Jets from Massive Unstable Particles: Top-Mass Determination

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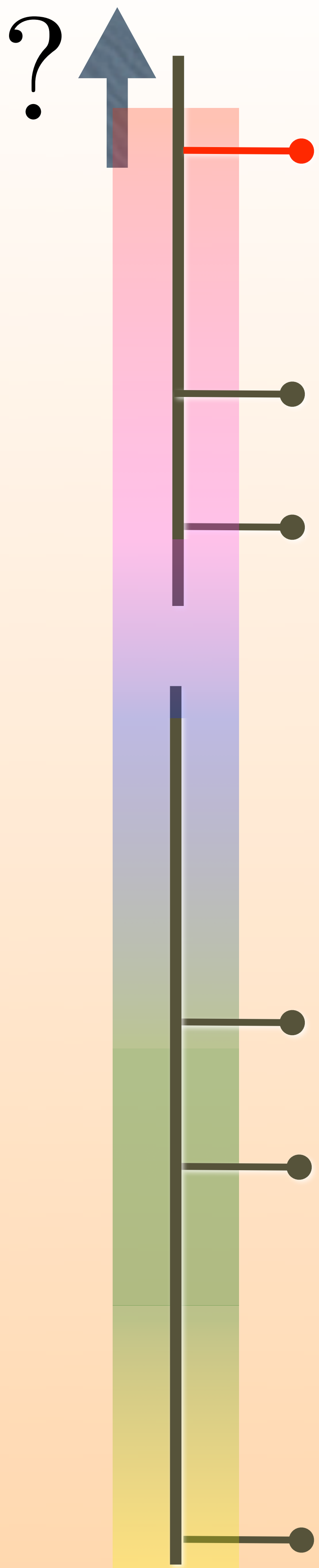
Based on work with:

Andre Hoang, Sean Fleming, & Sonny Mantry (hep-ph/0703207)

Outline

- Top mass measurements. Why do we want a precision m_t ?
- Which mass? Observables & Issues
- Effective Field Theories for Top-Jets: SCET and HQET
- Factorization theorem for Jet Invariant Masses
- Summation of Large Logs $Q \gg m_t \gg \Gamma_t$
- Predictions and Phenomenology
- Summary

Motivation



Q

- The top mass is a fundamental parameter of the Standard Model

$m_t = 171.4 \pm 2.1\,\mathrm{GeV}$ (already a 1% measurement!)

m_W

- Important for precision e.w. constraints

- Top Yukawa coupling is large. Top parameters are important for many new physics models

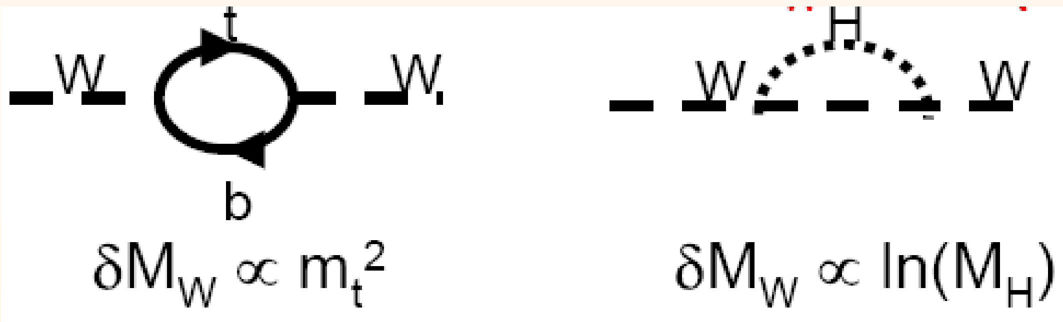
$\Gamma_t = 1.4\,\mathrm{GeV}$ from $t \to bW$

Λ_{QCD}

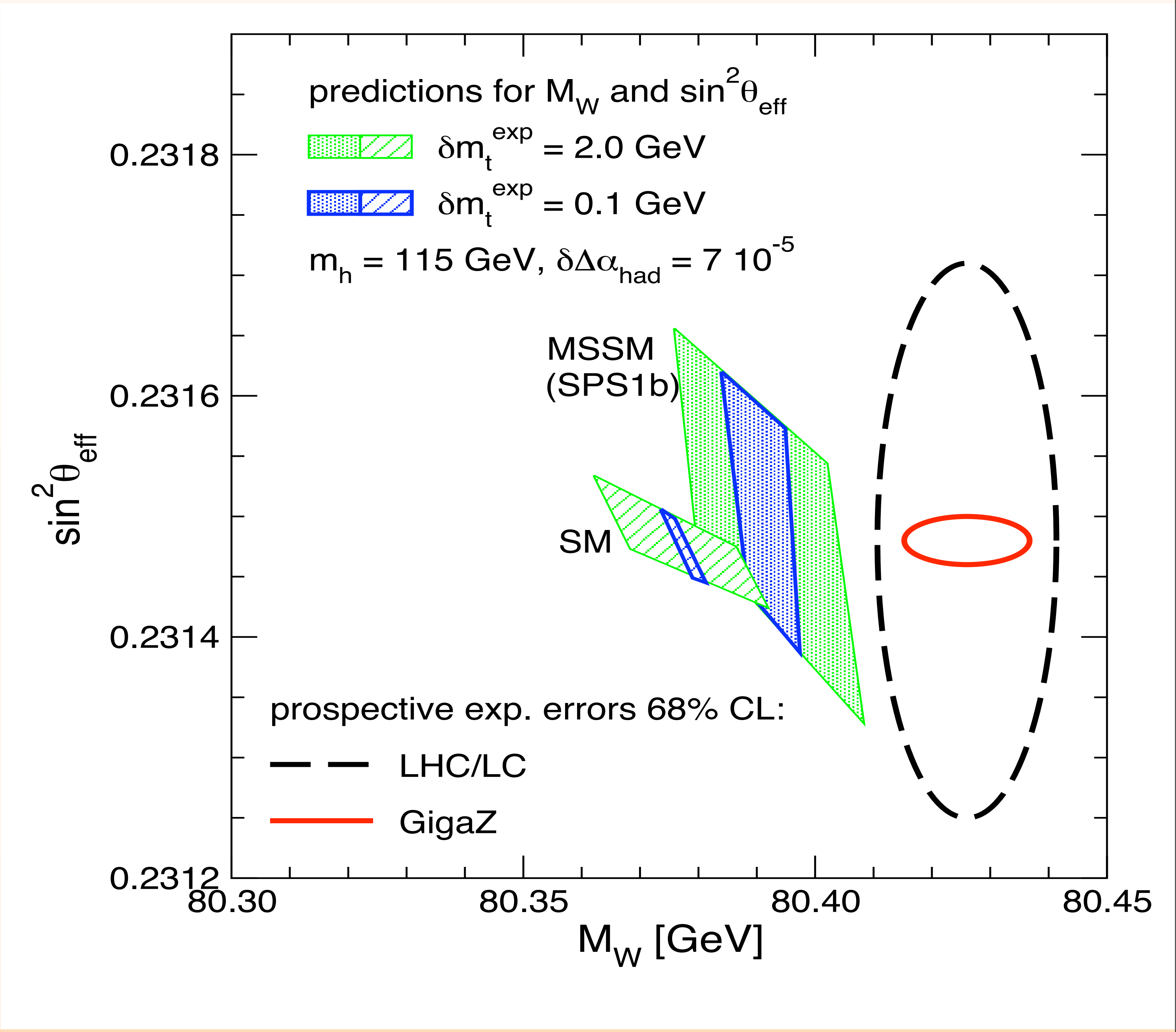
- Top is very unstable, it decays before it has a chance to hadronize. How does this effect jet observables involving top-quarks?

$m_{u,d}$

Electroweak precision observables



$$\sin^2\theta_W \times \Big(1+\delta(m_t, m_H, \ldots)\Big)$$
$$= \quad 1-\frac{m_W^2}{m_Z^2}$$

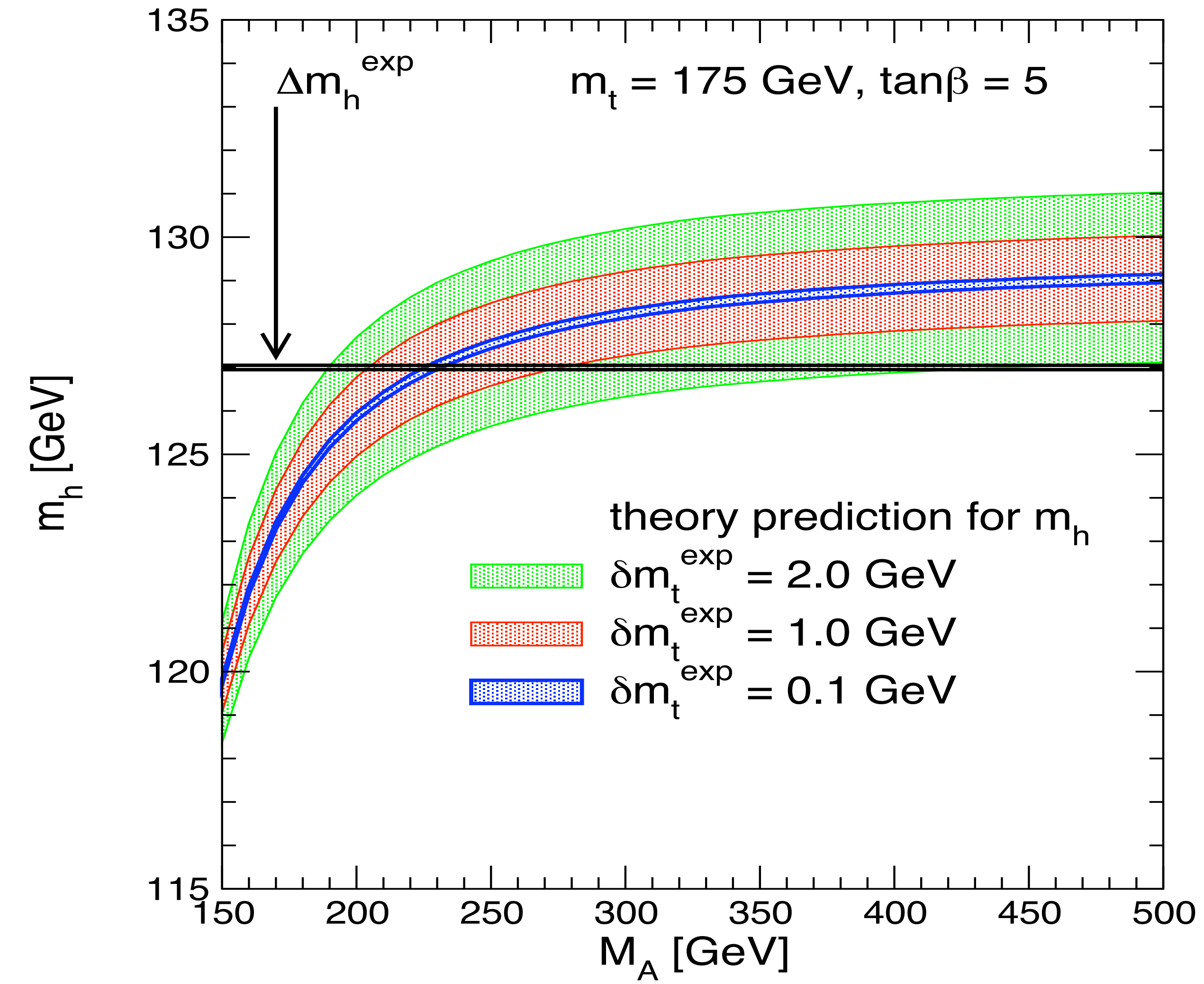


Heinemeyer et.al.

Mass of Lightest MSSM Higgs Boson

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

	LHC	LC
δm_h	1 GeV	50 MeV
needed δm_t	4 GeV	0.2 GeV
expected δm_t	1-2 GeV	~ 0.1 GeV



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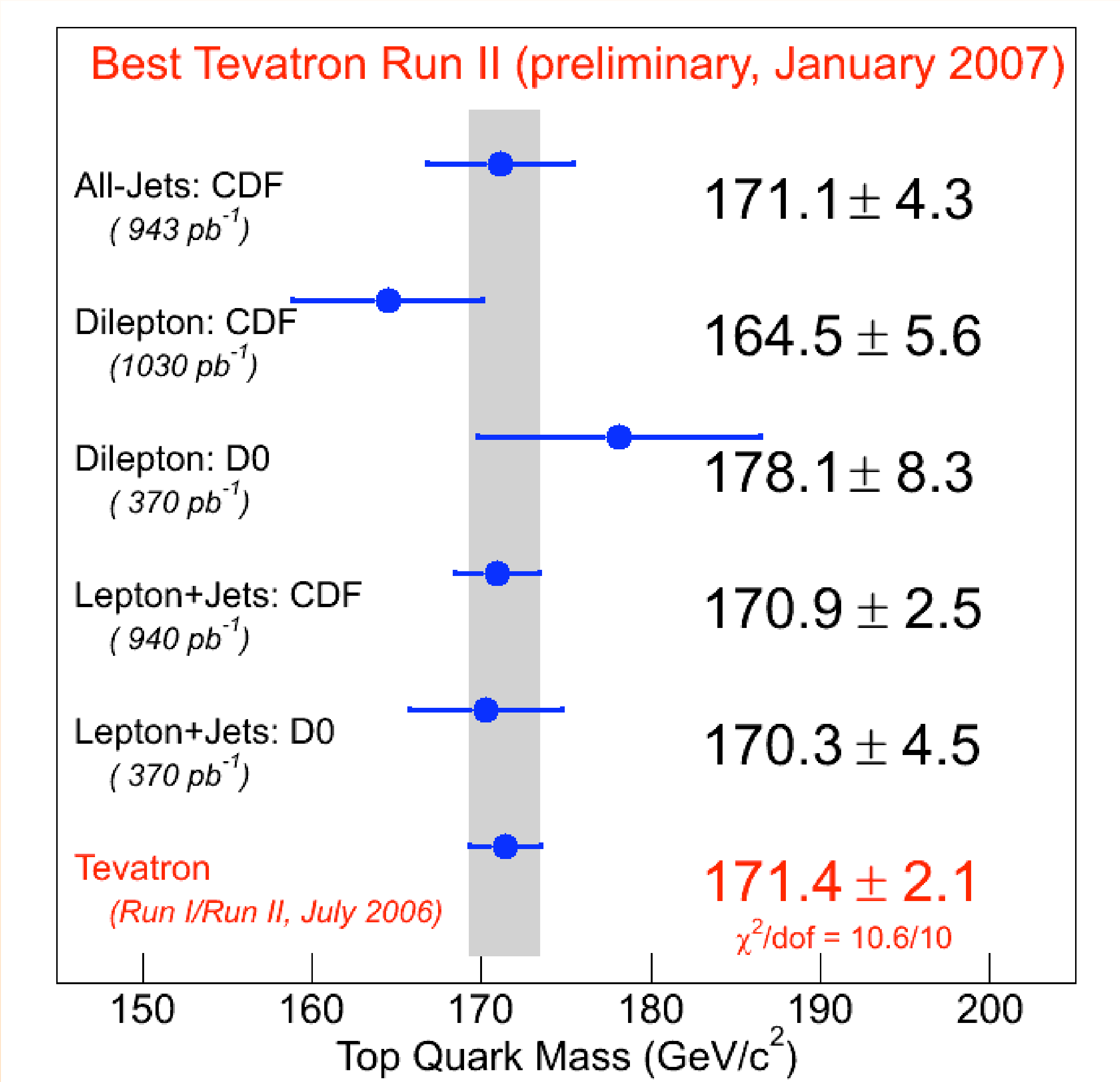
How is it the top-mass measured?



$$pp \xrightarrow{\sigma} t\bar{t}X$$

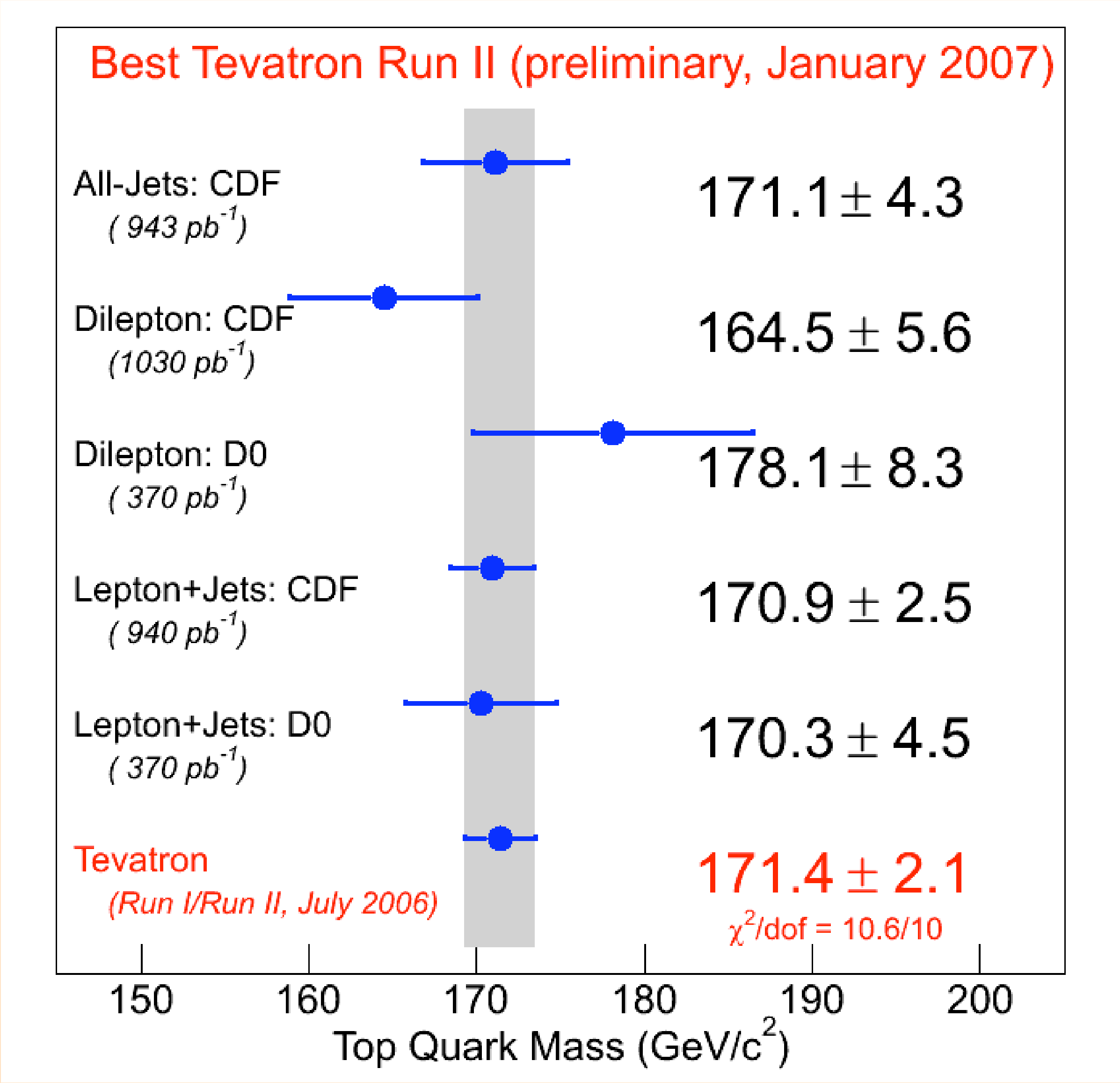
$$\Gamma: \; t \to bW \to e^+\nu, \quad \bar{t} \to \bar{b}W \to q\bar{q}'$$

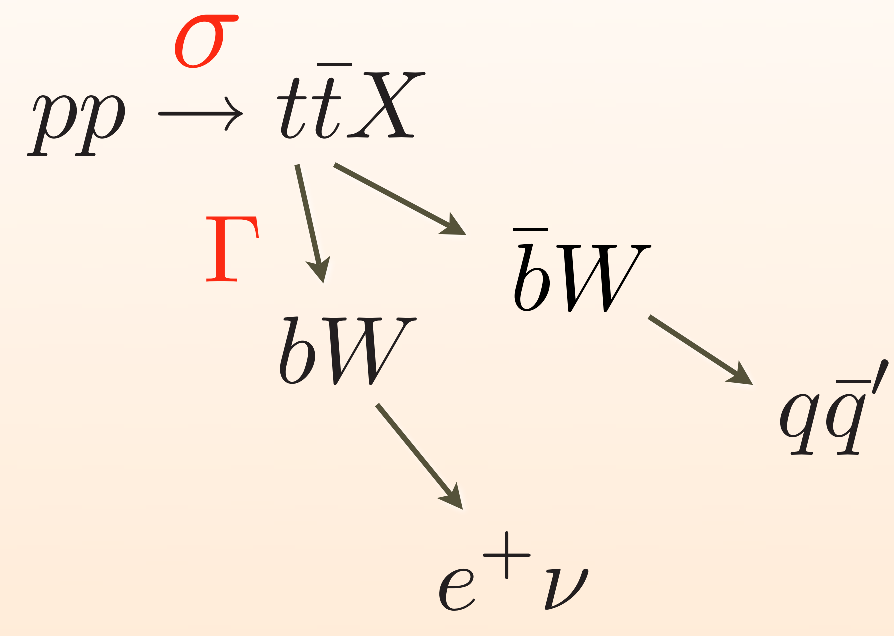
two b-jets + leptons

two b-jets + 2 jets+leptons

two b-jets + 4 jets

How is it the top-mass measured?





two b-jets + leptons
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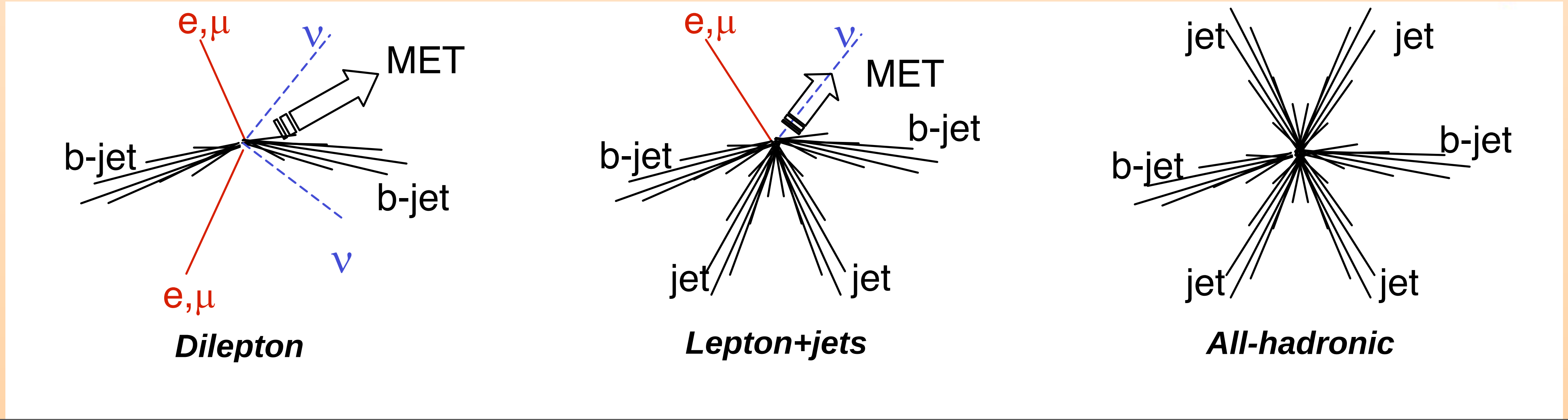


from A.Juste

Template Method (CDF II)

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

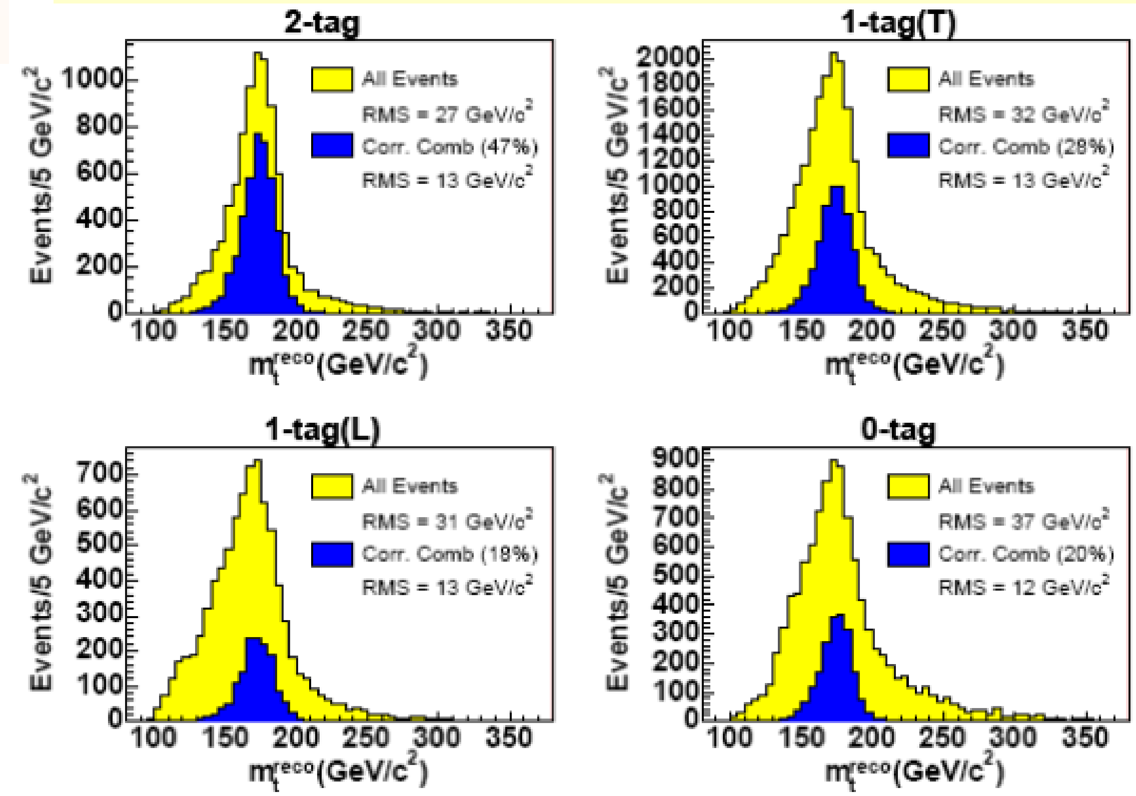
$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$

$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

- Build templates from MC for signal and background and compare to data.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method (D0 II)

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

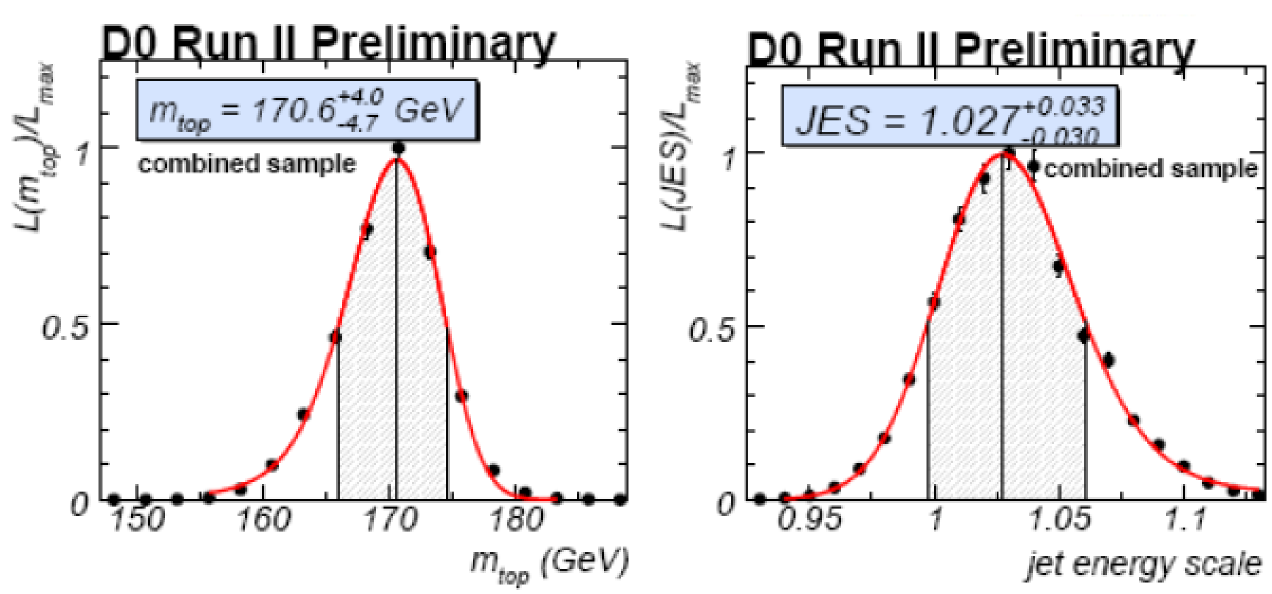
parton distribution functions

$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x|y)$$

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



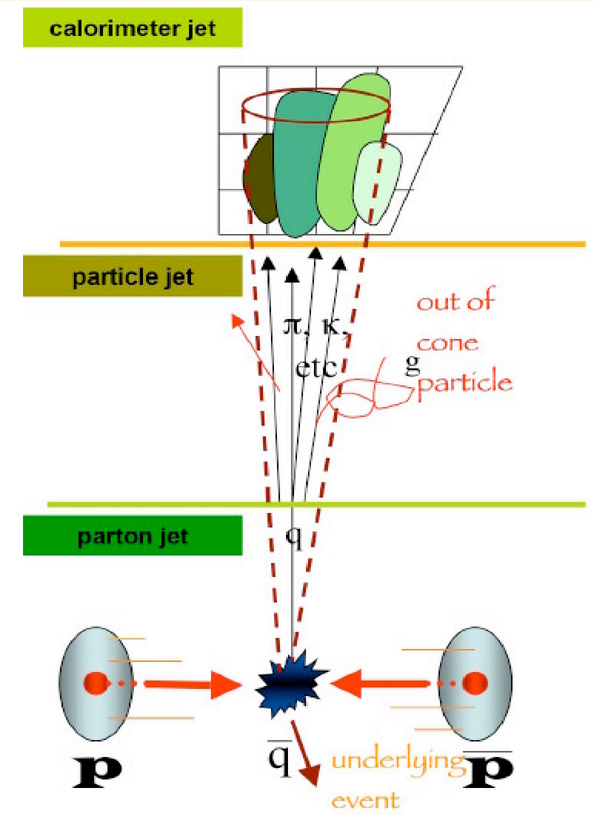
Uncertainties $m_t = 171.4 \pm 1.2\ (\mathrm{stat})\ \pm 1.8\ (\mathrm{syst})\ \mathrm{GeV}$

(eg. reconstruction)

- determine parton momentum of daughters, combinatorics
- jet-energy scale: calorimeter response, uninstrumented zones, multiple hard interactions, energy outside the jet "cone", underlying event (spectator partons)

W-mass helps

- initial & final state radiation, parton distribution functions, b-fragmentation
- which jet algorithm? which Monte-Carlo?
- background (W+jets), b-tagging efficiency
- Statistics



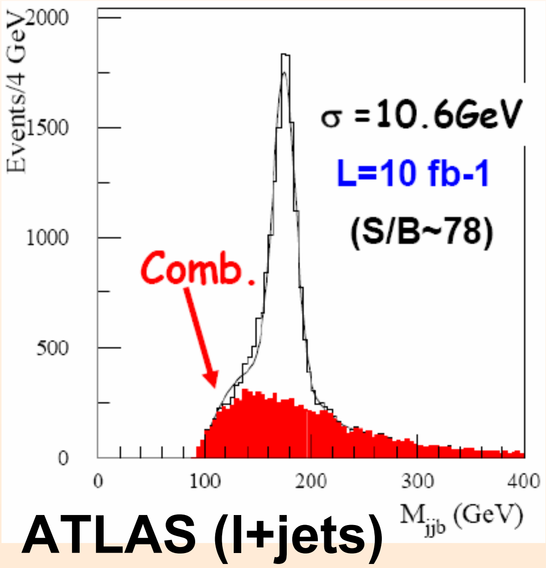
Current Uncertainties

$$m_t = 171.4 \pm 1.2 \text{ (stat) } \pm 1.8 \text{ (syst) GeV}$$

Future -LHC: $pp \to t\bar{t}X$

top factory, 8 million $t\bar{t}$ / year (at low luminosity)

$\delta m_t \sim 1\,\text{GeV}$ systematics dominated



Future -ILC: $e^+e^- \to t\bar{t}$

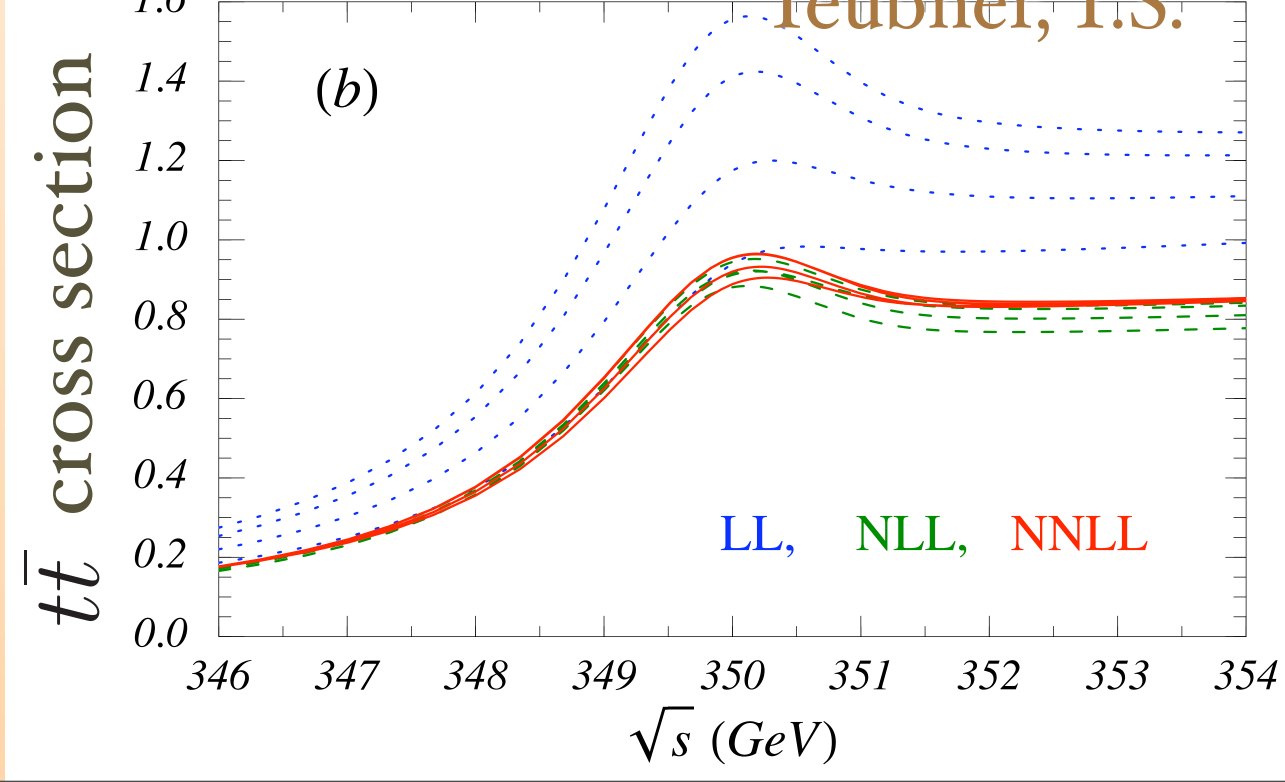
exploit threshold region

$$\sqrt{s} \simeq 2m_t$$

with high precision theory calculations

$\delta m_t \sim 0.1\,\text{GeV}$

Hoang, Manohar, Teubner, I.S.

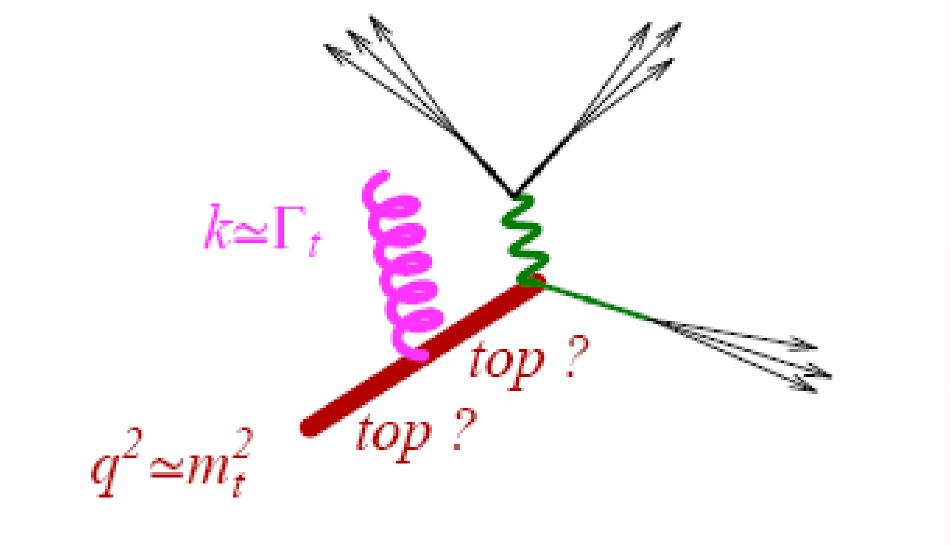


What mass is it?

$m = 171.4 \pm 1.2\ (\mathrm{stat})\ \pm 1.8\ (\mathrm{syst})\ \mathrm{GeV}$

- pole mass?
  - ambiguity $\delta m \sim \Lambda_{\mathrm{QCD}}$, linear sensitivity to IR momenta
  - poor behavior of α_s expansion
  - not used anymore for m_b, m_c

    e.g. $m_b^{1S} = (4.70 \pm 0.04)\,\mathrm{GeV}$



$\delta m \sim \alpha_s(\Gamma)\Gamma$

quark masses are Lagrangian parameters, use a suitable scheme

$m_q^{\mathrm{schemeA}} = m_q^{\mathrm{schemeB}}(1 + \alpha_s + \alpha_s^2 + \ldots)$

- top $\overline{\mathrm{MS}}$ mass? No

$$m^{\mathrm{pole}} - m^{\overline{\mathrm{MS}}}(m) \sim 8\,\mathrm{GeV}$$

some schemes are more appropriate than others

Theory Issues for $pp \to t\bar{t}X$

- jet observable
- suitable top mass for jets
- initial state radiation
- final state radiation
- underlying events
- color reconnection
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$

Theory Issues for $pp \to t\bar{t}X$

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying events
- color reconnection ★
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★

Here we'll study
$e^+e^- \to t\bar{t}X$
and the issues ★

We'll take this calculation seriously, it can be measured at a future ILC.

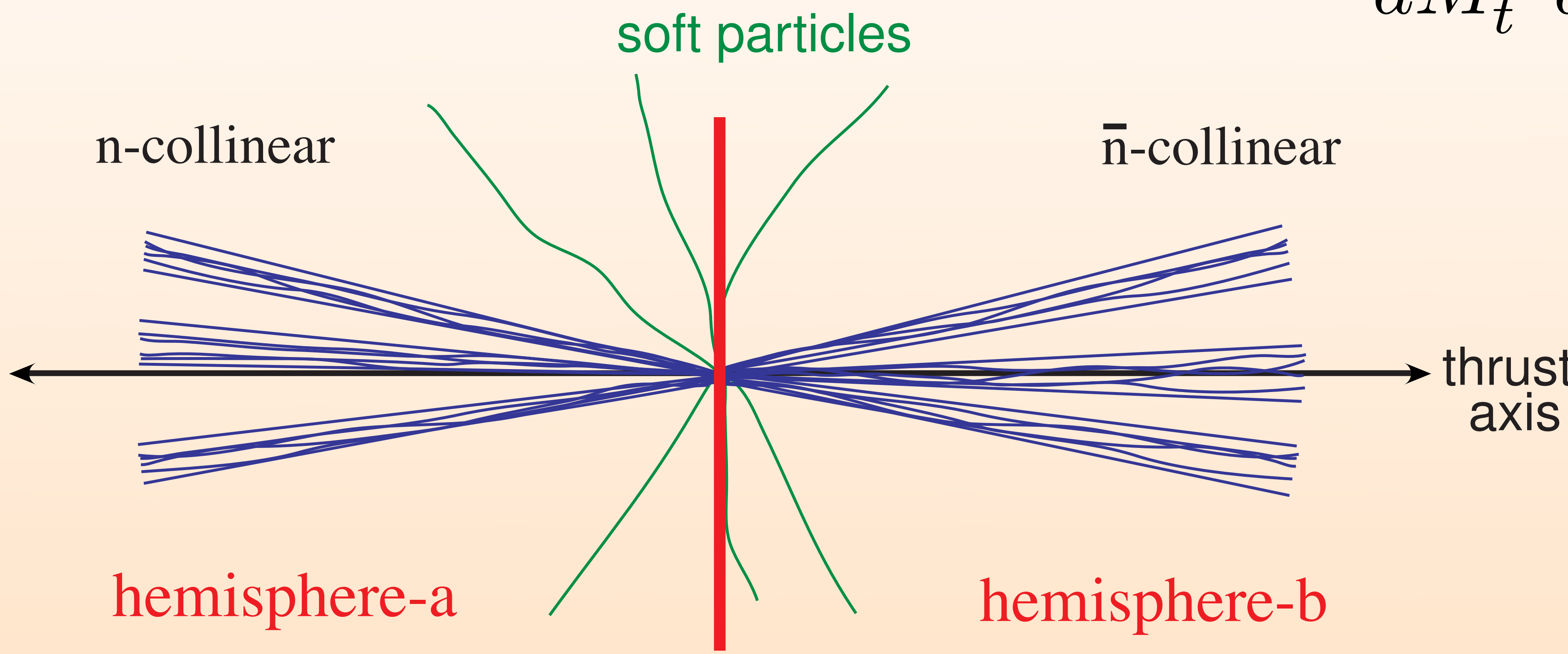
Goals Use Effective Field Theory to:

- Connect jet observables and a Lagrangian mass parameter
  (define a short-distance top-mass that is suitable for measurement with jets)
- Prove factorization: separation of length scales & dynamics
- Simultaneously treat top production and top decay
- Quantify non-perturbative and perturbative effects, universality, hopefully reduce experimental uncertainties

Measure what observable?

Hemisphere Invariant Masses

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$$



$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad M_{\bar{t}}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$

Measure what observable?

Hemisphere Invariant Masses

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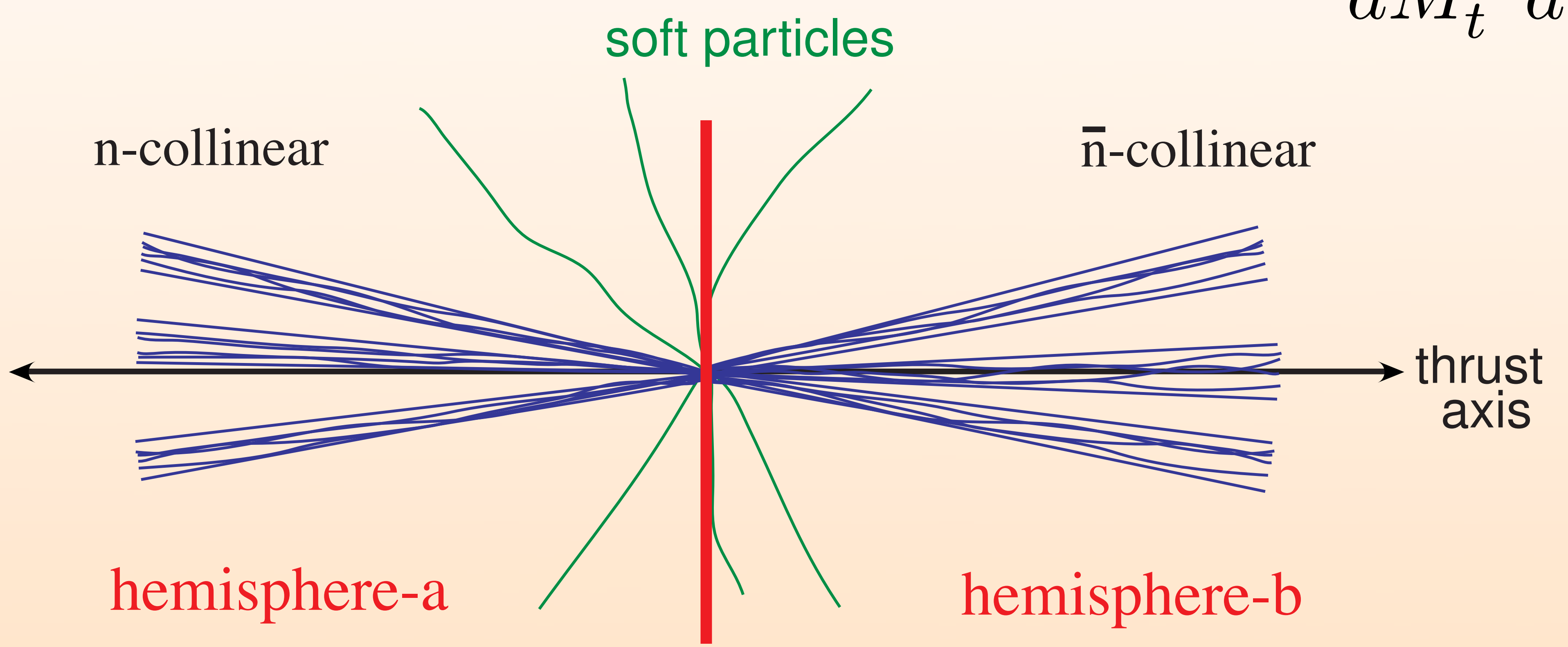

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad M_{\bar{t}}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$

Peak region:

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2 \qquad s_{\bar{t}} \equiv M_{\bar{t}}^2 - m^2 \sim m\Gamma \ll m^2$$

Invariant Mass Distribution

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$$

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$$

Invariant Mass Distribution $\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$$

- A first guess might be that the shape is a Breit Wigner

$$\frac{m\Gamma}{s_t^2 + (m\Gamma)^2} = \left(\frac{\Gamma}{m}\right) \frac{1}{\hat{s}_t^2 + \Gamma^2}$$

$$\hat{s}_t \equiv \frac{s_t}{m} \sim \Gamma$$

Invariant Mass Distribution

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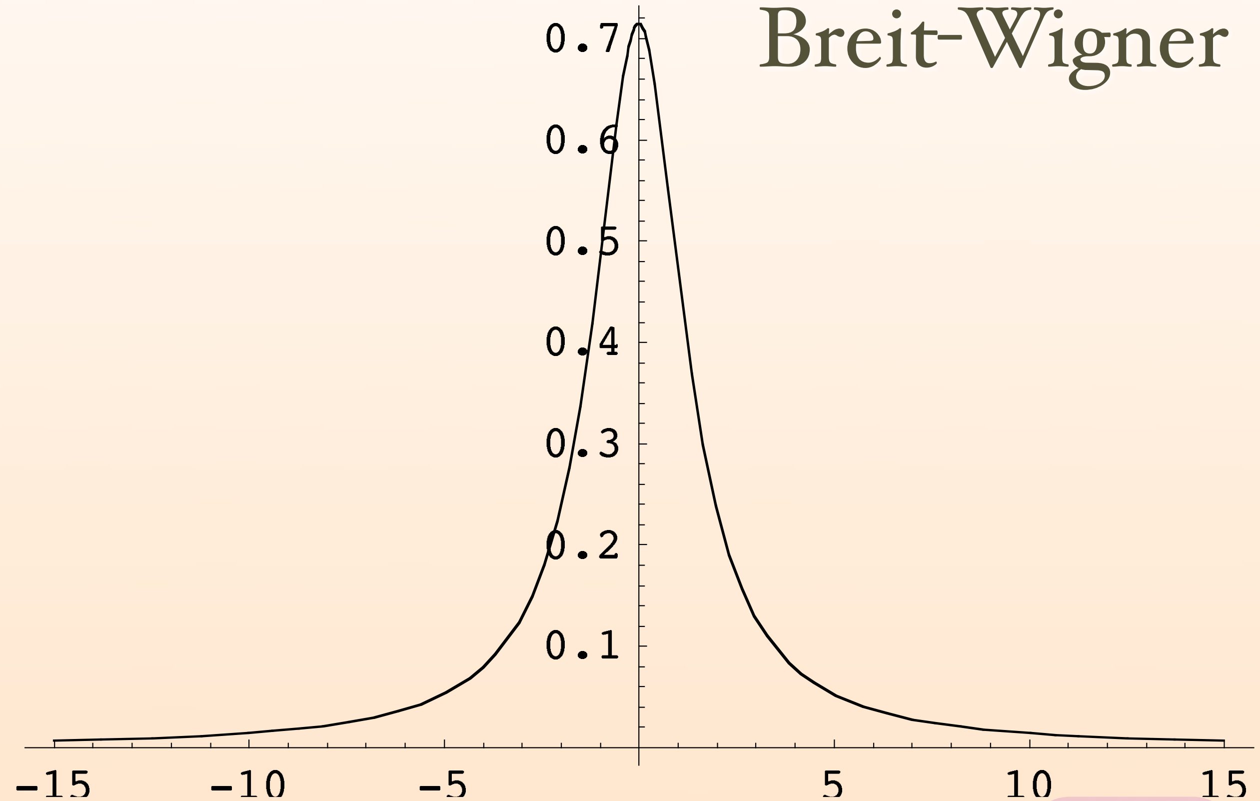
Breit-Wigner



$\hat{s}_t$

Invariant Mass Distribution

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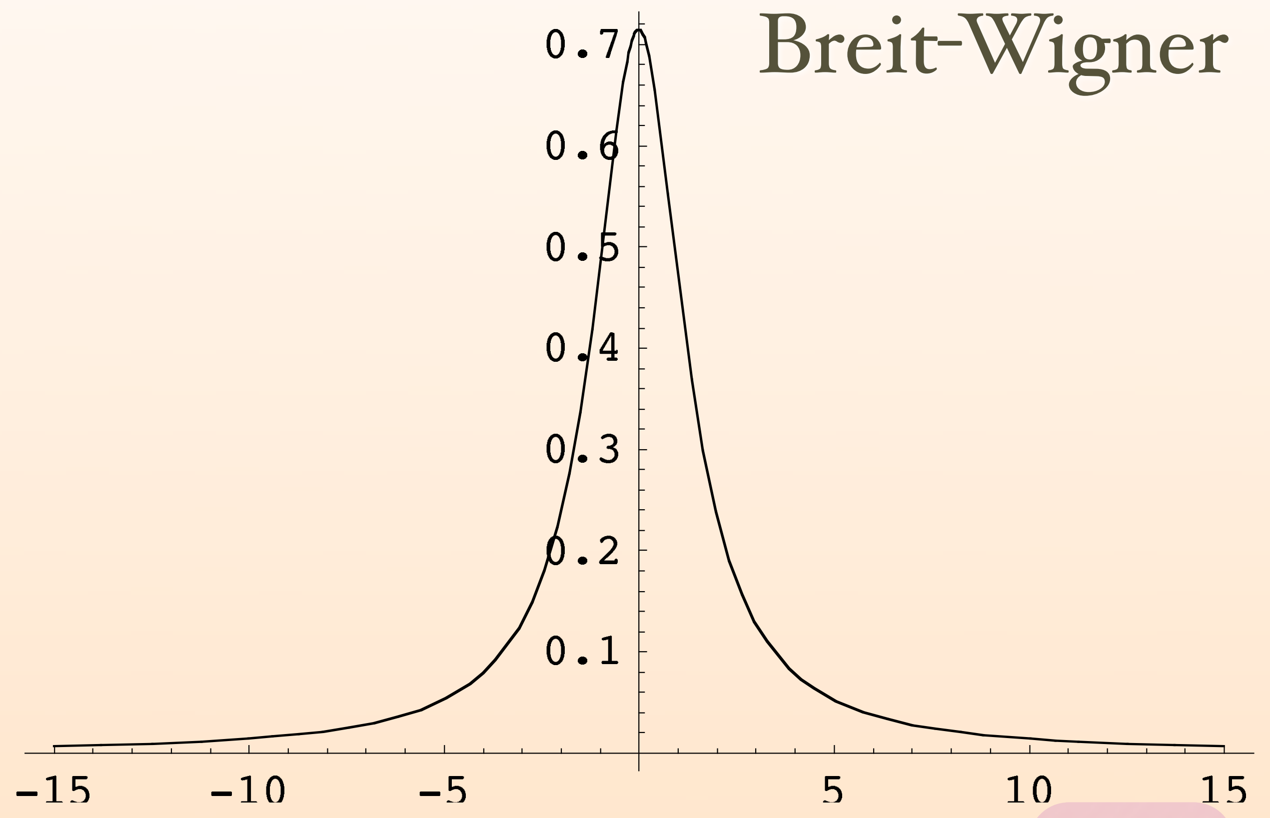
Breit-Wigner



$\hat{s}_t$

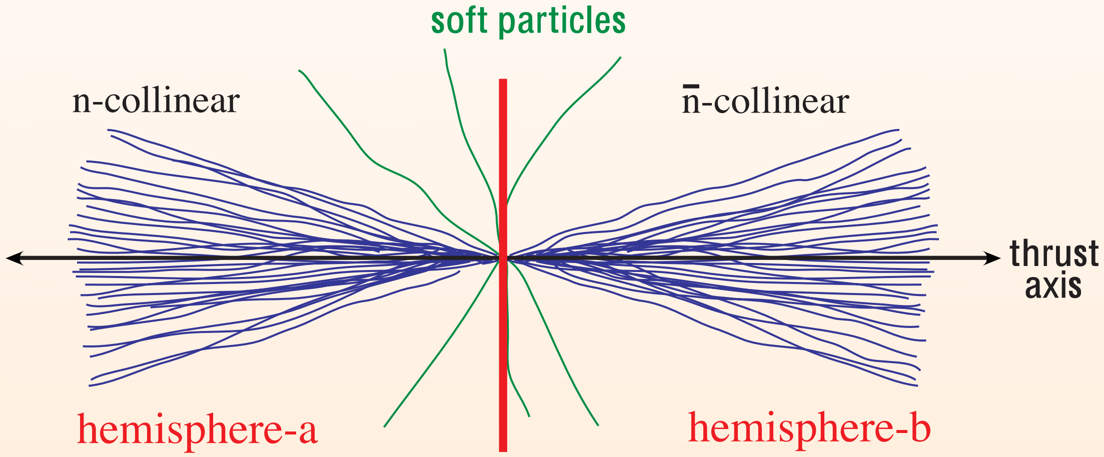
- Since $\Gamma \gg \Lambda_{\rm QCD}$ we can calculate it and see.
  Answer: not quite. Our guess is a bit too naive.

$$Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$$

Disparate Scales $\Rightarrow$ Effective Field Theory

$Q \gg m$ SCET = Soft Collinear Effective Theory

(Bauer, Pirjol, I.S.; Fleming, Luke)



Top quarks are collinear.

Soft radiation btwn. jets.

$Q \gg m$ SCET = Soft Collinear Effective Theory

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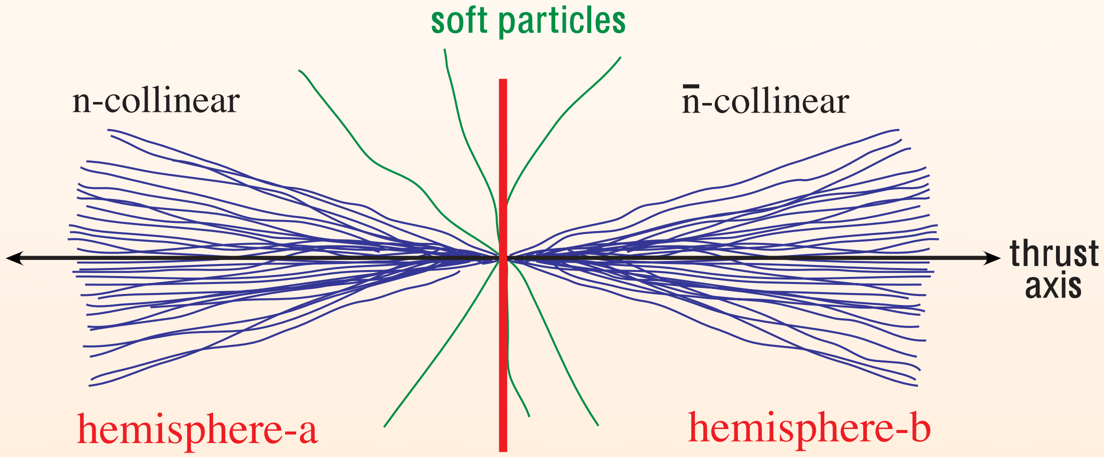
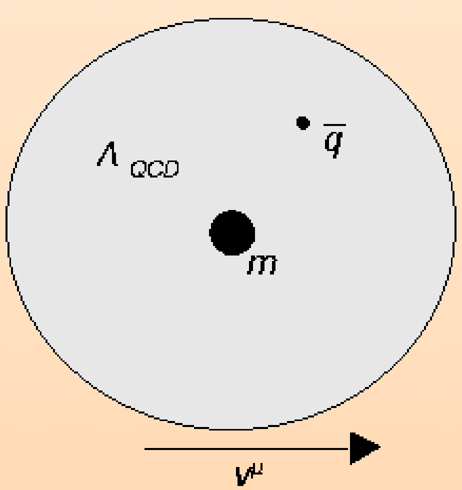


Top quarks are collinear.

Soft radiation btwn. jets.

$m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$ HQET = Heavy Quark Effective Theory

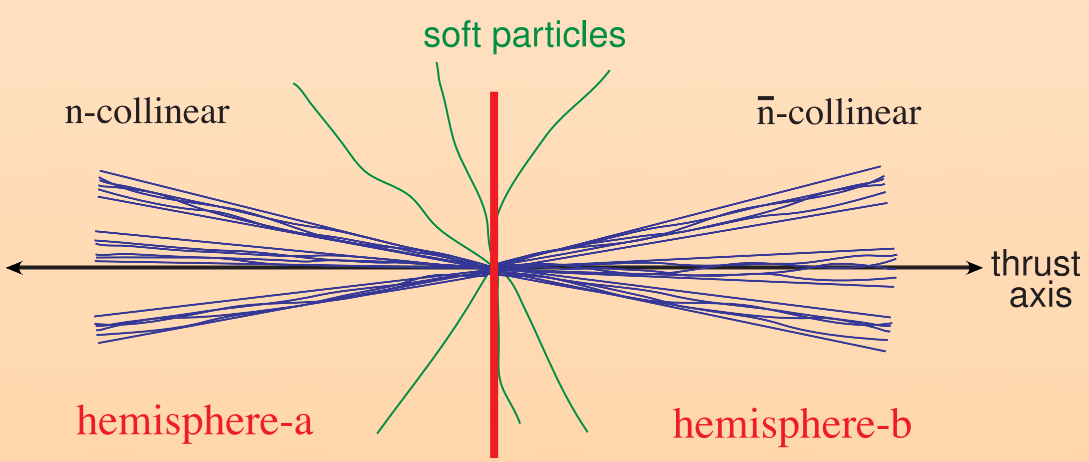
(Isgur, Wise, ...)



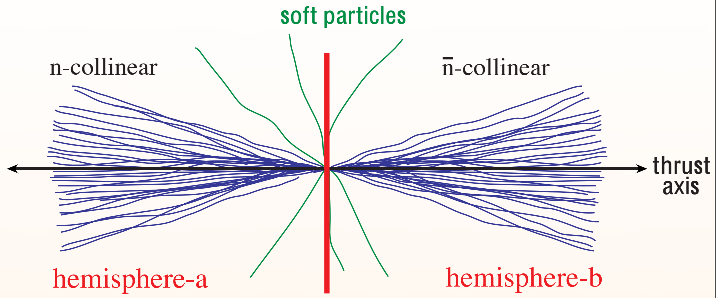
Fluctuations $\ll m$, tops act like static boosted color source

unstable particle EFT

Beneke, Chapovsky, Signer, Zanderighi



Brief Intro to SCET



Degrees of Freedom

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

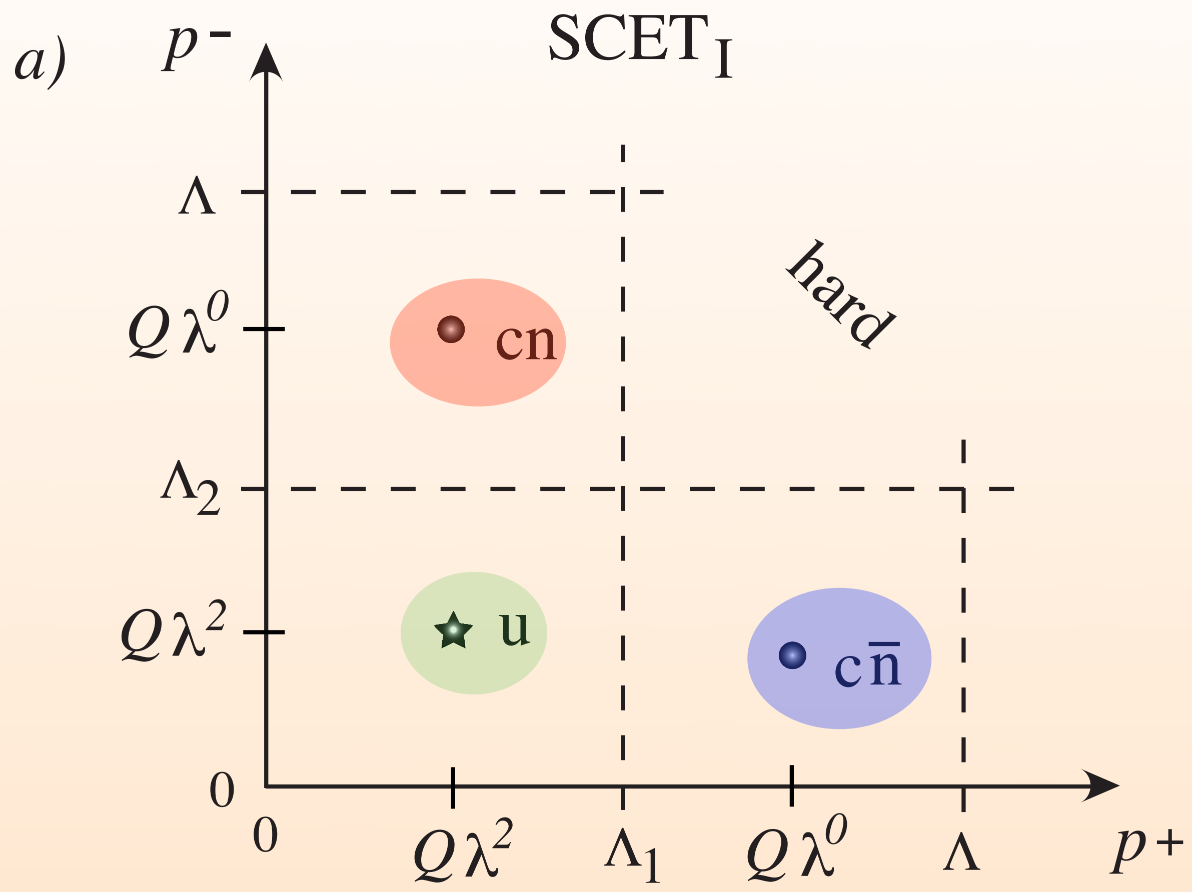
quark fields

gluon fields

$(+, -, \perp)$

light-cone coordinates

Soft - Collinear EFT



A formalism for jets.

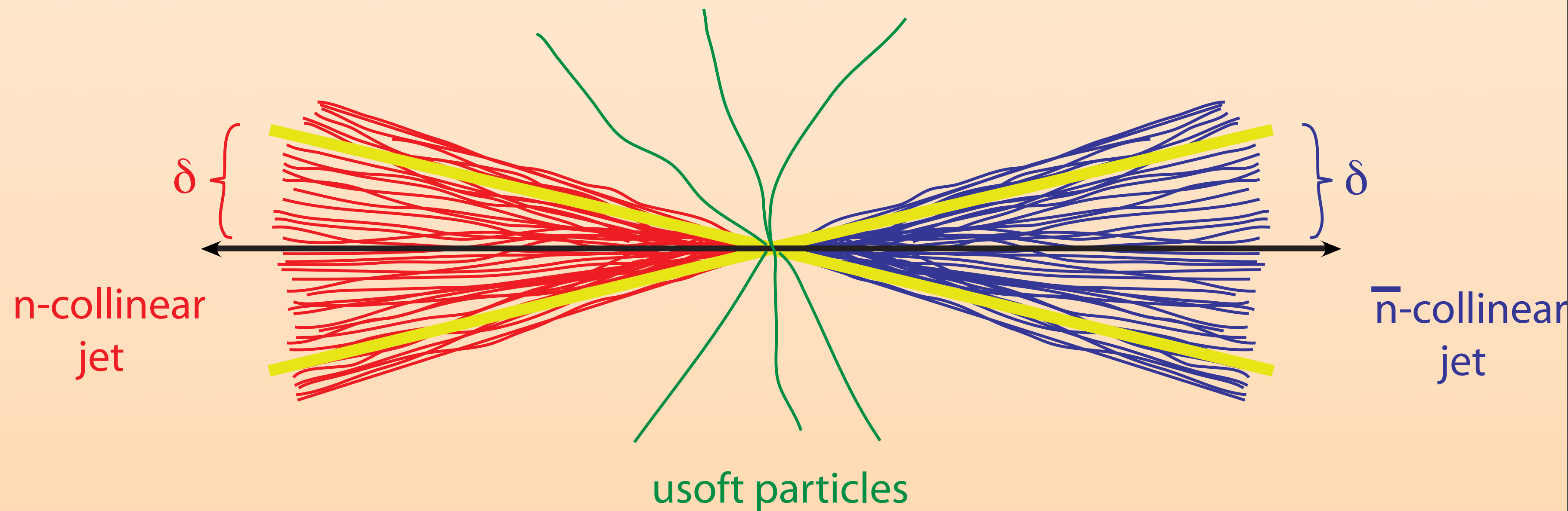
$$p^2 = p^+ p^- + p_\perp^2$$

eg. $e^+e^- \to 2$ jets

$$\lambda \sim \frac{\Delta}{Q}$$

$$m_X^2 \sim \Delta^2$$

$$\Lambda^2 \ll \Delta^2 \ll Q^2$$



Jet constituents : $p^\mu \sim \left(\frac{\Delta^2}{Q}, Q, \Delta\right) \sim Q(\lambda^2, 1, \lambda)$

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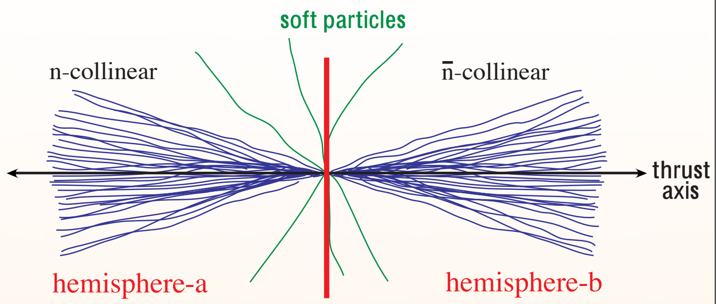

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LO collinear Lagrangian:

$$\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \Big[in \cdot D_s + gn \cdot A_n + (i\not{D}_c^\perp - m) W_n \frac{1}{\bar{n} \cdot \mathcal{P}} W_n^\dagger (i\not{D}_c^\perp + m) \Big] \frac{\not{\bar{n}}}{2} \xi_n$$

eikonal soft couplings

collinear Wilson line

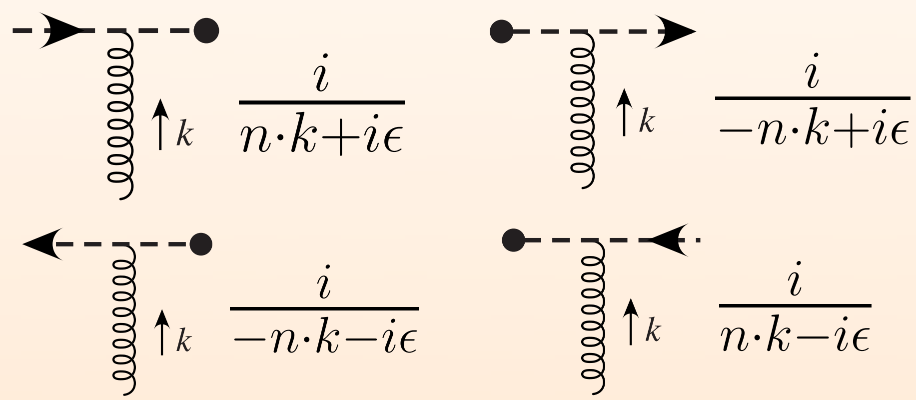
$$W_n = P \exp\left(ig \int_0^\infty ds\, \bar{n} \cdot A_n(s\bar{n}) \right)$$

Ultrasoft - Collinear Factorization

Multipole Expansion:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n \cdot iD_{us} + g n \cdot A_n + i\not{D}_\perp^c \frac{1}{i\bar{n} \cdot D_c} i\not{D}_\perp^c \Big\} \frac{\bar{\not{n}}}{2} \xi_n$$

usoft gluons have eikonal Feynman rules and induce eikonal propagators



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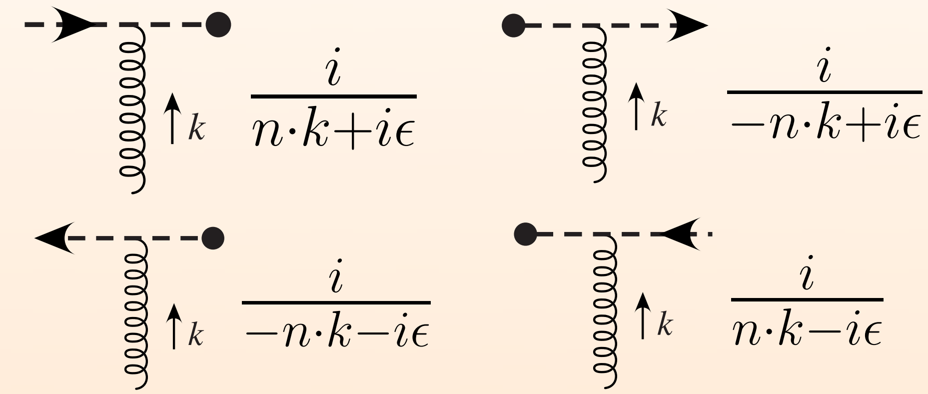

Field Redefinition:

$$\xi_n \to Y\xi_n \ , \quad A_n \to Y A_n Y^\dagger$$

$$Y(x) = P\exp\Big(ig\int_{-\infty}^0 ds\, n\cdot A_{us}(x+ns)\Big)$$

$$n\cdot D_{us} Y = 0,\ Y^\dagger Y = 1$$

choice of $\pm\infty$ here is irrelevant if one is careful

Ultrasoft - Collinear Factorization

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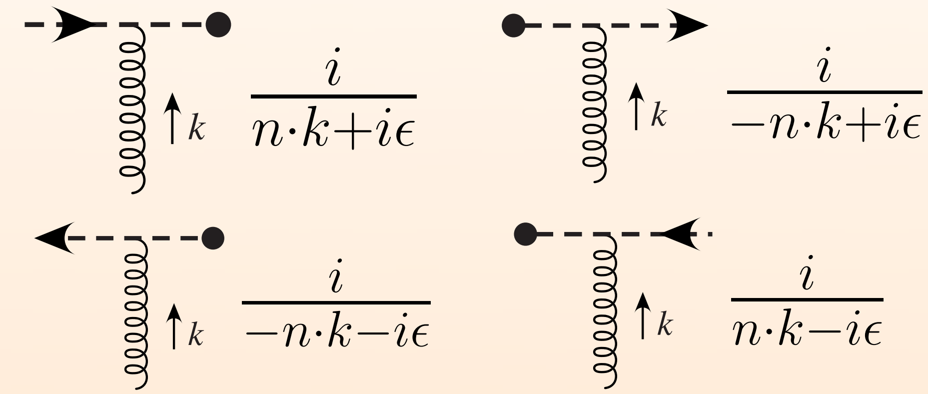

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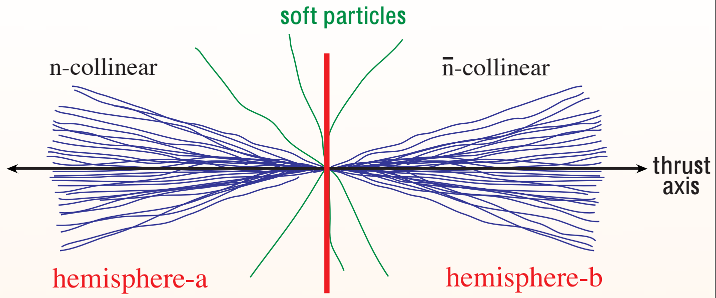
gives:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n\cdot iD_{\mathrm{us}} + \ldots \Big\} \frac{\not{\bar{n}}}{2} \xi_n \to \bar{\xi}_n \Big\{ n\cdot iD_c + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

Moves all usoft gluons to operators, simplifies cancellations

Brief Intro to SCET



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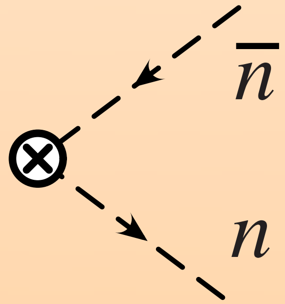
Production Current:



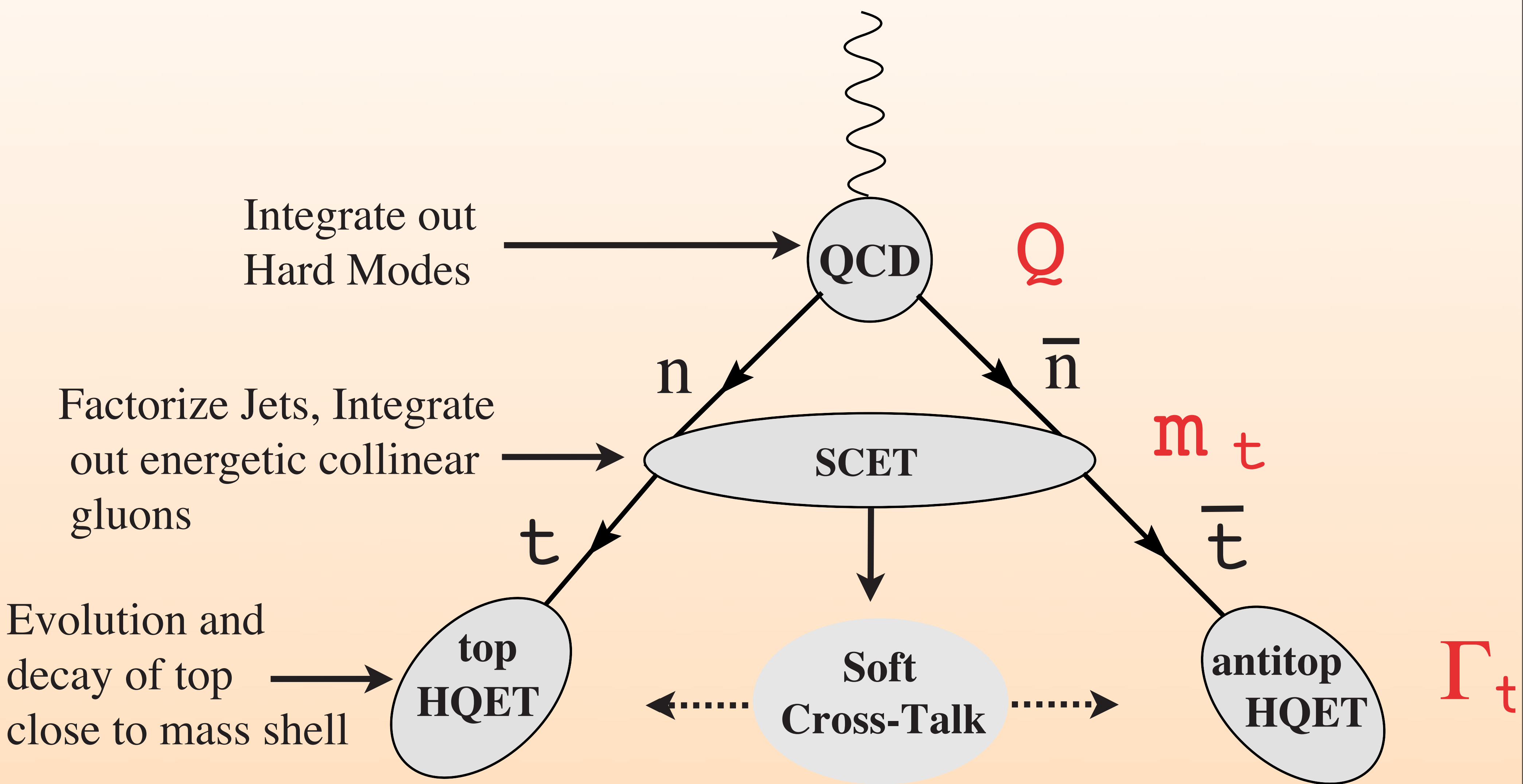
$$\underbrace{\bar\psi\,\Gamma^\mu\psi}_{\mathcal{J}_i^\mu} \to (\bar\xi_n W_n)_\omega\,\Gamma^\mu (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega} = (\bar\xi_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar n} (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$

Matching and Running

QCD

SCET

HQET



Brief Intro to unstable boosted HQET

$$v_+^\mu = \left(\frac{m}{Q}, \frac{Q}{m}, \mathbf{0}_\perp\right)$$

fluctuations beneath the mass

$$p^\mu = m v_+^\mu + k^\mu$$

collinear, but with smaller overall scale

one HQET for top

one HQET for antitop

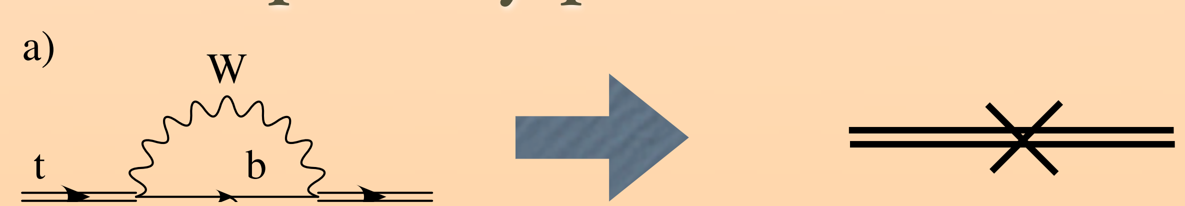
bHQET $[\Gamma/m \ll 1]$	
n-ucollinear $(h_{v_+},\ A^\mu_{v_+})$	$k^\mu \sim \Gamma(\lambda^2, 1, \lambda)$
$\bar{n}$-ucollinear $(h_{v_-},\ A^\mu_{v_-})$	$k^\mu \sim \Gamma(1, \lambda^2, \lambda)$
same soft $(q_s,\ A^\mu_s)$	$p^\mu_s \sim (\Delta, \Delta, \Delta)$

$$\mathcal{L}_+ = \bar{h}_{v_+}\big(iv_+\cdot D_+ - \delta m + \frac{i}{2}\Gamma\big)h_{v_+}, \qquad \mathcal{L}_- = \bar{h}_{v_-}\big(iv_-\cdot D_- - \delta m + \frac{i}{2}\Gamma\big)h_{v_-}$$

mass scheme choice

$$\delta m = m^{\text{pole}} - m$$

our observable is inclusive in top decay products



We are ready
to derive the
Factorization Theorem

In QCD: The full cross-section is

a restricted set of states: $s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$

$$\sigma = \sum_X^{res.} (2\pi)^4 \, \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \, \langle 0 | \mathcal{J}_i^{\nu\dagger}(0) | X \rangle \langle X | \mathcal{J}_i^{\mu}(0) | 0 \rangle$$

lepton tensor, γ & Z exchange

by using EFT's we will be able to move these restrictions into the operators

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In SCET:

$$\mathcal{J}_i^{\mu}(0) = \int d\omega \, d\bar{\omega} \, C(\omega, \bar{\omega}, \mu) J_i^{(0)\mu}(\omega, \bar{\omega}, \mu)$$

Wilson coefficient

SCET current

$$(\bar{\xi}_n W_n)_\omega \, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$$

$$\equiv \bar{\chi}_{n,\omega} \, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} \chi_{\bar{n},\bar{\omega}}$$

Momentum conservation:

$$\to C(Q, Q, \mu)$$

SCET cross-section:

$$|X\rangle = |X_n X_{\bar n} X_s\rangle$$

$$\sigma = K_0 \sum_{\vec n} \sum_{X_n X_{\bar n} X_s}^{res.\,\prime} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \langle 0|\overline{Y}_{\bar n}\, Y_n|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger|0\rangle$$

$$\times\ |C(Q,\mu)|^2 \langle 0|\hat{\bar{n}}\!\!\!/\,\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle \langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\hat{n}\!\!\!/\,\chi_{\bar n,\bar\omega}|0\rangle$$

all-orders

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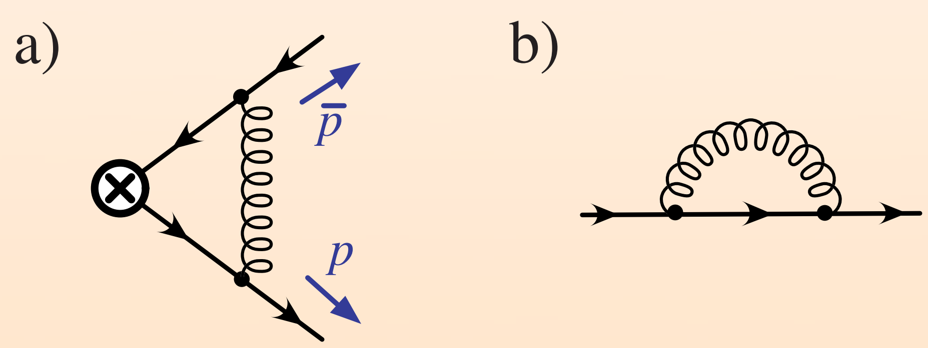
$$\times\ |C(Q,\mu)|^2 \langle 0|\bar{\not{n}}\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle\langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\not{n}\chi_{\bar n,\bar\omega}|0\rangle$$

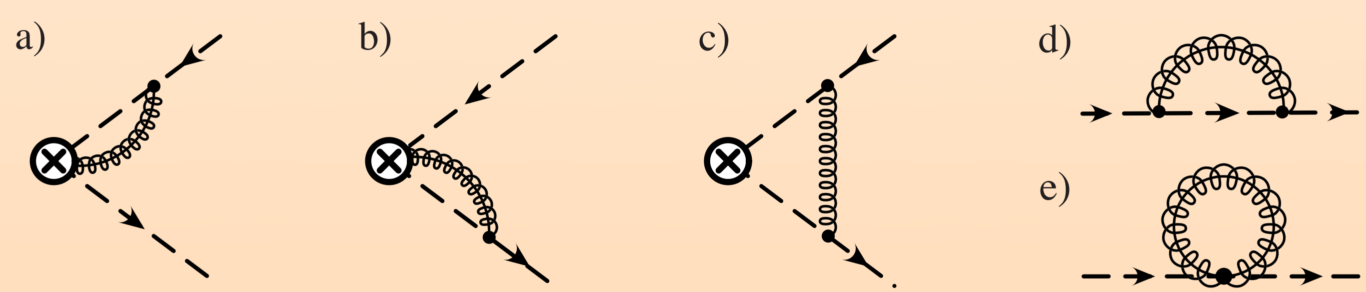
all-orders

QCD



one-loop

SCET



gives $$C(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\log\frac{-Q^2 - i0}{\mu^2} - \log^2\frac{-Q^2 - i0}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

Specify hemisphere invariant masses for the jets:

total soft momentum is the sum of momentum in each hemisphere

$$K_{X_s} = k_s^a + k_s^b$$

$$\hat{P}_a \, |X_s\rangle = k_s^a \, |X_s\rangle, \quad \hat{P}_b \, |X_s\rangle = k_s^b \, |X_s\rangle$$

hemisphere projection operators

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hemisphere projection operators

Insert: $$1 = \int dM_t^2\,\delta\big((p_n + k_s^a)^2 - M_t^2\big)\int dM_{\bar{t}}^2\,\delta\big((p_{\bar{n}} + k_s^b)^2 - M_{\bar{t}}^2\big)$$

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hemisphere projection operators

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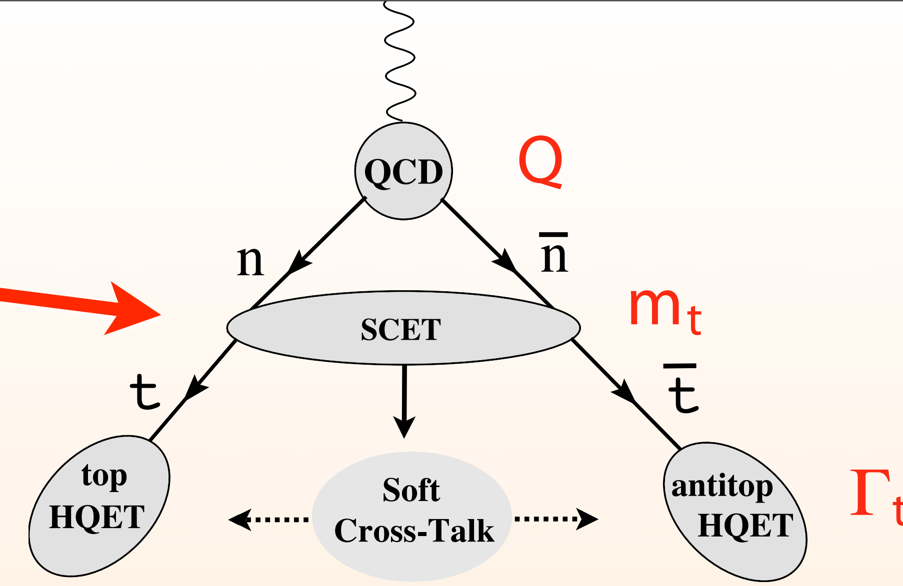
... Some Algebra ...

SCET factorization Theorem:

we're here



$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Hard Function

$$H_Q(Q,\mu) = |C(Q,\mu)|^2$$

Top Jet Function

Anti-top Jet Function

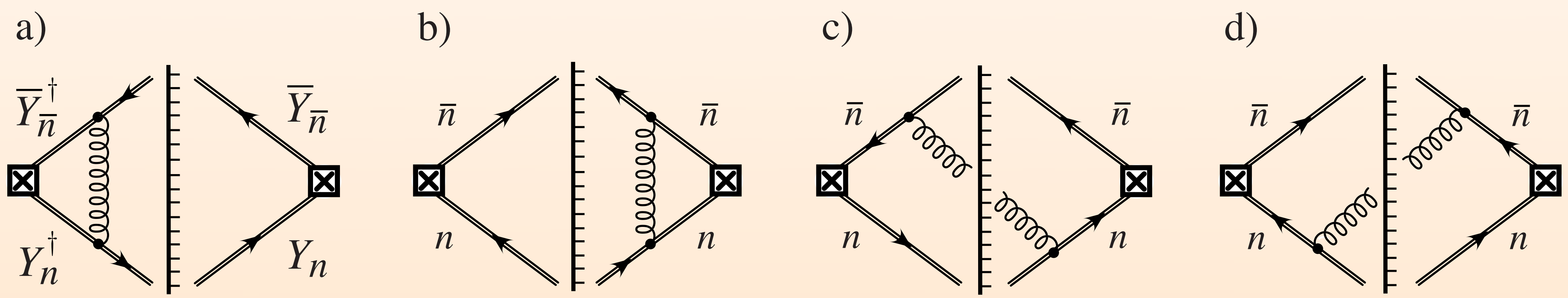
depend on m

Soft radiation Function

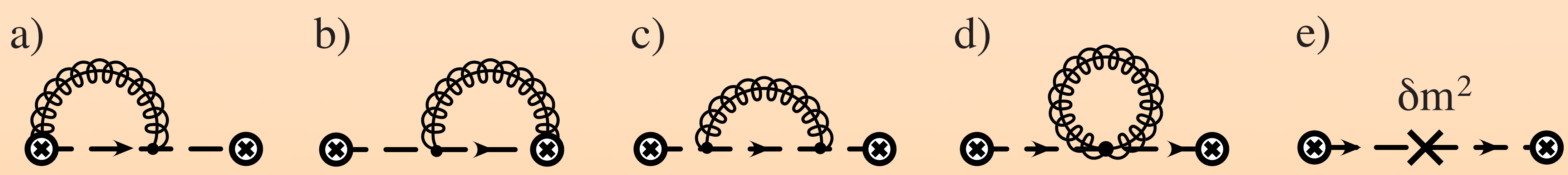
universal

Soft function is nonperturbative, but universal,
it also appears in massless dijets

$$\boxed{S_{\rm hemi}(\ell^+,\ell^-,\mu)} = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$$



Jet function: $\boxed{J_n(Qr_n^+ - m^2)} = \dfrac{-1}{2\pi Q}\displaystyle\int d^4x\, e^{ir_n\cdot x}\,{\rm Disc}\,\langle 0|{\rm T}\{\overline{\chi}_{n,Q}(0)\bar{\not n}\chi_n(x)\}|0\rangle$



is perturbative

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu)\int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

$\hat s_t = s_t/m \ll m$

match onto HQET

$$J_n(m\hat s,\Gamma,\mu_m) = T_+(m,\mu_m)\, B_+(\hat s,\Gamma,\mu_m)$$

SCET jet fn.

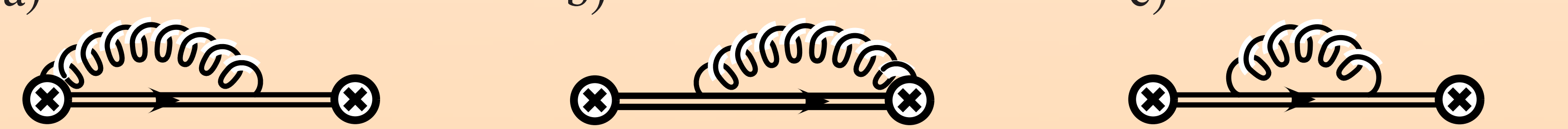
Wilson coefficient

HQET jet fn.

Integrate out mass scale

$$B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\int d^4x\, e^{ik\cdot x}\, \mathrm{Disc}\, \langle 0|\mathrm{T}\{\bar h_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\}|0\rangle$$

a) b) c)



Matching:

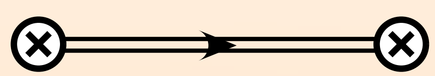
$$T_\pm(\mu,m) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right)$$

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

done!

$$H_m = T_+ T_-$$

Everything but S_hemi is calculable, and it has been measured using massless event shapes

At tree level: 

$$B_\pm^{\rm tree}(\hat s, \Gamma) = \frac{-1}{8\pi N_c m}(-2N_c)\, {\rm Disc}\Big(\frac{i}{v_\pm \cdot k + i\Gamma/2}\Big) = \frac{1}{4\pi m}{\rm Im}\Big(\frac{-2}{v_\pm \cdot k + i\Gamma/2}\Big)$$
$$= \frac{1}{\pi m}\, \frac{\Gamma}{\hat s^2 + \Gamma^2}\,.$$

our Breit-Wigner

- B.W. receives calculable perturbative corrections
- cross-section depends on non.pert. soft function, not just B.W.'s

  ** the B.W. is only a good approx. for collinear top & gluons **
- in the fact. thm. we remove largest component of soft momentum from the inv.mass. to get the argument for the B.W.

A Short-Distance Top-Mass for Jets

- First, why not $\overline{\text{MS}}$? $\delta\overline{m} \sim \alpha_s \overline{m} \gg \Gamma$

when we switch to a short-distance mass scheme we must expand in α_s

$$B_+(\hat{s},\mu,\delta\overline{m}) = \frac{1}{\pi\overline{m}}\left\{\frac{\Gamma}{\left[\frac{(M_t^2-\overline{m}^2)^2}{\overline{m}^2}+\Gamma^2\right]} + \frac{(4\,\hat{s}\,\Gamma)\,\delta\overline{m}}{\left[\frac{(M_t^2-\overline{m}^2)^2}{\overline{m}^2}+\Gamma^2\right]^2}\right\}$$

$\sim 1/(\overline{m}\Gamma)$ $\sim \alpha_s/\Gamma^2$

not a correction!
it swamps the 1st term

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$$\sim 1/(\overline{m}\Gamma) \qquad\qquad \sim \alpha_s/\Gamma^2$$

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- Jet mass scheme $\quad m_J(\mu) \qquad \delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma$

define the scheme by holding the B.W.
peak position fixed

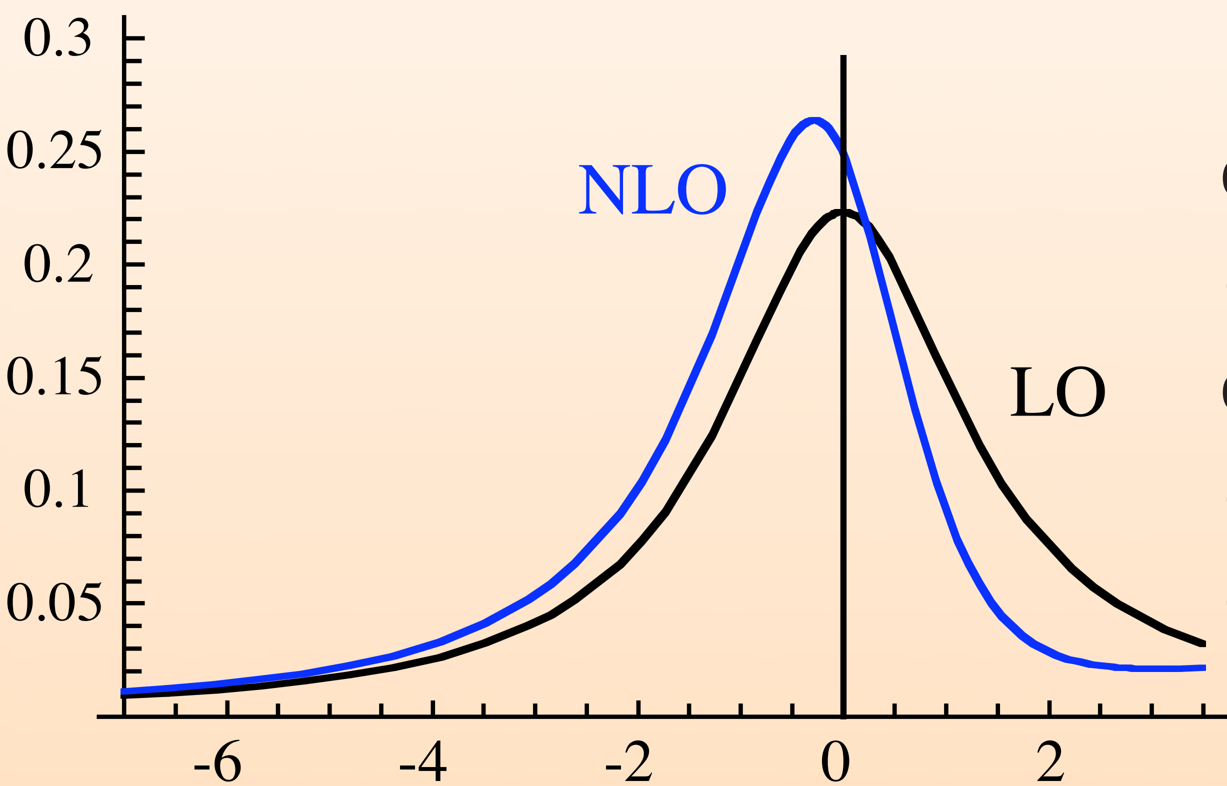
$$\frac{dB_+(\hat{s}, \mu, \delta m_J)}{d\hat{s}}\bigg|_{\hat{s}=0} = 0$$

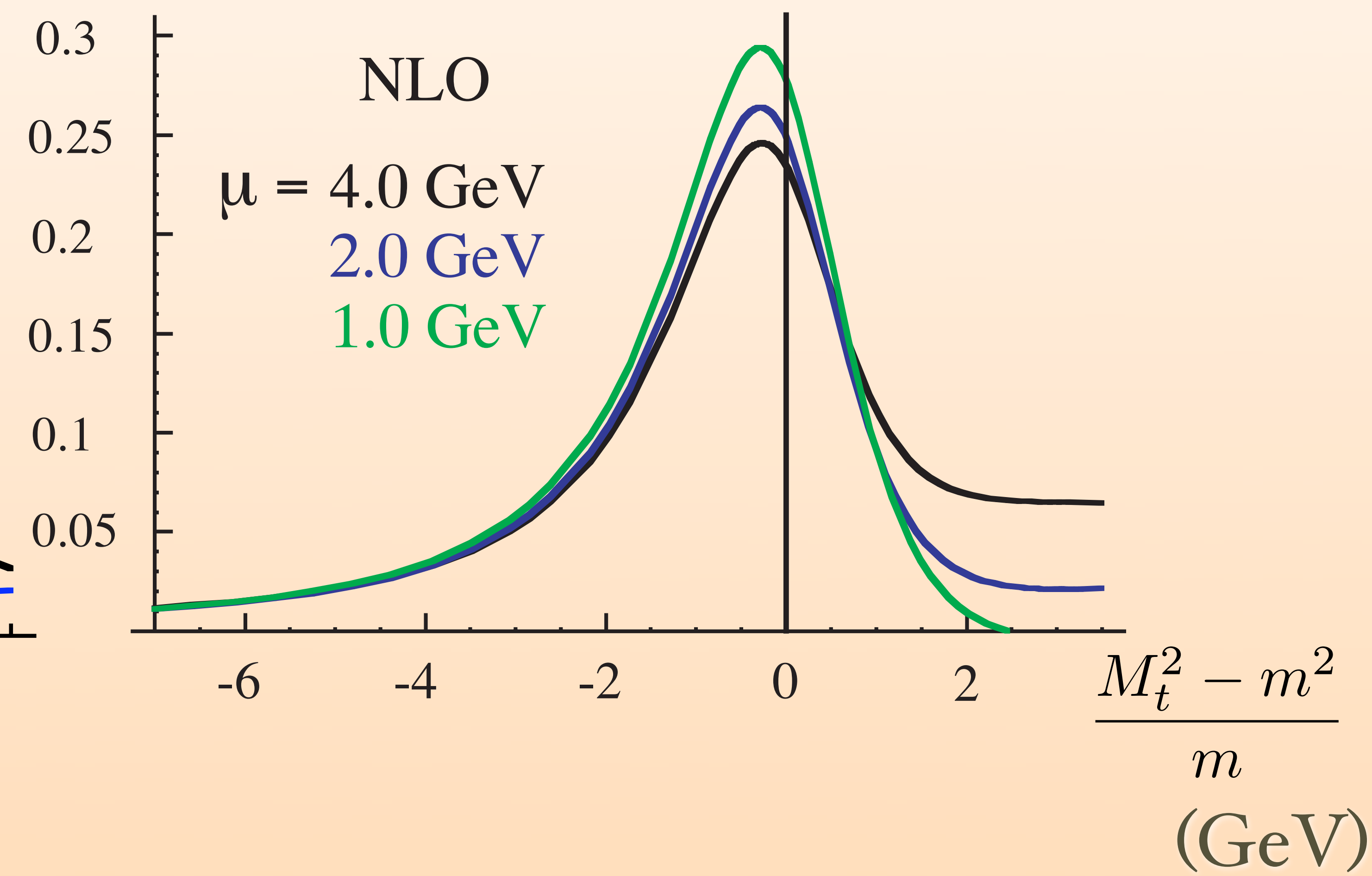
$$\begin{aligned} m_J(\mu) &= m_{\text{pole}} - \delta m_J \\ &= m_{\text{pole}} - \Gamma \frac{\alpha_s(\mu)}{3}\left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2}\right] \end{aligned}$$

Perturbative Peak Shifts

NLO Corrections

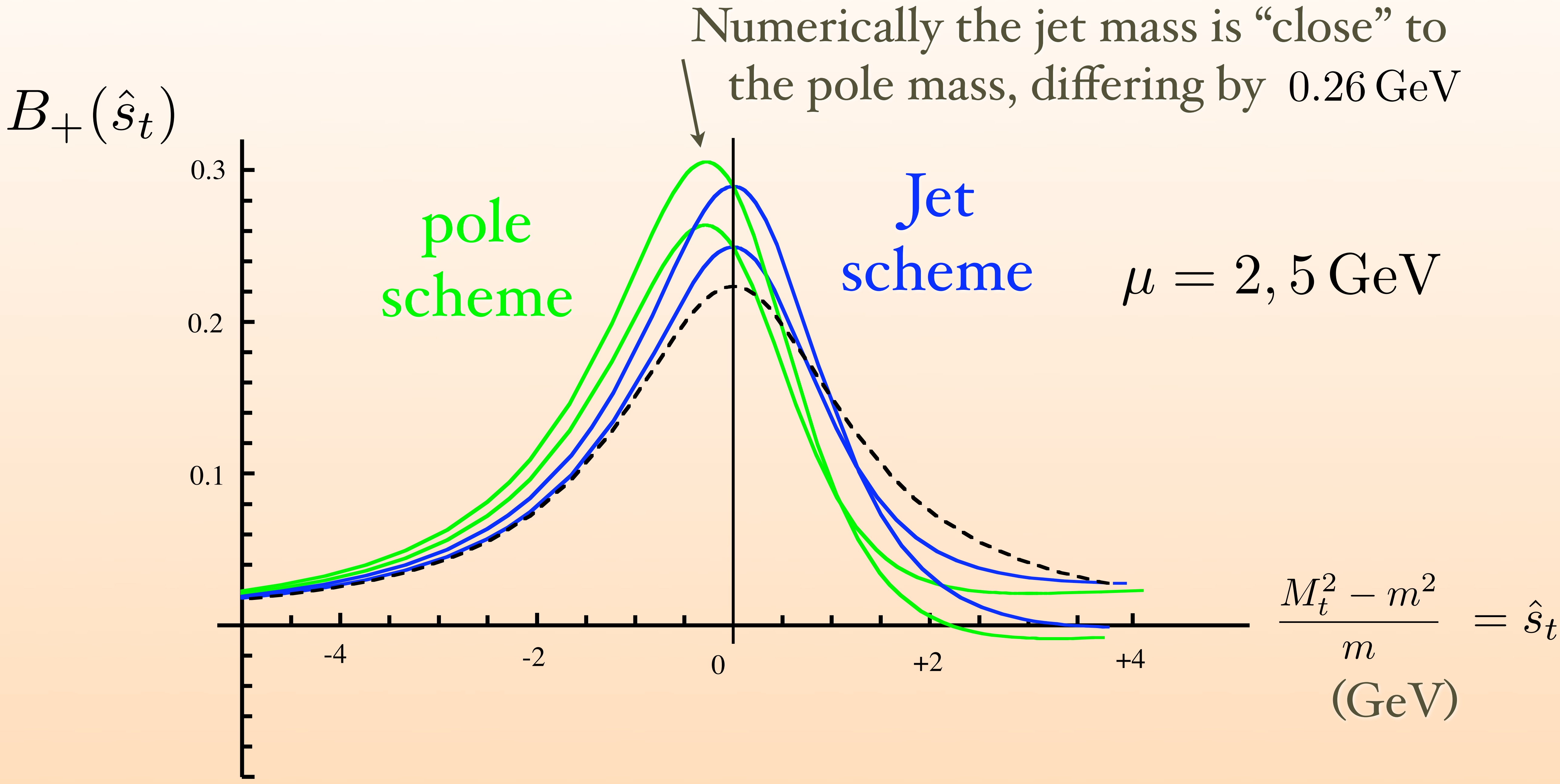
pole mass scheme





We can define a short distance mass scheme, δm, for jets by demanding that the peak of the jet function does not get shifted by perturbation theory.

Short - Distance Jet mass scheme



There is no theoretical obstacle to measuring this jet mass to accuracy better than Λ_{QCD}

Hard Production modes integrated out

“Hard” collinear

Final cross-section with short-dist. mass

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m_J, \frac{Q}{m_J}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\ \tilde{B}_+\left(\hat{s}_t - \frac{Q\ell^+}{m_J}, \Gamma, \mu\right) \tilde{B}_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m_J}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

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$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; \tilde B_+\!\left(\hat s_t - \frac{Q\ell^+}{m_J}, \Gamma, \mu\right) \tilde B_-\!\left(\hat s_{\bar t} - \frac{Q\ell^-}{m_J}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

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Non-perturbative Cross talk

Lets first study the phenomenological implications.

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"Hard" collinear

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Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

Lets first study the phenomenological implications.

I will then come back to prove that the summation of large logs does not significantly affect this phenomenology.

despite the large hierarchy! $Q \gg m \gg \Gamma$

Plots and Analysis

Plots and Analysis

- Soft function is nonperturbative. Can be modeled

$$S_{\text{hemi}}^{\text{M1}}(\ell^+,\ell^-) = \theta(\ell^+)\theta(\ell^-)\frac{\mathcal{N}(a,b)}{\Lambda^2}\left(\frac{\ell^+\ell^-}{\Lambda^2}\right)^{a-1}\exp\left(\frac{-(\ell^+)^2-(\ell^-)^2-2b\ell^+\ell^-}{\Lambda^2}\right)$$

and extracted from massless dijets using universality.

Korchemsky & Tafat
hep-ph/0007005

massless dijet
event shapes

fit soft fn.

$$a = 2\,, \quad b = -0.4$$

$$\Lambda = 0.55\,\mathrm{GeV}$$

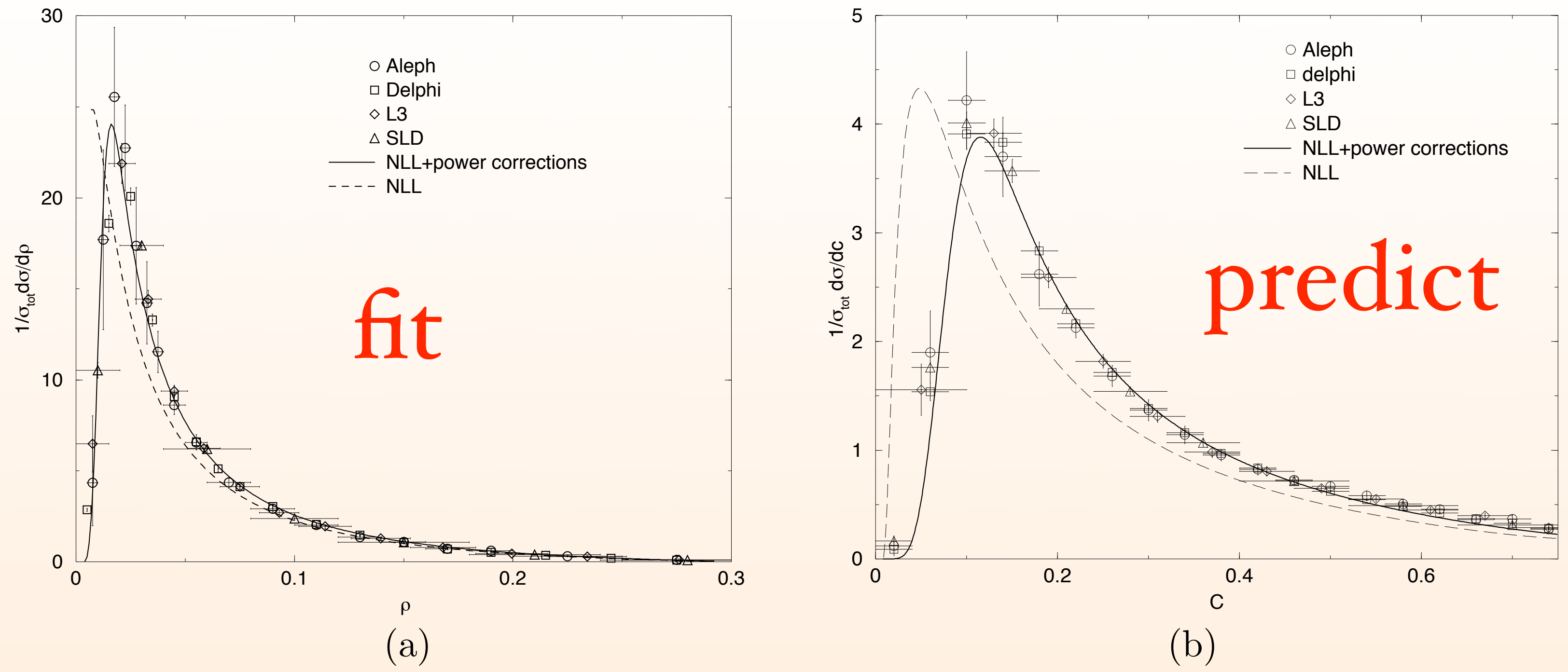


Figure 1: Heavy jet mass (a) and $C-$parameter (b) distributions at $Q = M_Z$ with and without power corrections included.

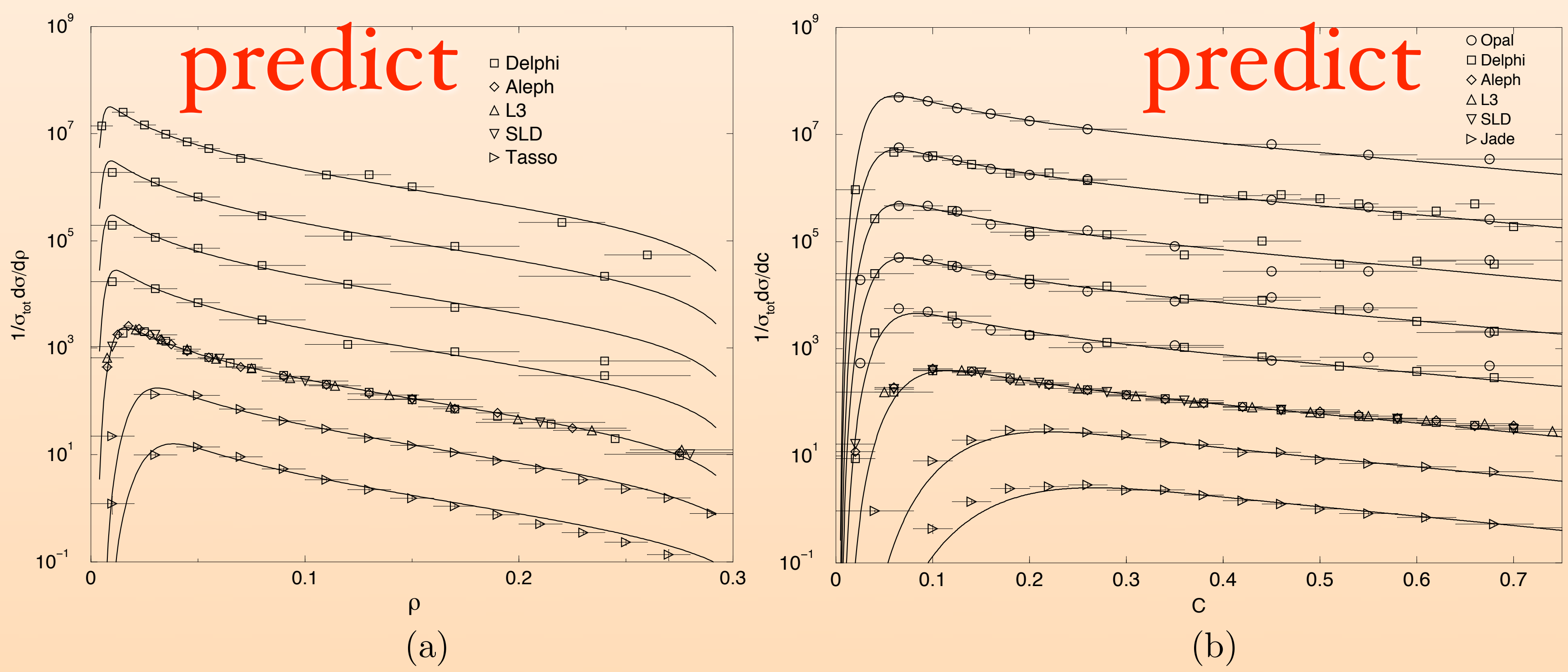
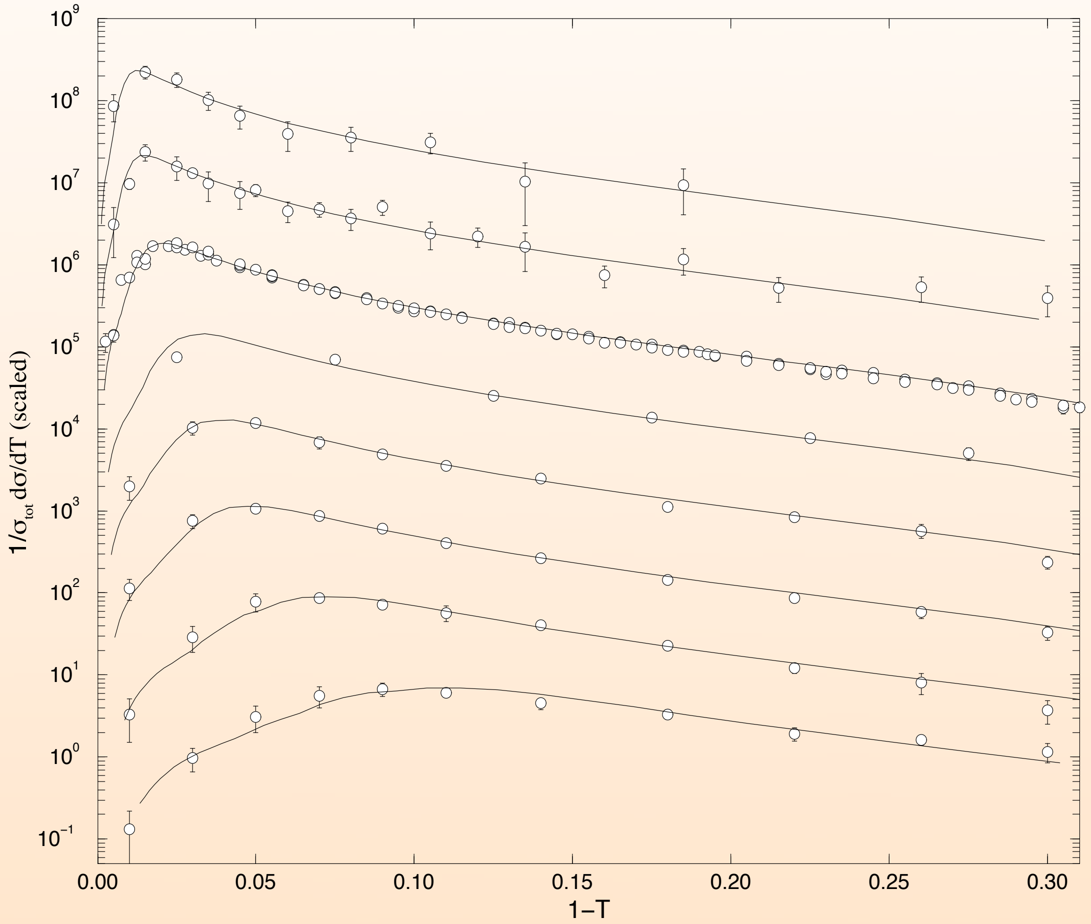


Figure 2: Comparison of the QCD predictions for the heavy jet mass (a) and $C-$parameter (b) distributions with the data at different center-of-mass energies (from bottom to top): $Q/\mathrm{GeV} = 35\,, 44\,, 91\,, 133\,, 161\,, 172\,, 183\,, 189$, based on the shape function.

and thrust too



Korchemsky & Sterman

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

So we can use it to predict the top-invariant mass distribution

$$\frac{d^2\sigma}{dM_t\,dM_{\bar t}} = 4M_t M_{\bar t}\,\sigma_0^H \int_{-\infty}^{\infty} d\ell^+\,d\ell^-\,\tilde B_+\Big(\hat s_t - \frac{Q\ell^+}{m_J},\Gamma,\mu\Big)\tilde B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m_J},\Gamma,\mu\Big) S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

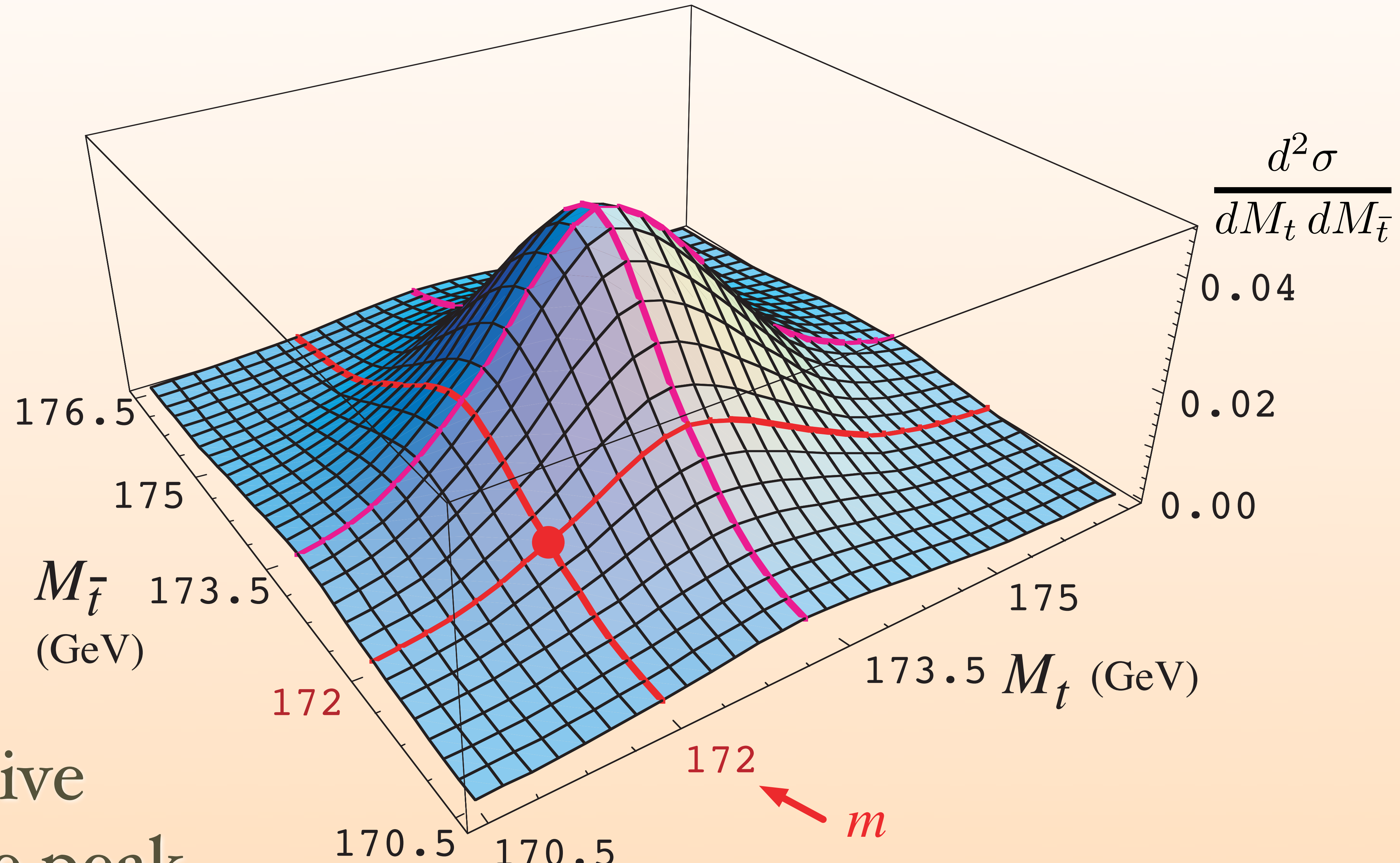
$$\hat s_t = 2M_t - 2m_J\,, \qquad \hat s_{\bar t} = 2M_{\bar t} - 2m_J\,,$$

Start with lowest order

$$\tilde B_+(\hat s_t) = \frac{2}{(m_J\Gamma)}\,\frac{1}{(\hat s_t/\Gamma)^2+1}\,, \qquad \tilde B_-(\hat s_{\bar t}) = \frac{2}{(m_J\Gamma)}\,\frac{1}{(\hat s_{\bar t}/\Gamma)^2+1}$$

Double Differential Invariant Mass Distribution

$Q = 750\,\text{GeV}$

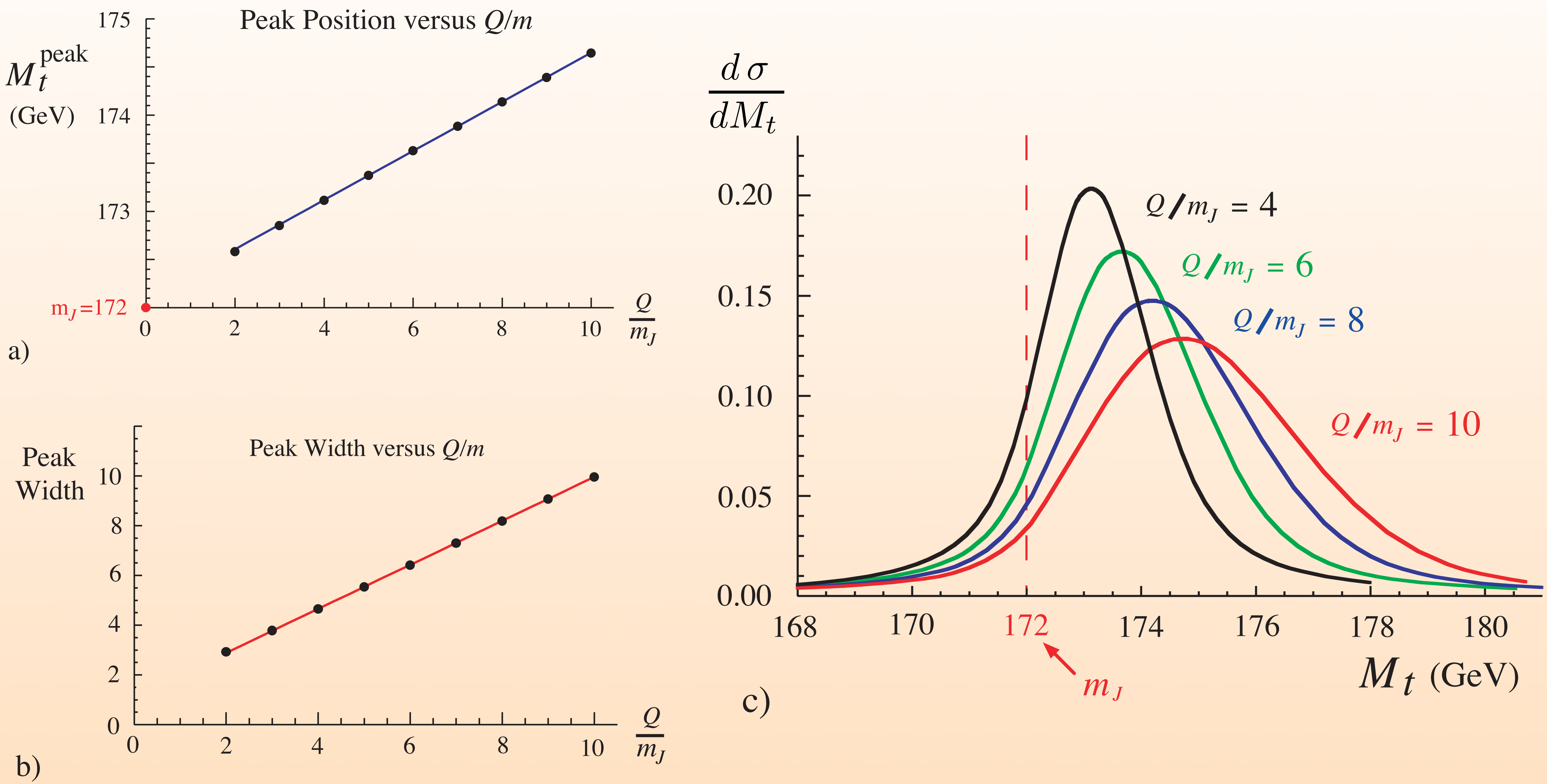


Non-perturbative effects shift the peak position, and broadens the distribution

Simple soft model: very narrow Gaussian centered at $\ell^\pm = \ell_0^\pm$

peak occurs at $M_{t,\bar{t}} \sim m_J + Q\ell_0^\pm/(2m_J)$

Nonperturbative Peak & Width Shifts with Q



Linear growth with Q!

This can be understood analytically:

Mean of distribution: $2L \gg Q\Lambda$

$$F^{(1)} \equiv \frac{1}{m_J^2\Gamma^2}\int_{-L}^{L} ds_t\, \frac{\hat{s}_t}{2}\int_{-\infty}^{\infty} ds_{\bar{t}}\; F(M_t, M_{\bar{t}}) = \int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t\, \frac{\hat{s}_t}{2}\, \tilde{B}_+\Big(\hat{s}_t - \frac{Q\ell^+}{m_J}\Big) \int_{-\infty}^{\infty} d\ell^- S_{\rm hemi}(\ell^+, \ell^-)$$

$$\simeq \frac{1}{2}\int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t\, \Big(\hat{s}_t + \frac{Q\ell^+}{m_J}\Big)\, \tilde{B}_+(\hat{s}_t) \int_{-\infty}^{\infty} d\ell^- S_{\rm hemi}(\ell^+, \ell^-)$$

$$= \frac{Q}{2m_J} S_{\rm hemi}^{(1,0)}$$

slope is $S_{\rm hemi}^{(1,0)} = \int d\ell^+ d\ell^-\; \ell^+ S_{\rm hemi}(\ell^+, \ell^-)$

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$$\simeq \frac{1}{2}\int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t\,\Big(\hat{s}_t + \frac{Q\ell^+}{m_J}\Big)\,\tilde{B}_+(\hat{s}_t)\int_{-\infty}^{\infty} d\ell^- S_{\rm hemi}(\ell^+, \ell^-)$$

$$= \frac{Q}{2m_J} S_{\rm hemi}^{(1,0)}$$

Peak of distribution:

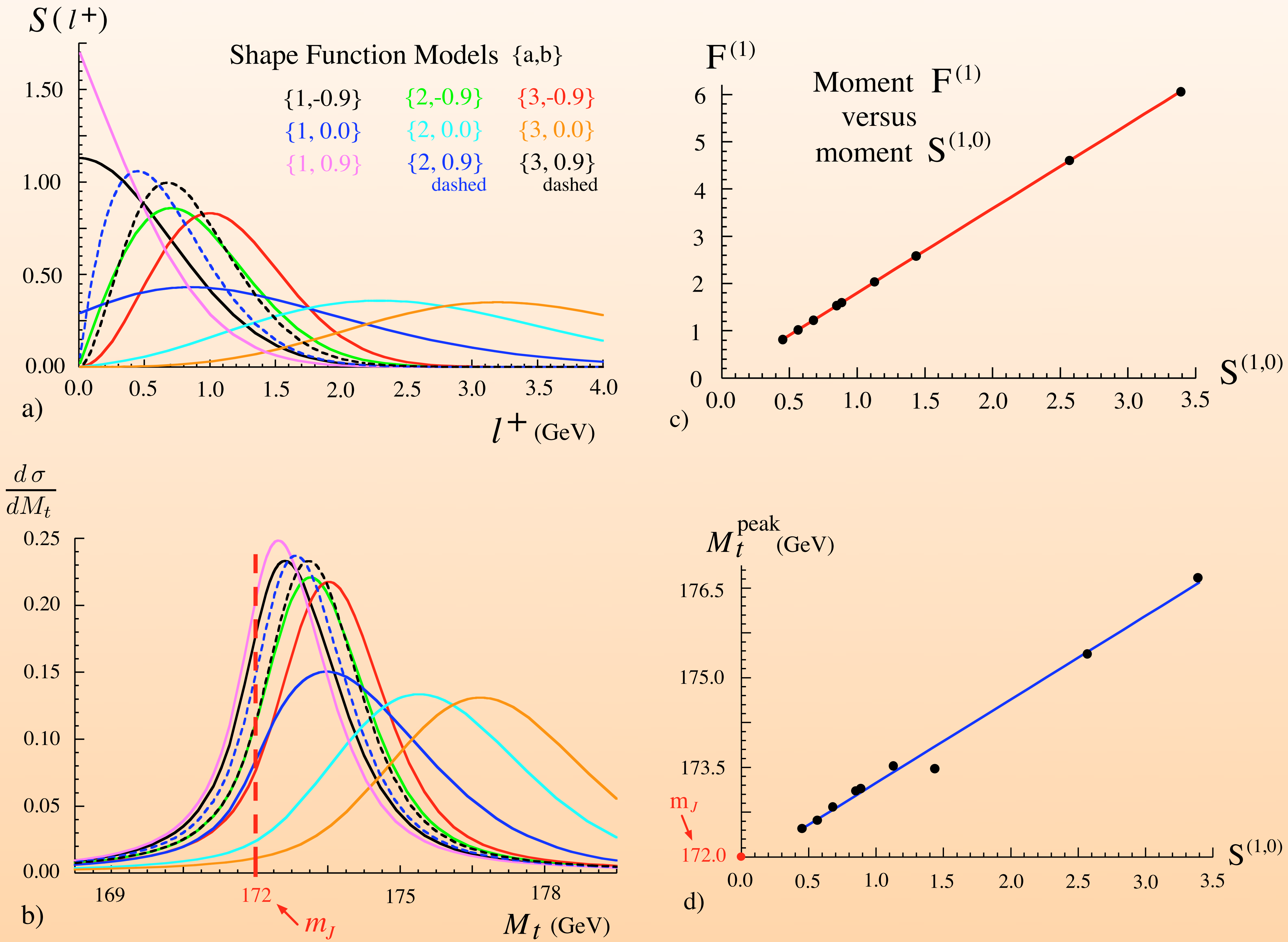
$$0 = \frac{1}{m_J^2\Gamma^2}\int_{-\infty}^{\infty} d\hat{s}_{\bar{t}}\,\frac{dF(M_t, M_{\bar{t}})}{d\hat{s}_t} = \int_{-\infty}^{\infty} d\ell^+ \tilde{B}'_+\Big(\hat{s}_t - \frac{Q\ell^+}{m_J}\Big)\int_{-\infty}^{\infty} d\ell^- S_{\rm hemi}(\ell^+, \ell^-)$$

$$= \int_{-\infty}^{\infty} d\ell^+ \bigg[\Big(\hat{s}_t - \frac{Q\ell^+}{m_J}\Big)\tilde{B}''_+(0) + \frac{1}{3!}\Big(\hat{s}_t - \frac{Q\ell^+}{m_J}\Big)^3 \tilde{B}_+^{(4)}(0) + \ldots\bigg]\int_{-\infty}^{\infty} d\ell^- S_{\rm hemi}(\ell^+, \ell^-)$$

$$\Rightarrow \quad M_t^{\rm peak} \simeq m_J + Q/(2m_J)\, S_{\rm hemi}^{(1,0)}.$$

slope is $S_{\rm hemi}^{(1,0)} = \int d\ell^+ d\ell^-\; \ell^+ S_{\rm hemi}(\ell^+, \ell^-)$

If for some (eg. experimental) reason the universality of the soft function was not applicable then we would need to fit the soft function as well:



Finally, other observables can be projected out from ours.

Thrust $$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

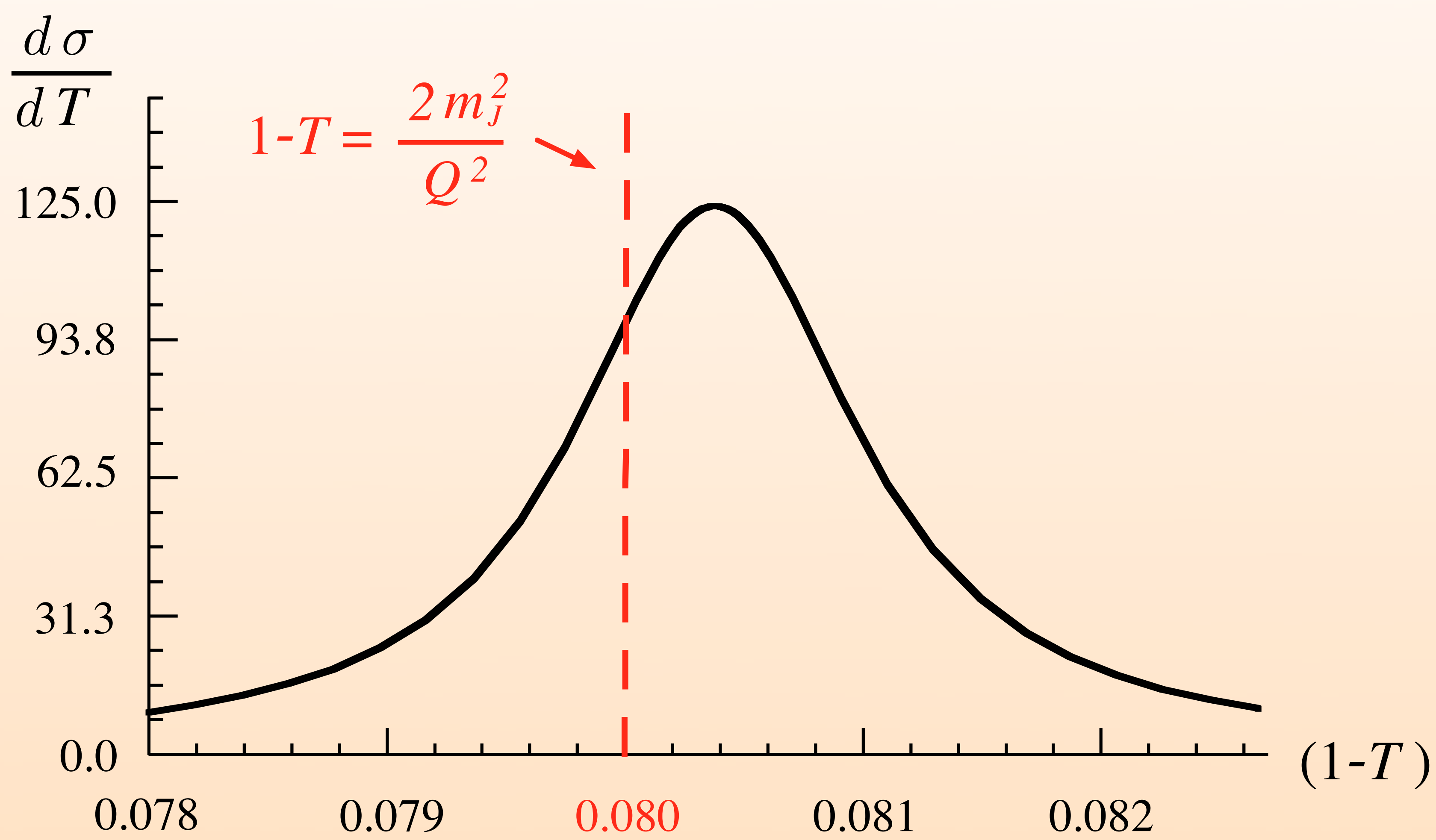
2 massive particles: $T = \sqrt{Q^2 - 4m^2}/Q = 1 - 2m^2/Q^2 + \mathcal{O}(m^4/Q^4)$

Insert: $$1 = \int dT \; \delta\Big(1 - T - \frac{M_t^2 + M_{\bar{t}}^2}{Q^2}\Big)$$

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t \, ds_{\bar{t}} \;\; \tilde{B}_+\Big(\frac{s_t}{m_J}, \Gamma, \mu\Big) \, \tilde{B}_-\Big(\frac{s_{\bar{t}}}{m_J}, \Gamma, \mu\Big) \, S_{\text{thrust}}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar{t}})}{Q^2}, \mu\Big)$$

$$S_{\text{thrust}}(\tau, \mu) = \int_0^{\infty} d\ell^+ \, d\ell^- \delta\Big(\tau - \frac{(\ell^+ + \ell^-)}{Q}\Big) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

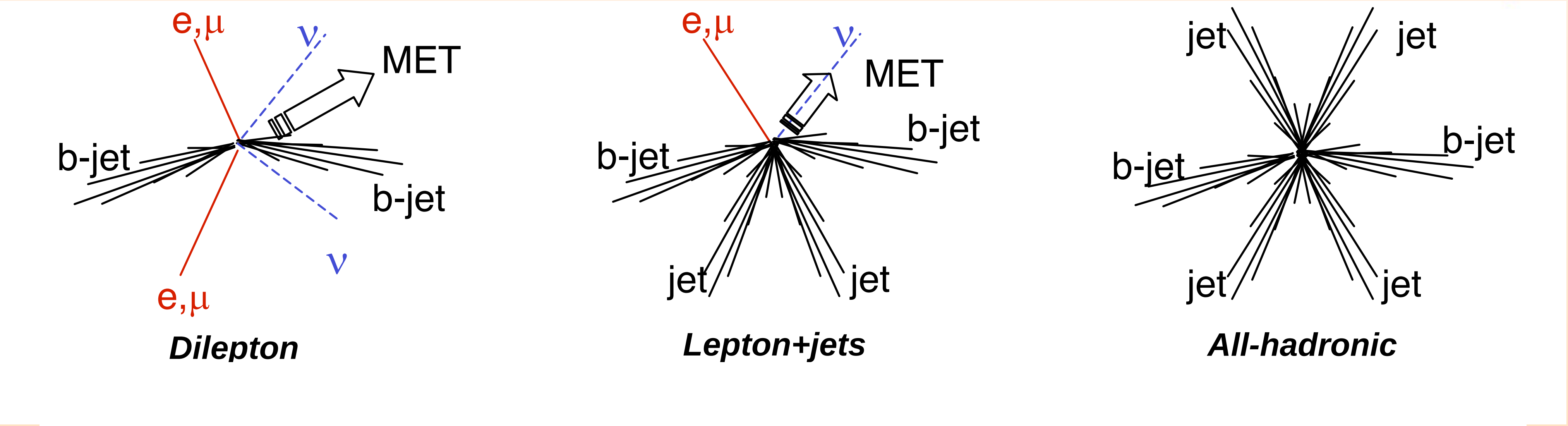
Thrust Distribution



What about using a Jet Algorithm?

If all soft radiation is grouped into the jets (inclusive mode) then the factorization theorem is the same, but has a different soft function.

What about using a Jet Algorithm?

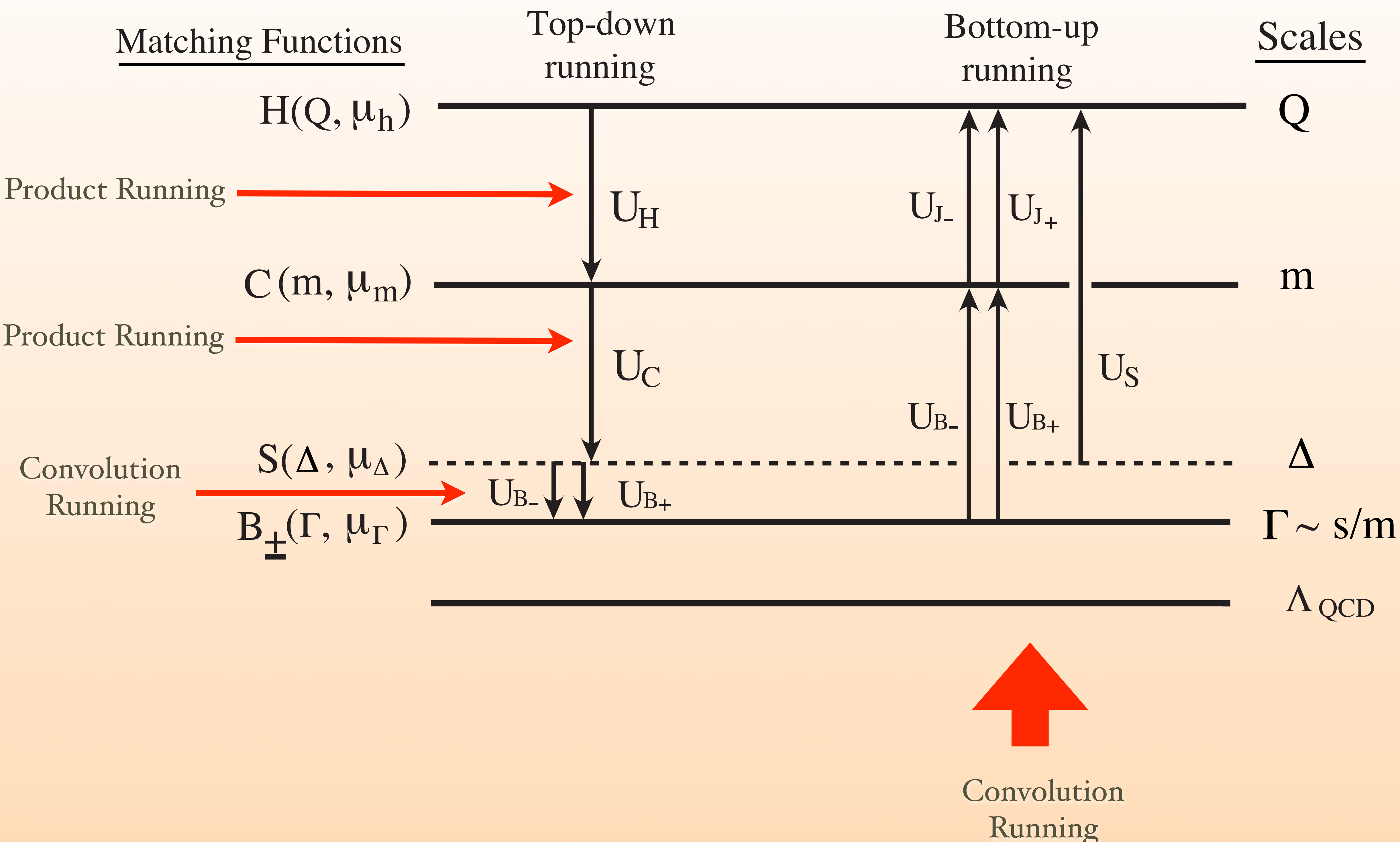


If all soft radiation is grouped into the jets (inclusive mode) then the factorization theorem is the same, but has a different soft function.

Log resummation

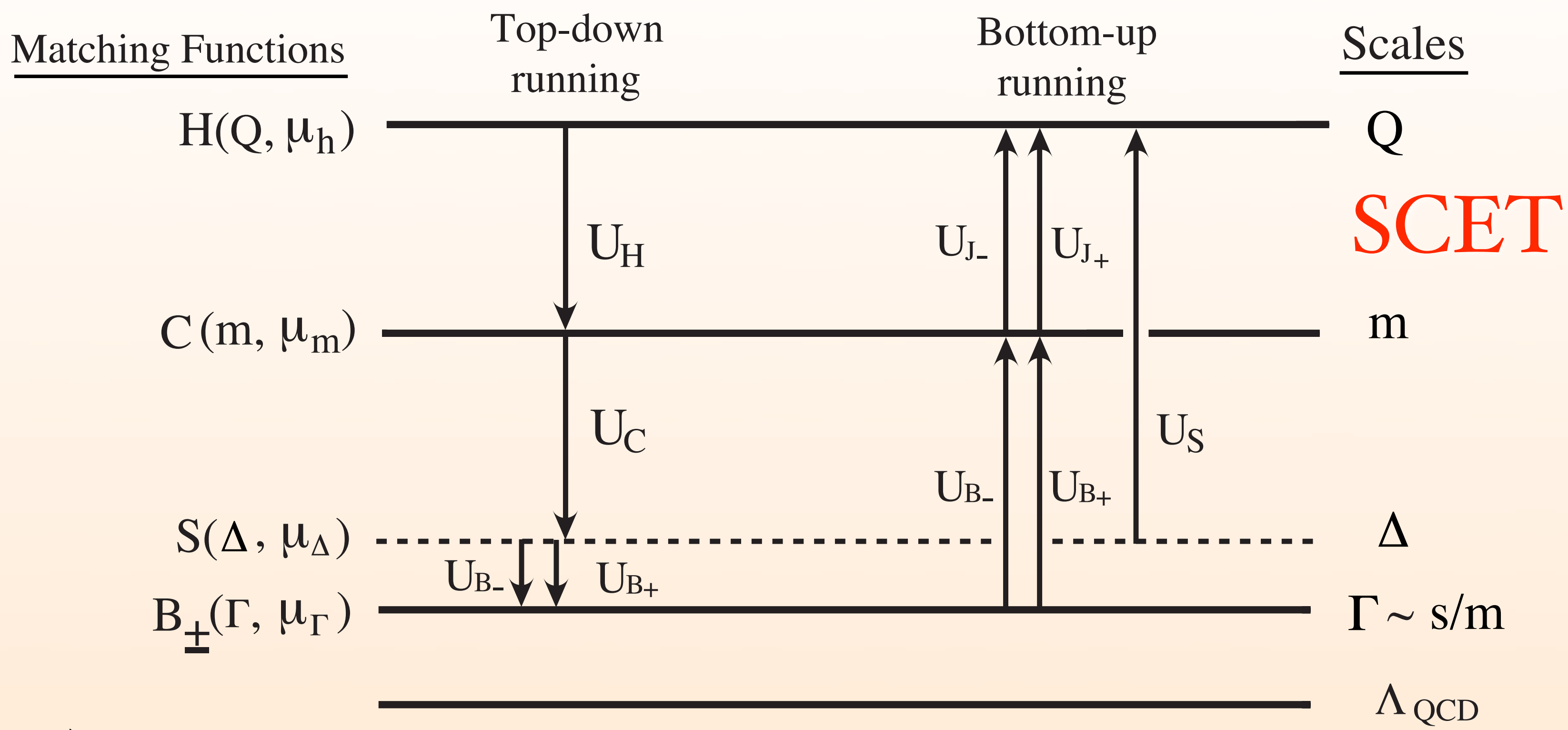
from renormalization of UV divergences in the effective field theories, which induce anomalous dimensions.

Log resummation



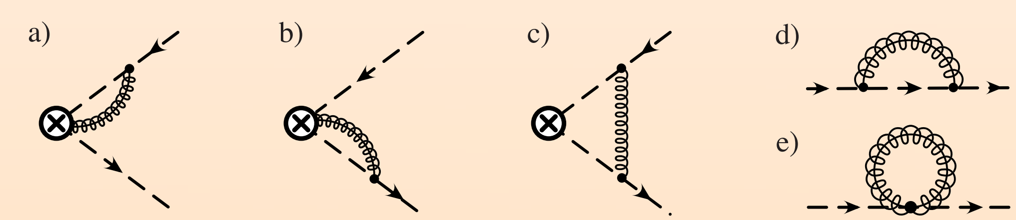
$$\mu\frac{d}{d\mu}B_{\pm}(\hat{s},\mu) = \int d\hat{s}'\,\gamma_{B_{\pm}}(\hat{s}-\hat{s}')\,B_{\pm}(\hat{s}',\mu)$$

SCET Log resummation



top-down:

$$\mu \frac{d}{d\mu} H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$$

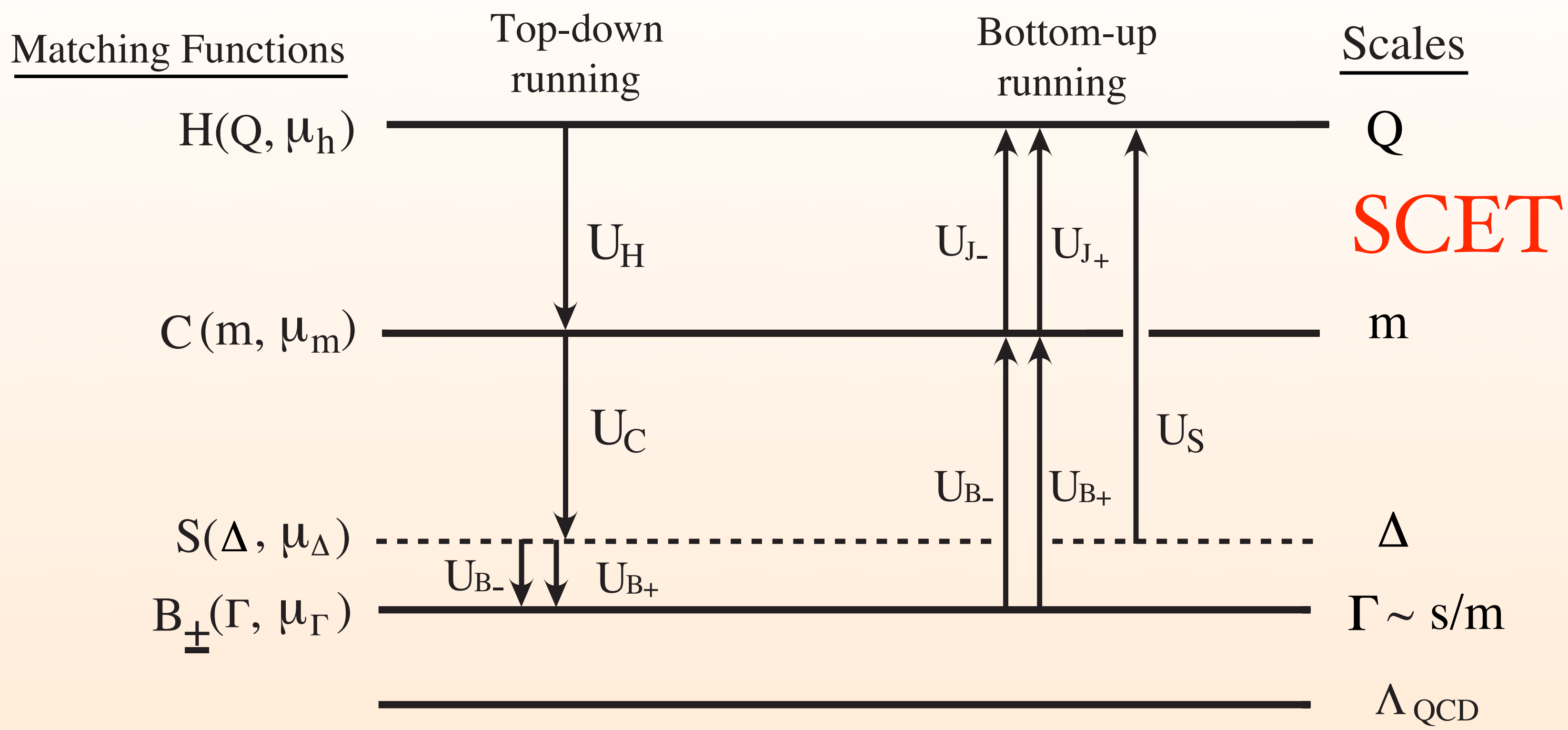


$$H_Q(Q,\mu) = U_{H_Q}(\mu,\mu_h)\, H_Q(Q,\mu_h)$$

$$U_{H_Q}(\mu,\mu_Q) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_Q)}\right]^{\frac{-16\pi C_F}{\beta_0^2\,\alpha_s(\mu_Q)}+\frac{6C_F}{\beta_0}} \left(\frac{\mu}{\mu_Q}\right)^{-8C_F/\beta_0} \left(\frac{\mu_Q}{Q}\right)^{\frac{8C_F}{\beta_0}\ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_Q)}\right)}$$

- Product of soft and collinear jet functions run locally all the way down to the low scale.
- This local running only affects the normalization of the distribution.

SCET Log resummation



bottom-up:

$$\mu\frac{d}{d\mu}J_{n,\bar{n}}(s,\mu) = \int ds'\, \gamma_{J_{n,\bar{n}}}(s-s')\, J_{n,\bar{n}}(s',\mu)$$

$$J_n(s,\mu) = \int ds'\, U_{J_n}(s-s',\mu,\mu_m)\, J_n(s',\mu_m)\,,$$

$$U_{J_n}(s-s',\mu,\mu_m) = \frac{e^{L_1}\left(\mu_m^2\, e^{\gamma_E}\right)^{\omega_1}}{\Gamma(-\omega_1)}\left[\frac{\theta(s-s')}{(s-s')^{1+\omega_1}}\right]_+$$

$$\omega_1(\mu,\mu_m) = -\frac{4C_F}{\beta_0}\,\ln\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_m)}\right]$$

$$\mu\frac{d}{d\mu}S(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^-\, \gamma_S(\ell^+-\ell'^+,\ell^- -\ell'^-)S(\ell'^+,\ell'^-,\mu)$$

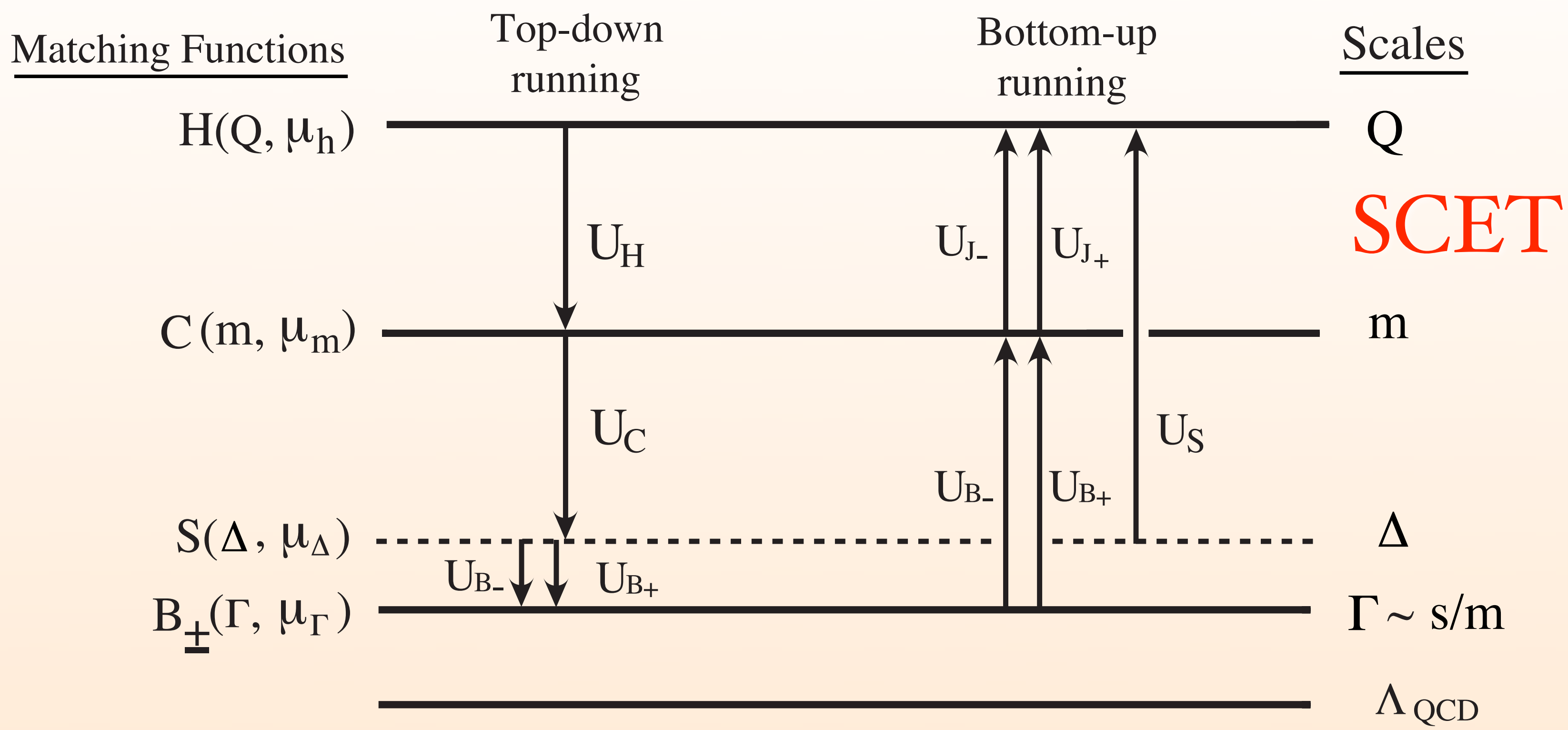
$$\omega_1+\omega_2=0$$

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^-\, U_S(\ell^+-\ell'^+,\ell^- -\ell'^-,\mu,\mu_m)\, S_{\rm hemi}(\ell'^+,\ell'^-,\mu_m)$$

$$U_S(\ell^+,\ell^-,\mu,\mu_0) = \frac{e^{2L_2}\left(\mu_m\, e^{\gamma_E}\right)^{2\omega_2}}{\Gamma(-\omega_2)^2}\left[\frac{\theta(\ell^+)}{(\ell^+)^{1+\omega_2}}\right]_+\left[\frac{\theta(\ell^-)}{(\ell^-)^{1+\omega_2}}\right]_+$$

$$\omega_2(\mu,\mu_m) = \frac{4C_F}{\beta_0}\,\ln\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_m)}\right]$$

SCET Log resummation



consistency:

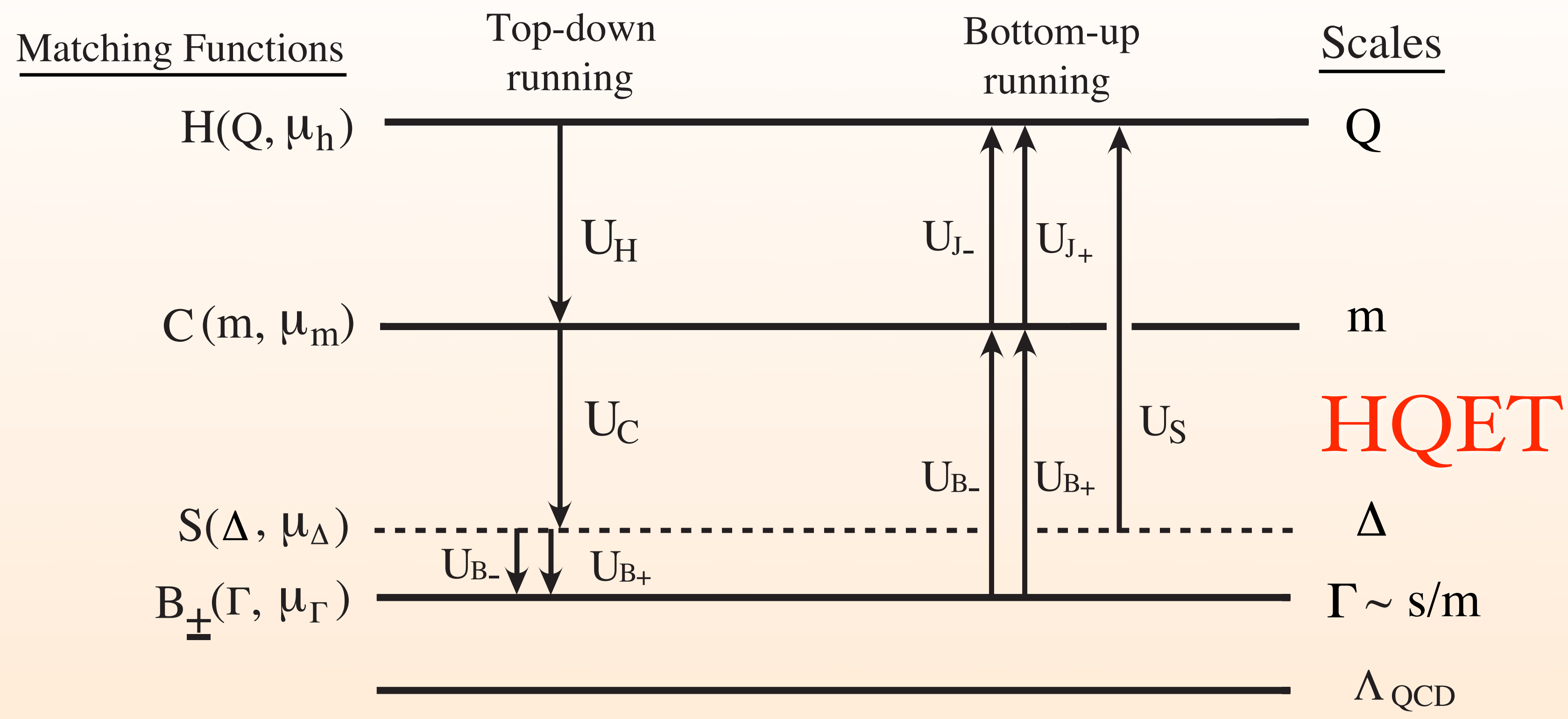
$$U_{H_Q}(\mu,\mu_m)\,\delta(s-Q\ell'^{+})\,\delta(\bar{s}-Q\ell'^{-})$$

$$\omega_1+\omega_2=0$$

$$=\int d\ell^{+}d\ell^{-}\,U_{J_n}(s-Q\ell^{+},\mu,\mu_m)U_{J_{\bar n}}(\bar{s}-Q\ell^{-},\mu,\mu_m)U_S(\ell^{+}-\ell'^{+},\ell^{-}-\ell'^{-},\mu,\mu_m)$$

cancellation between soft & collinear factors

HQET Log resummation



top-down:

$$\mu \frac{d}{d\mu} H_m\Big(m, \frac{Q}{m}, \mu\Big) = \gamma_{H_m}\Big(\frac{Q}{m}, \mu\Big)\, H_m\Big(m, \frac{Q}{m}, \mu\Big)$$

$$H_m(\mu) = U_{H_m}(\mu, \mu_m) H_m(\mu_m)$$

bottom-up:

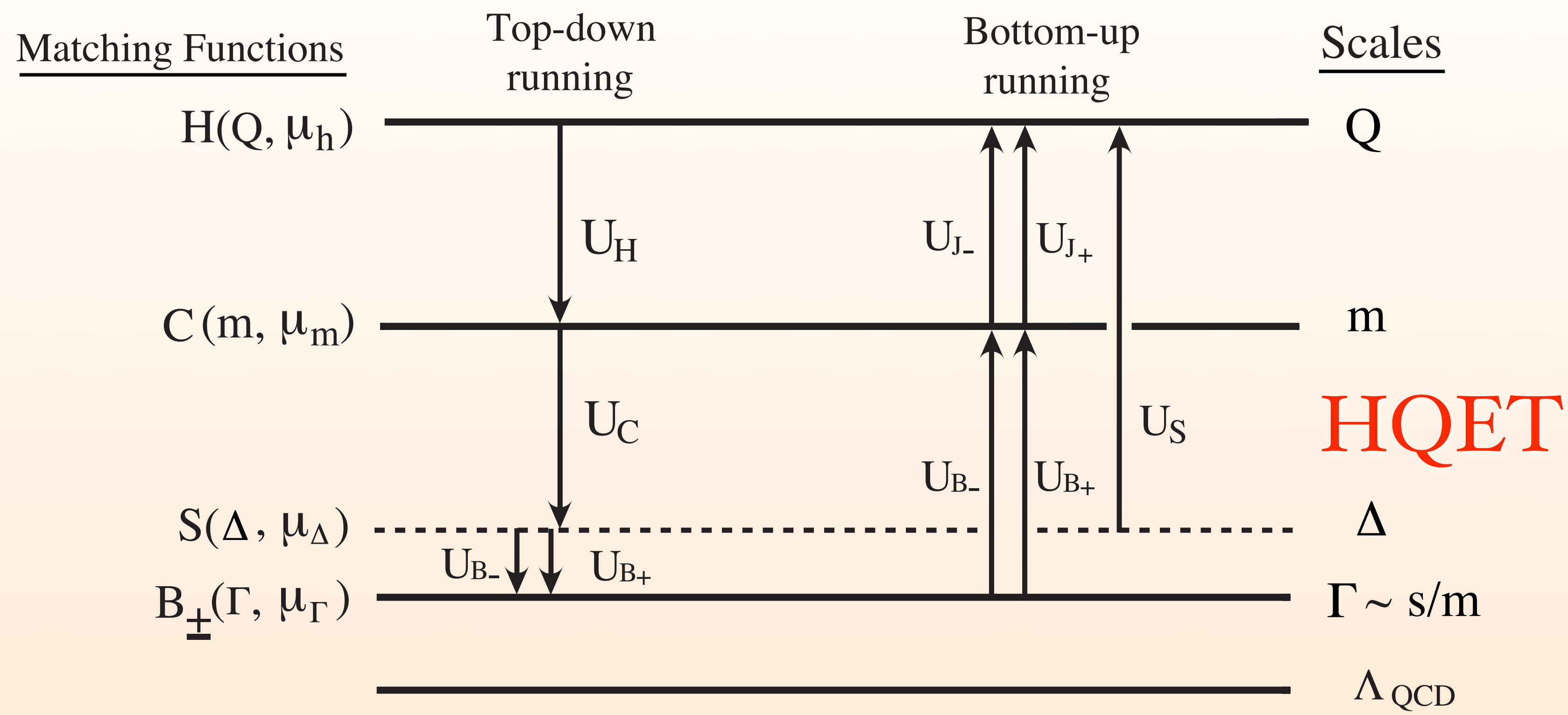
$$B_\pm(\hat{s}, \mu) = \int d\hat{s}'\; U_{B_\pm}(\hat{s}-\hat{s}', \mu, \mu_\Gamma)\; B_\pm(\hat{s}', \mu_\Gamma)$$

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^-\; U_S(\ell^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_m)\; S_{\rm hemi}(\ell'^+, \ell'^-, \mu_m)$$

similar to SCET

$$\omega_1 + \omega_2 = 0$$

HQET Log resummation



consistency:

$$U_{H_m}(\mu,\mu_\Delta)\,\delta\Big(\hat{s}-\frac{Q\ell'^+}{m}\Big)\,\delta\Big(\hat{\bar{s}}-\frac{Q\ell'^-}{m}\Big)$$

$$\omega_1+\omega_2=0$$

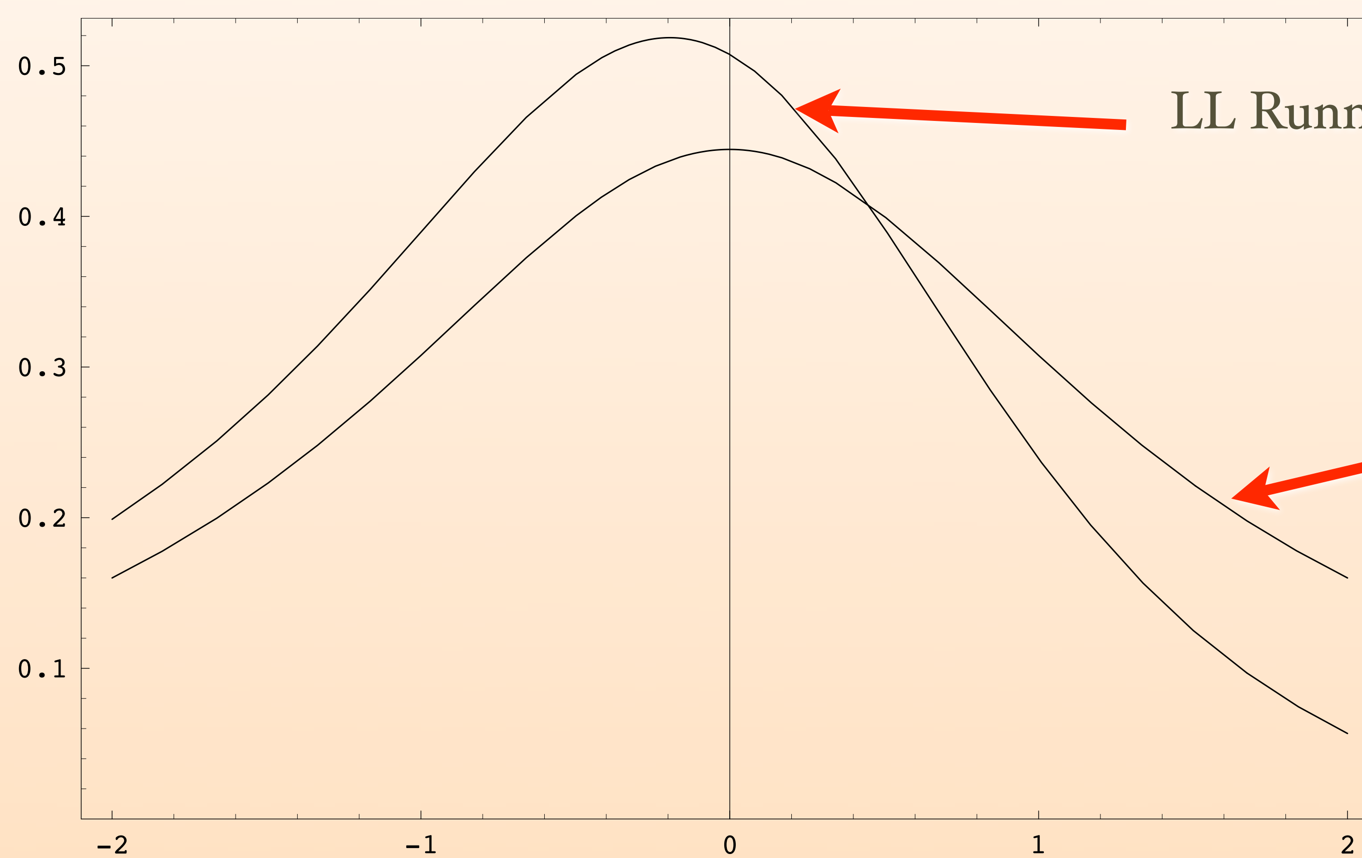
$$=\int d\ell^+ d\ell^-\; U_{B_+}\Big(\hat{s}-\frac{Q\ell^+}{m},\mu,\mu_\Delta\Big)U_{B_-}\Big(\bar{s}-\frac{Q\ell^-}{m},\mu,\mu_\Delta\Big)U_S(\ell^+-\ell'^+,\ell^--\ell'^-,\mu,\mu_\Delta)$$

cancellation between soft & collinear factors again

an observable that did not account for the soft radiation would not have this property.

BHQET Jet Function $B_\pm(\hat{s},\mu)$

LL running in our case large logs do not effect the normalization



$$\mu = \Gamma$$

$$\frac{M_t^2 - m^2}{m} \quad \text{(GeV)}$$

$$B_\pm(\hat{s},\mu) = \int d\hat{s}'\; U_{B_\pm}(\hat{s}-\hat{s}',\mu,\mu_\Gamma)\; B_\pm(\hat{s}',\mu_\Gamma)$$

$$U_{B_\pm}(s-s',\mu,\mu_i) = \frac{e^{L_2(\mu,\mu_i)}\left(m\,\mu_i e^{\gamma_E}\right)^{\omega_1}}{\Gamma(-\omega_1)}\left[\frac{\theta(s-s')}{(s-s')^{1+\omega_1}}\right]_+ , \qquad \omega_1(\mu,\mu_i) = -\frac{4C_F}{\beta_0}\ln\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_i)}\right]$$

Lessons, Implications, and Conclusion

- Factorization allows us to keep track of how the observable effects corrections from other categories (hadronization, final state radiation, etc.)
- In our analysis the inclusive nature of the hemisphere mass definition reduces the uncertainty from hadronization. The jet functions sum over hadronic states up to $m\Gamma$ and are perturb. The soft functions is universal. If we switch observables (eg. like thrust) we can in some cases relate the soft functions.
- Gluon radiation between the decay products is power suppressed
- Summation of Large Logs, control of final state radiation
- Definition of a short-distance mass scheme for jets
- Results are observable dependent and will be different for the LHC. The corr. analysis may help reduce uncertainties.

The END