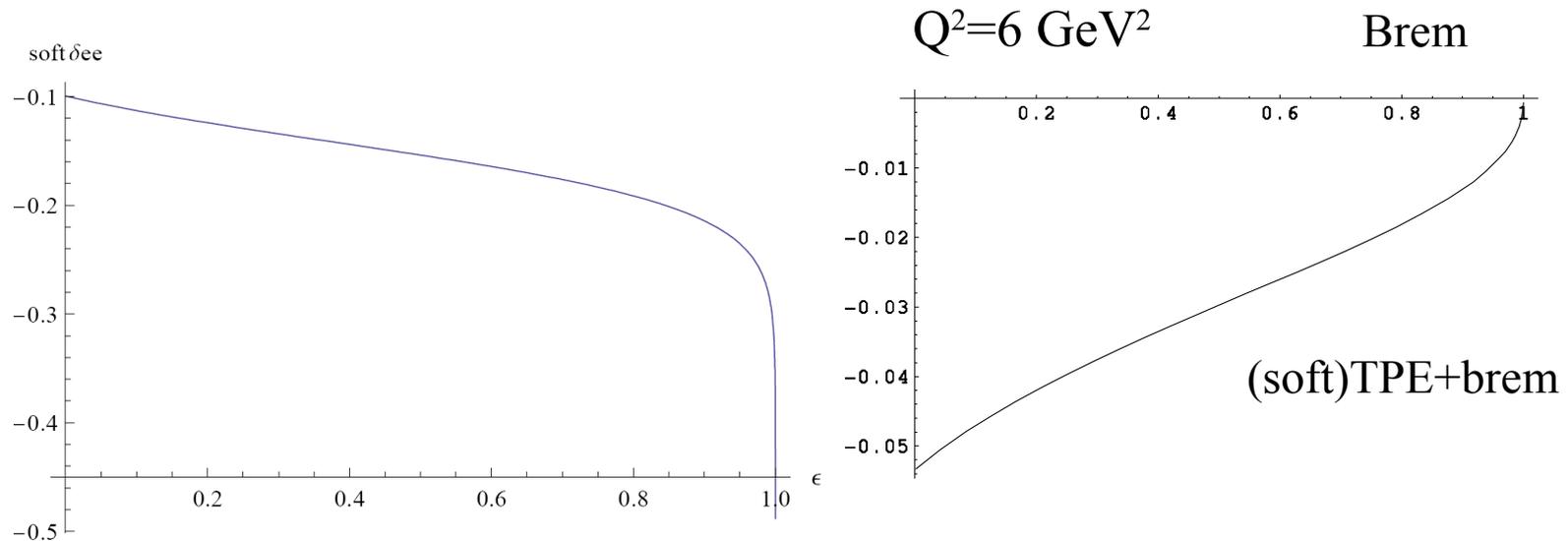
Five orders of magnitude enhancement in HAPPEX kinematics due to excitation of inelastic intermediate states in 2γ -exchange (AA, Merenkov; use Compton data from Erevan)



RC for Elastic ep-scattering at EIC

- Large beam energy, fixed $Q^2 \Rightarrow$ need high epsilon ~ 1 to maintain reasonable count rates (keep in mind luminosity 10^{34} e-N/cm²/s)
 - E.g., for $Q^2=6\text{GeV}^2$, value of $1-\epsilon$ is in the 10^{-4} - 10^{-5} range
 - Consider behavior of rad.correction at large epsilon

$$\sigma_{\text{exp}} = \sigma_{\text{Born}} (1 + \delta)$$



Rad. Corrections Changes very rapidly at high epsilon
 For $1-\epsilon \sim 10^{-4}$ - 10^{-5} , RC(brem)=-39% to -44%; RC(TPE) $\sim 10^{-4}$



Observable in large-epsilon limit

- Observables in terms of 2g-exchange form factors

$$\sigma_R = |G'_M|^2 + \frac{\epsilon}{\tau} |G'_E|^2 + \sqrt{\frac{(1+\tau)(1-\epsilon^2)}{\tau}} G_M \operatorname{Re}(G'_A) + O(\alpha^2)$$

$$P_n \sigma_R = A_n \sigma_R = \sqrt{\frac{2\epsilon(1+\epsilon)}{\tau}} \left[\operatorname{Im}(G'_E {}^* G'_M) + \sqrt{\frac{(1+\tau)(1-\epsilon)}{\tau(1+\epsilon)}} G_E \operatorname{Im}(G'_A) + O(\alpha^2) \right]$$

$$P_s \sigma_R = A_s \sigma_R = -P_e \sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}} \left[\operatorname{Re}(G'_E {}^* G'_M) + \sqrt{\frac{(1+\tau)(1+\epsilon)}{\tau(1-\epsilon)}} G_E \operatorname{Re}(G'_A) + O(\alpha^2) \right]$$

$$P_l \sigma_R = A_l \sigma_R = P_e \left[\sqrt{1-\epsilon^2} |G'_M|^2 + 2 \sqrt{\frac{1+\tau}{\tau}} G_M \operatorname{Re}(G'_A) + O(\alpha^2) \right]$$

- Factors of $\frac{1}{\sqrt{1-\epsilon}}$ are common; they have an infinite slope when $\epsilon \rightarrow 1$

- Studies of elastic ep-scattering at EIC may be possible at forward scattering angles

- 2gamma correction is suppressed, but may be rapidly variable with epsilon
- Standard RC is enhanced; also varies rapidly with epsilon



Summary: SSA in Elastic ep-Scattering

- Collinear photon exchange present in (light particle) beam SSA
- Models violating EM gauge invariance **encounter collinear divergence** for target SSA
- VCS amplitude in *beam asymmetry* is enhanced in different kinematic regions compared to *target asymmetry*
- *Beam asymmetry unsuppressed in forward angles, important systematic effect for PREX, Q_{weak}*
- Strong-interaction dynamics for Mott asymmetry in small-angle ep-scattering above the resonance region is *soft diffraction*
 - *For the diffractive mechanism A_n is a) not suppressed with beam energy and b) does not grow with Z ($\sim A/Z$)*
 - *Confirmed experimentally (SLAC E158) → first observation of diffractive component in elastic electron-hadron scattering*



Two-photon exchange for electron-nucleon scattering

- Model calculations of 2γ -exchange radiative corrections bring into agreement the results of polarization transfer and Rosenbluth techniques for Gep measurements
- Full treatment of brem corrections removes $\sim 25\%$ of R/P discrepancy in addition to $2\gamma \rightarrow$ Important to compute conventional corrections accurately
- Experimental tests of two-photon exchange
 - C-violation in electron vs positron elastic scattering (JLab E04-116, E-07-005)
 - Measurement of nonlinearity of Rosenbluth plot (JLab E05-017)
 - Search for deviation of angular dependence of polarization and/or asymmetries from Born behavior at fixed Q^2 (JLab E04-019)
 - Elastic single-spin asymmetry or induced polarization (JLab E05-015)
 - Extended to inelastic (e,e') in E-07-013
 - 2γ normal beam asymmetry measurements parallel to parity-violating experiments (HAPPEX, G0, PREX)

*Objectives: a) Testing precision of the electromagnetic probe
b) Double-virtual VCS studies*



Implications for EIC

- *Elastic ep-scattering studies may be possible with EIC*
- *Limited to forward region: covers previously unexplored high-epsilon region*
- *QED corrections essential; detailed feasibility studies needed*

