Perturbing gauge/gravity duals by Romans mass

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Introduction

Our main character: Romans mass F_0

originally introduced as a 'cosmological constant' in IIA supergravity [Romans '86]

Later understood as RR flux; equal dignity as any F_k [Polchinski '95]

[for example, mixed with the others by T-duality]

It also helps produce many more vacua with stabilized moduli

[deWolfe, Giryavets, Kachru, Taylor '05]

internal flux F_k contributes to the 4d potential $\sim \dfrac{e^{4\phi_4}}{r^{6-2k}}$

Nevertheless, still mysterious:

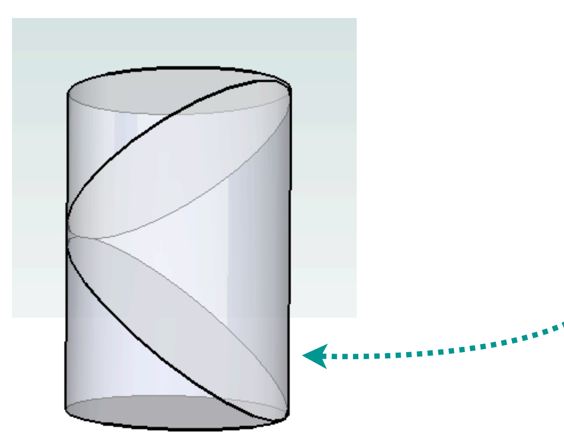
- its M-theory interpretation is not known
- couples to D8-branes [whose gravity grows with distance]
- field strength with no potential…

On some backgrounds, a nonperturbative understanding of string theory:

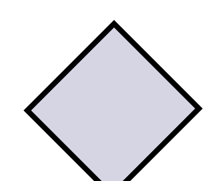
gauge/gravity correspondence

it relates [for example]

a gravity theory in AdS_{d+1}



a conformal theory in $\mathrm{Minkowski}_d$



F_0 has a coupling to D2-branes $\int F_0 CS(\mathcal{A})$

but the near-horizon limit of D2 branes is **not** $\mathrm{AdS}_4\times$ anything

the n.h. limit of **M2** branes is $\mathrm{AdS}_4 \times S^7$ ... but again, we don't know what F_0 is in M-theory.

Fortunately:

a Chern-Simons-matter theory (with explicit Lagrangian) $\overset{\text{is dual to}}{\longleftrightarrow}$ old $\mathrm{AdS}_4 \times \mathbb{CP}^3$ solution (with $F_0 = 0$)

[Aharony, Bergman, Jafferis, Maldacena '08]

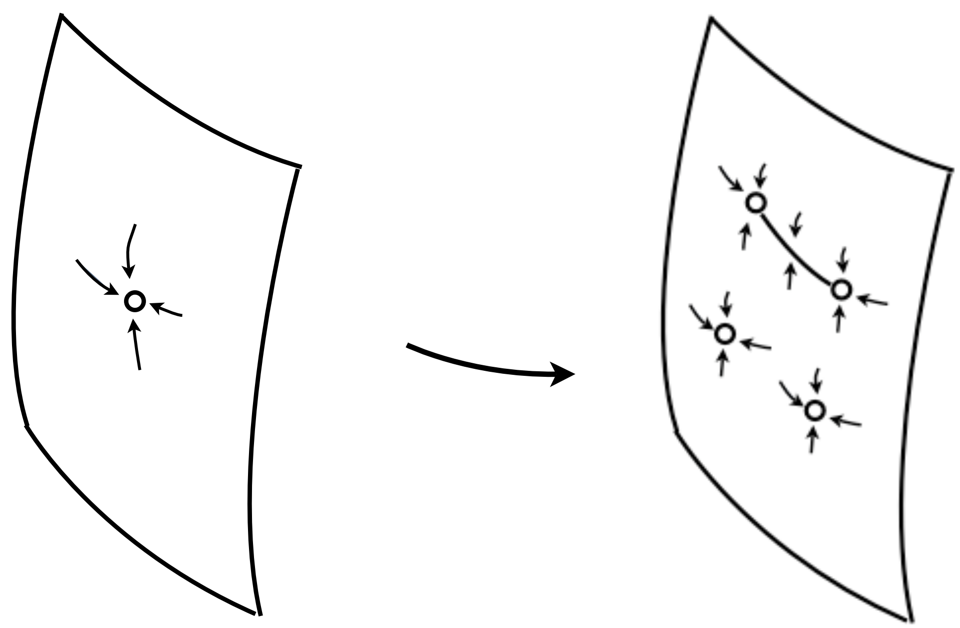
- We will achieve gauge/gravity duals with lower supersymmetry:

one CFT

$\mathcal{N} = 6$

several CFTs

$0 \leq \mathcal{N} \leq 3$



$F_0 = 0$

$F_0 \neq 0$

[Gaiotto's talk here]

- The new theories will help us find new string vacua!

We will find the gravity duals
perturbatively in F_0

At first order:

- superpotential drives the solution
- already some constraints on allowed superpotentials
- automatic procedure; also possible for more general topologies

Plan

- Review: Chern-Simons-matter theories

  ABJM theory, and its lower susy deformations

- Review: some of the geometry behind supersymmetry

- The perturbative procedure

Chern-Simons-matter CFTs

- in 2d the conformal group is ∞-dimensional
- in 4d, gauge couplings are dimensionless
- in 3d?

Chern-Simons action:

another gauge action
with dimensionless coupling:

integer!

$$S = \frac{k}{4\pi} \int \mathrm{Tr}\Big(A\, dA + \frac{2}{3} A^3\Big)$$

unfortunately: this
theory is topological

EoM:

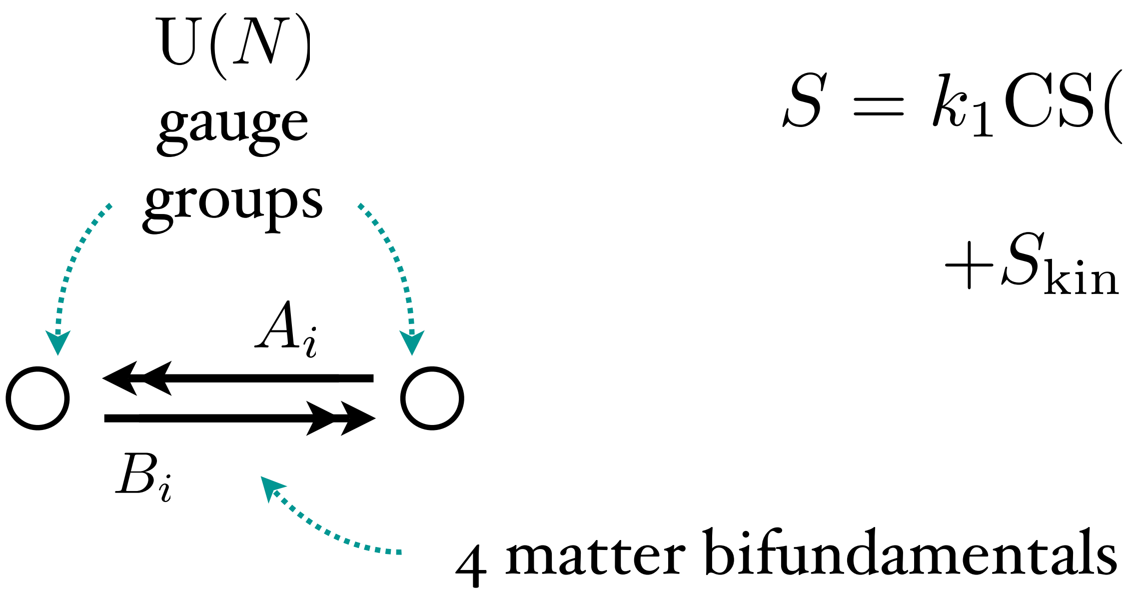
$$F = 0$$

But: susy adds matter $\longrightarrow$ nontrivial CFT3

[Avdeev, Kazakhov,Kondrashuk '93;
Kao, Lee '92; Gaiotto, Yin '07]

The $\mathcal{N}=6$ theory [“preaching to the pope”]

[Aharony, Bergman, Jafferis, Maldacena‘08]



$$S = k_1 \mathrm{CS}(\mathcal{A}_1) + k_2 \mathrm{CS}(\mathcal{A}_2)$$
$$+S_{\text{kin}}(A_i, B_i) \quad + \frac{2\pi}{k} \int d^2\theta \Big[\mathrm{Tr}(A_i B_i)^2 - \mathrm{Tr}(B_i A_i)^2 \Big]$$

If $\boxed{\begin{array}{l} k_1 = k \\ k_2 = -k \end{array}}$

this theory has $\mathcal{N}=6$

- It is also possible to write the theory with $\mathcal{N}=1$ superfields:

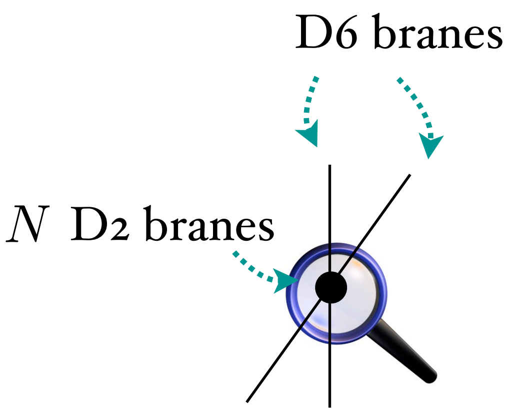
introduce $X^I = \begin{pmatrix} A_1 \\ A_2 \\ B_1^\dagger \\ B_2^\dagger \end{pmatrix}$ then $\mathcal{N}=1$ superpotential is

$$\frac{2\pi}{k}\int d^2\theta (X_I^\dagger X^I X_J^\dagger X_J - X_I^\dagger X^J X_J^\dagger X^I - 2\omega^{IK}\omega_{JL} X_I^\dagger X^J X_K^\dagger X_L)$$

in this form, $\mathrm{Sp}(2) \cong \mathrm{SO}(5)$ is manifest.

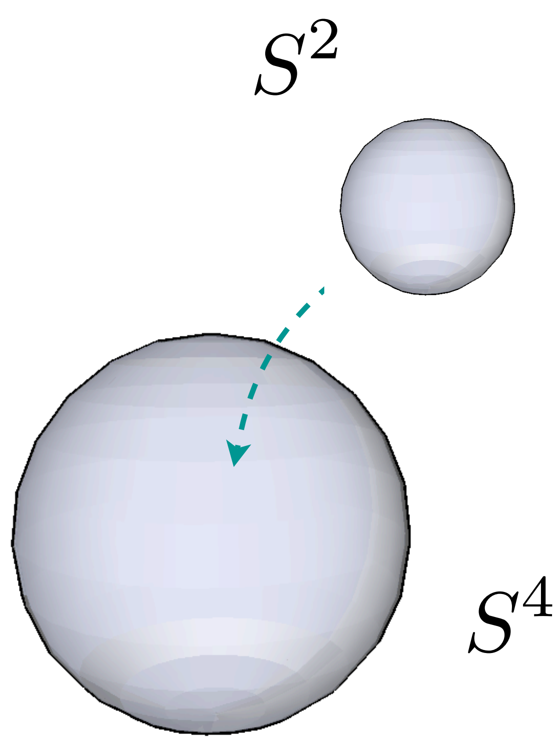
duality with $\mathrm{AdS}_4 \times \mathbb{CP}^3$

- by zooming in on brane systems whose effective action is known.



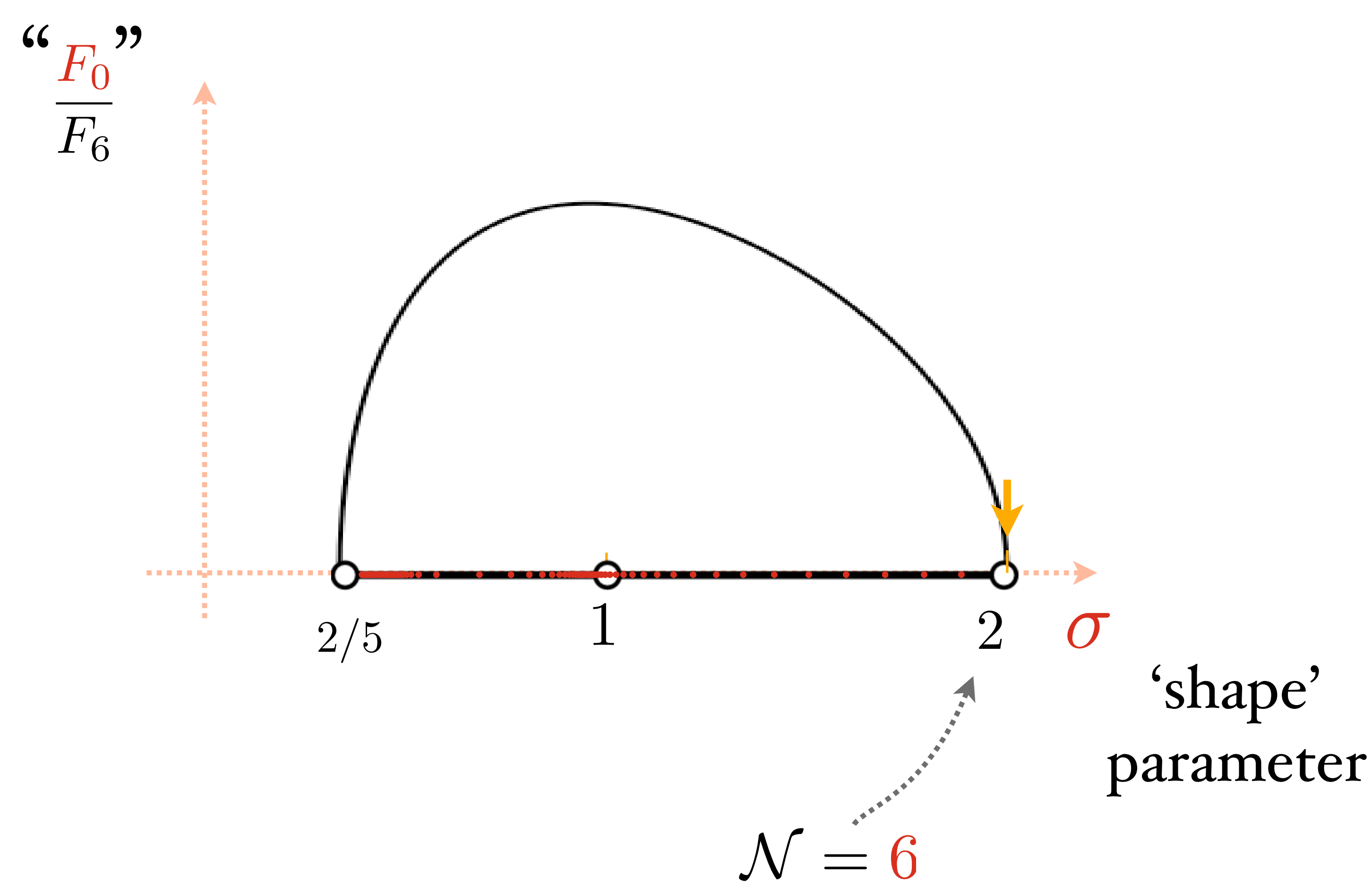
- There are actually more vacua on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ [AT, '07]
  if one allows for non-zero Romans mass

$\mathbb{CP}^3$ is a sphere fibration:



supersymmetry equations
boil down to:

$$\frac{\text{“}F_0\text{”}}{F_6} = \frac{\sqrt{(\sigma - \frac{2}{5})(2-\sigma)}}{\sigma + 2}$$



F_0 has a coupling to D2-branes $\qquad \int F_0 CS(\mathcal{A})$

one can show:

$$\begin{tikzcd} \text{CS level } k \arrow[r, dotted] & \bigcirc \arrow[r, shift right, twoheadrightarrow] & \bigcirc \arrow[l, shift right, twoheadrightarrow] & \text{CS level } -k \boxed{+F_0} \arrow[l, dotted] \end{tikzcd}$$

[Gaiotto, AT '09]

the enhancement to $\mathcal{N} = 6$ doesn't work now;

accidental $\mathrm{SO}(5)$ and R-symmetry $\mathrm{SO}(1)$ now commute.

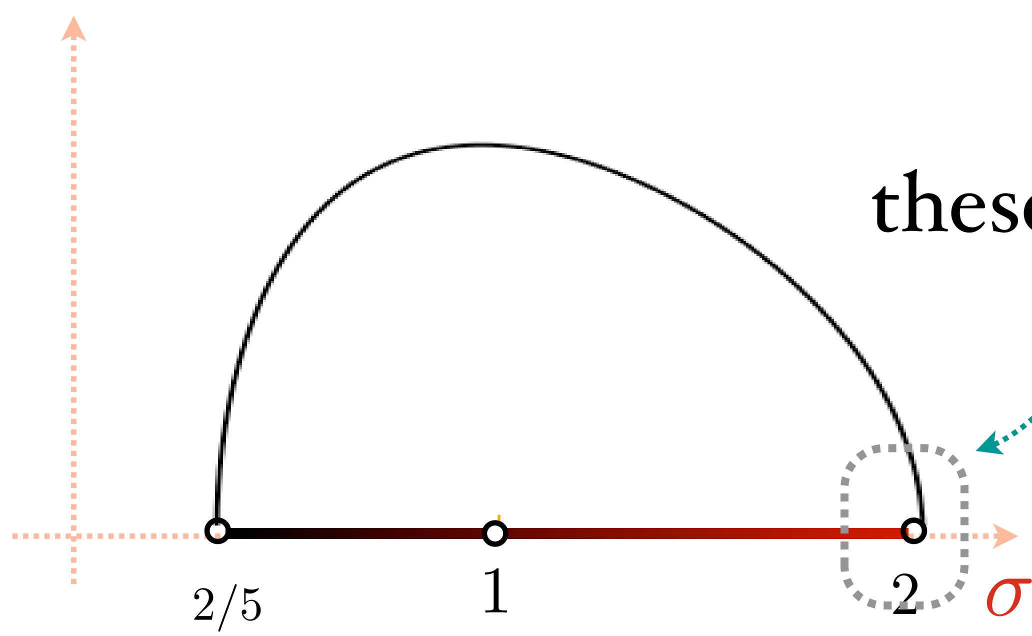
However: If $\boxed{F_0 \ll k}$

one can still argue for a fixed point with these symmetries.

$\mathcal{N}=1$
superpotential:

$$\int d^2\theta (c_1 X_I^\dagger X^I X_J^\dagger X_J + c_2 X_I^\dagger X^J X_J^\dagger X^I + c_3 \omega^{IK}\omega_{JL} X_I^\dagger X^J X_K^\dagger X_L)$$

So the proposal is
that these theories
should be dual to



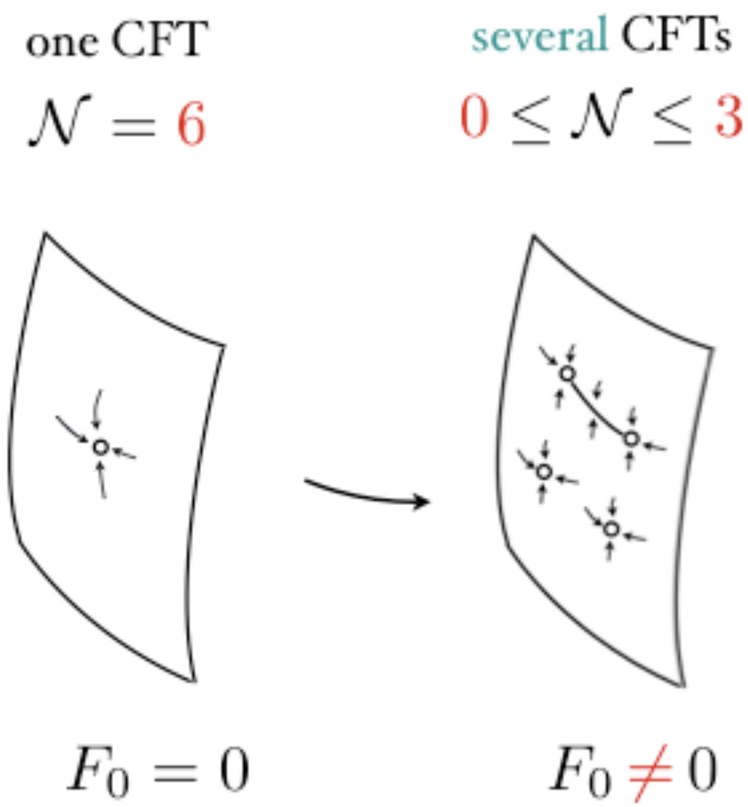
these vacua.

$F_0 \ll$ other fluxes;

small perturbation
around $\mathcal{N}=6$ solution.

This logic also leads to other theories.

supersymmetry	global symmetry
$\mathcal{N}=0$	SO(6)
$\mathcal{N}=1$	SO(5)
$\mathcal{N}=2$	$\mathrm{SO(2)_R}\times$ SO(4)
$\mathcal{N}=3$	$\mathrm{SO(3)_R}\times$ SO(3)



the last two are connected by a line:

$$W_{\mathcal{N}=2}=c_1\mathrm{Tr}(A_iB_i)^2+c_2\mathrm{Tr}(B_iA_i)^2$$

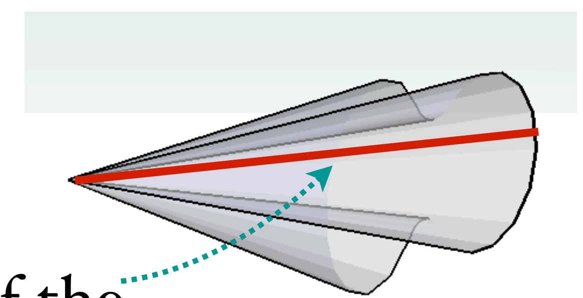
Can we find their gravity duals?

Supersymmetry

The superpotential $$W_{\mathcal{N}=2} = c_1 \mathrm{Tr}(A_i B_i)^2 + c_2 \mathrm{Tr}(B_i A_i)^2$$

doesn't vanish even in the abelian case.

Moduli space has dimension 4!



some subspace of the original 8d cone.

This is the hallmark of
generalized complex geometry (GCG)

[Hitchin '02; Gualtieri '04; Graña, Minasian, Petrini, AT '05]

It is a way of writing the supersymmetry conditions

in terms of Φ_+ and Φ_-

differential forms
with some constraints
[“pure spinors”]

- For example: “SU(3) structure” case

two-form

almost like a CY:

$$\Phi_+ = e^{i\theta} e^{-iJ}$$

$$\Phi_- = \Omega$$

“holomorphic”
three-form

- More generally:

“SU(3)×SU(3)
structure”

one-form

$$\Phi_- = v \wedge e^{\omega}$$

“holomorphic”
two-form

This general case is the one of interest to us.

One susy equation is a first-order equation purely on the geometry:

$$d\Phi_+ = -2\sqrt{-\Lambda}\, e^{-A}\, \mathrm{Re}\Phi_-$$

warping

cosmological constant

[There is one more susy equation; and Bianchi]

$$\Phi_+ = \rho\, e^{i\theta} e^{-iJ}$$
$$\Phi_- = v \wedge e^{\omega}$$

In particular:

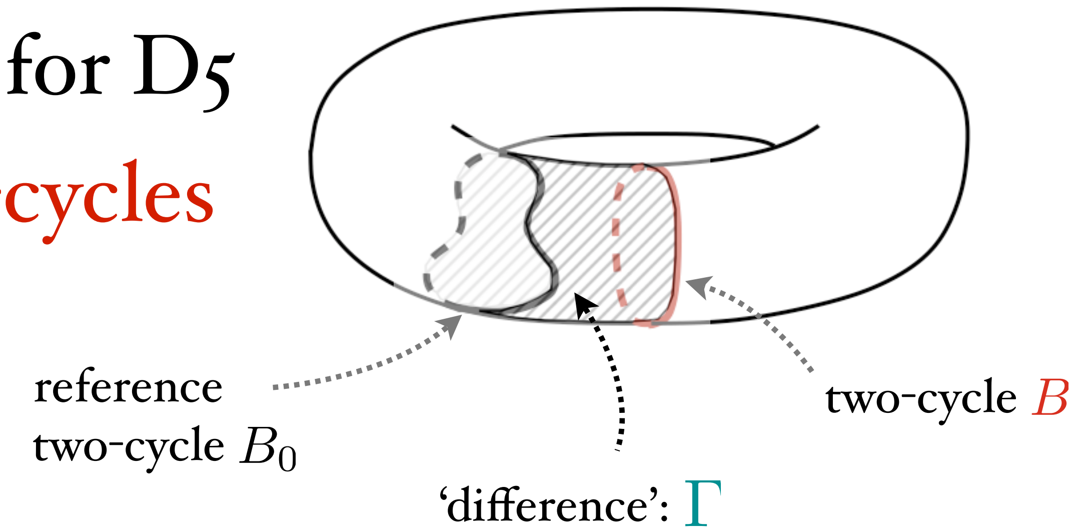
$$\frac{d\theta}{\sin^2(\theta)} = -2\sqrt{-\Lambda}\, e^{-A}\, \mathrm{Re} v$$

These forms are related to the brane superpotentials...

- Step back: superpotential for D5 wrapping two-cycles

$$W = \int_\Gamma \Omega$$

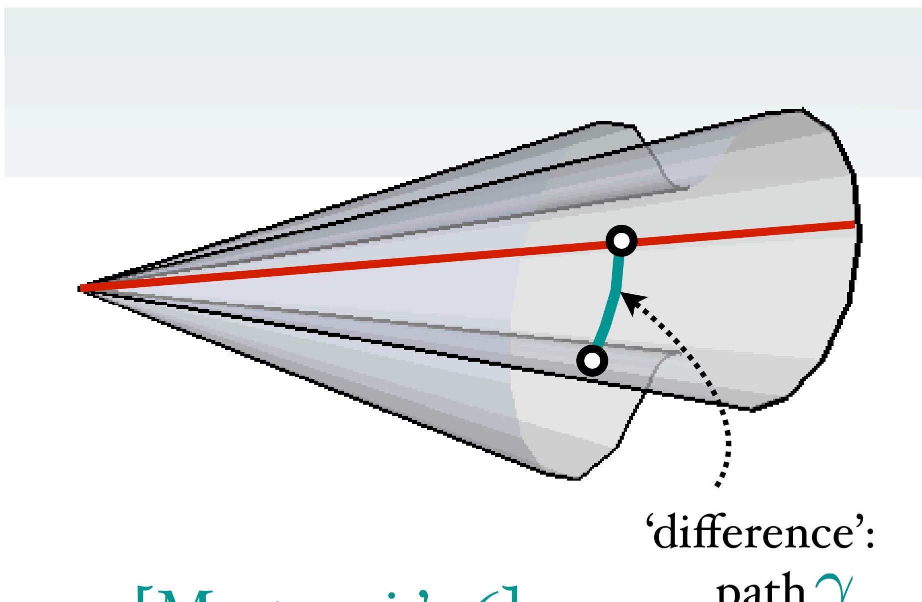
[Witten '97]



$$\partial\Gamma = B - B_0$$

- same for D2 wrapping points

$$\partial\gamma = p - p_0$$



[Martucci '06]

$$W = \int_\gamma \mathrm{Re}\Phi_-$$

$$= \int_\gamma \mathrm{Re}v \quad \propto \int_\gamma d(\cot(\theta))$$

$$= \cot(\theta)$$

- Susy conditions in terms of ‘pure spinors’

- A small piece of these ‘pure spinors’ is the superpotential of the CFT

This is not enough to find explicit solutions. But:

Perturbative solutions

[Gaiotto, AT '09]

perturbative parameter: $F_0 = k_1 + k_2$ [sum of CS levels]

$$\theta = \frac{\pi}{2} + F_0 \delta\theta + O(F_0^2) \quad \Rightarrow \quad W = F_0 \delta\theta + O(F_0^2)$$

We find:

W $\Rightarrow$ GCG machinery $\Rightarrow$ solution (inf. perturbation in F_0)

...if W is correct!

[For AdS_5 , any W works at first order [Graña, Polchinski '00]]

At first order:

- metric doesn't change
- F_2, F_6 don't change
- B-field changes (becomes $\neq 0$)
- F_0, F_4 change (become $\neq 0$)

w.r.t. almost
complex structure

$$W \longrightarrow v = \partial W \longrightarrow \omega = v \llcorner \Omega$$

$$\Phi_- = v \wedge e^{\omega}$$

$$B = W J + \mathrm{Im}\omega$$

$$* F_4 = d\mathrm{Im} v + B$$

...it remains
to check Bianchi.

So, on $\mathbb{CP}^3$, to summarize:

supersymmetry	global symmetry	gravity dual
$\mathcal{N} = 0$	SO(6)	
$\mathcal{N} = 1$	SO(5)	SU(3) structure: $\Phi_- = \Omega$
$\mathcal{N} = 2$	$\mathrm{SO}(2)_\mathrm{R} \times \mathrm{SO}(4)$	SU(3)×SU(3) structure: $\Phi_- = v \wedge e^{\omega}$
$\mathcal{N} = 3$	$\mathrm{SO}(3)_\mathrm{R} \times \mathrm{SO}(3)$	

What about other spaces?

- Most other IIA solutions with $F_0 = 0$ have nonconstant dilaton

$\Downarrow$

no $F_0 \neq 0$ deformation with $\mathrm{SU}(3)$ structure

However, no obstruction to $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure

An example has already appeared [Petrini, Zaffaroni '09]

Hopefully, most Freund-Rubin vacua $(F_0 = 0)$

will spawn many more vacua upon switching on F_0

Conclusions

- AdS/CFT: Romans mass = “overall” Chern-Simons coupling
- New CFT3’s with low supersymmetry
- many new string theory vacua