New Phenomena in 2d String Theory

Nathan Seiberg
Rutgers 2005

N.S.	hep-th/0502156
J.L. Davis, F. Larsen, N.S.	hep-th/0505081, and to appear
J. Maldacena, N.S.	hep-th/0506141

Low Dimensional String Theories

Matrix models give complete nonperturbative definitions of some string theories – only known well defined string theories which are exactly solvable.
They can be used as laboratories for new stringy effects.

Matrix model/Traditional string description (worldsheet) duality. Only example where the two sides of the duality are calculable.

Minimal String Theories ($c < 1$)

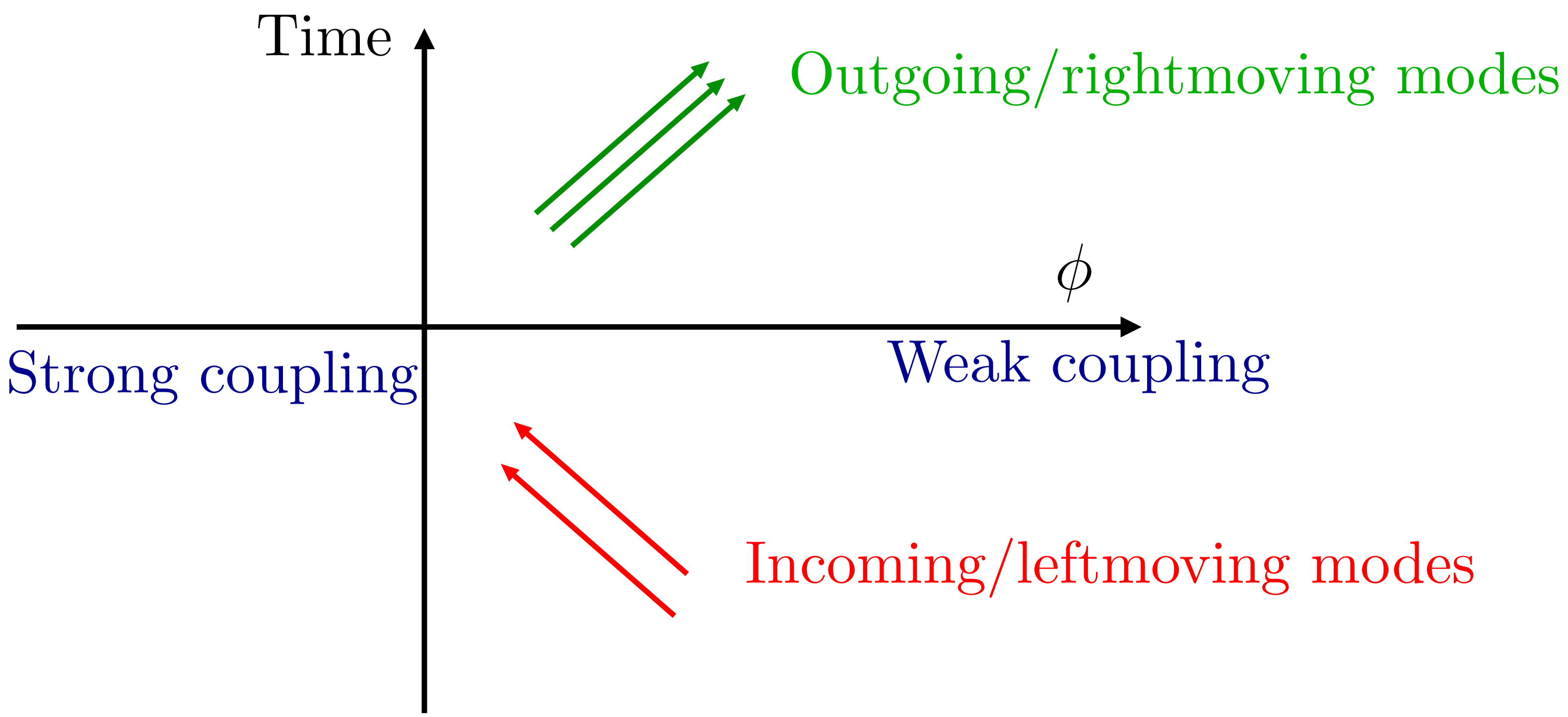
They describe strings in one Euclidean dimension (many examples).

They exhibit: D-branes, holography, RR-flux, connections to integrable systems, topological strings...

Two Dimensional Theories

The two dimensional theories have time and hence are richer.

Large spectrum of theories (many of them are new). Some of them have matrix model descriptions, some don't.



Scattering is from and to null infinity at the weak coupling end (the strong coupling region is effectively compact).

Some of the lessons of recent work

We can focus on the weak coupling asymptotic end of the target space. The local physics there is correctly described by weak coupling (worldsheet) methods.

There is no other asymptotic region of spacetime – the strong coupling end is effectively compact.

A worldsheet parameter μ plays the role of $\hbar$ ($\hbar \sim 1/\mu^2$). In nonperturbatively stable theories all the observables are smooth as μ changes from positive to negative – the $\mu = 0$ theory is not singular.

The bosonic, 0A and 0B string theories have known formulations in terms of matrix models which allow us to explore their strong coupling region.

We will discuss some of their peculiar properties:

- Massless nonperturbative excitations
- Interpretation of background RR flux
- Necessity of background long strings

We will discuss also other theories: IIA, IIB, HO, HE and THO (no known matrix model). We will focus on the simple physics in the weak coupling region.

They raise many issues including:

- The excitations visible in the worldsheet cannot have a unitary S-matrix – need massless solitons.
- New anomaly cancellation mechanism.
- New stringy phase transitions – peculiar thermodynamics?

Spectrum of the Simplest Theories

Bosonic: massless “tachyon” $T(p)$

0A: massless “tachyon” $T(p)$

0B: massless “tachyon” $T(p)$
massless RR scalar $C(p)$
(nonperturbative massless solitons of C)

More details about 0B

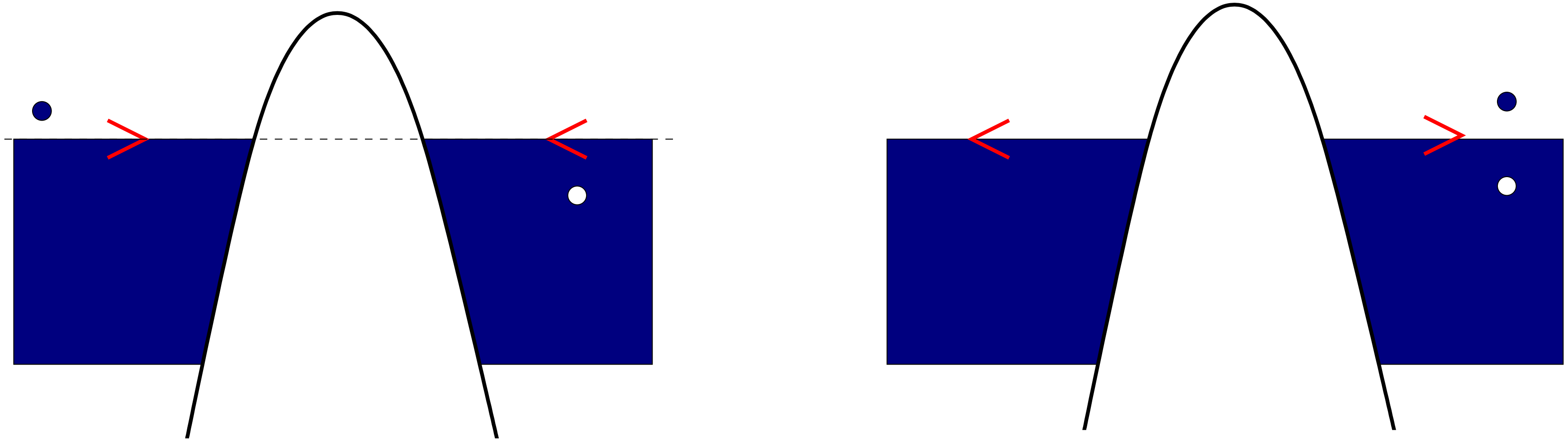
The massless RR scalar C is compact at the selfdual radius [Douglas, Klebanov, Kutasov, Maldacena, Martinec, N.S.].

Therefore, the two dimensional theory has two incoming and two outgoing "solitonic excitations" $e^{\pm i\sqrt{2}C_{left}}$, $e^{\pm i\sqrt{2}C_{right}}$.

They carry RR charges $q_{in,out} = \oint \partial_\pm C$. Unlike other such nonperturbative excitations, they are masselss.

Matrix model description

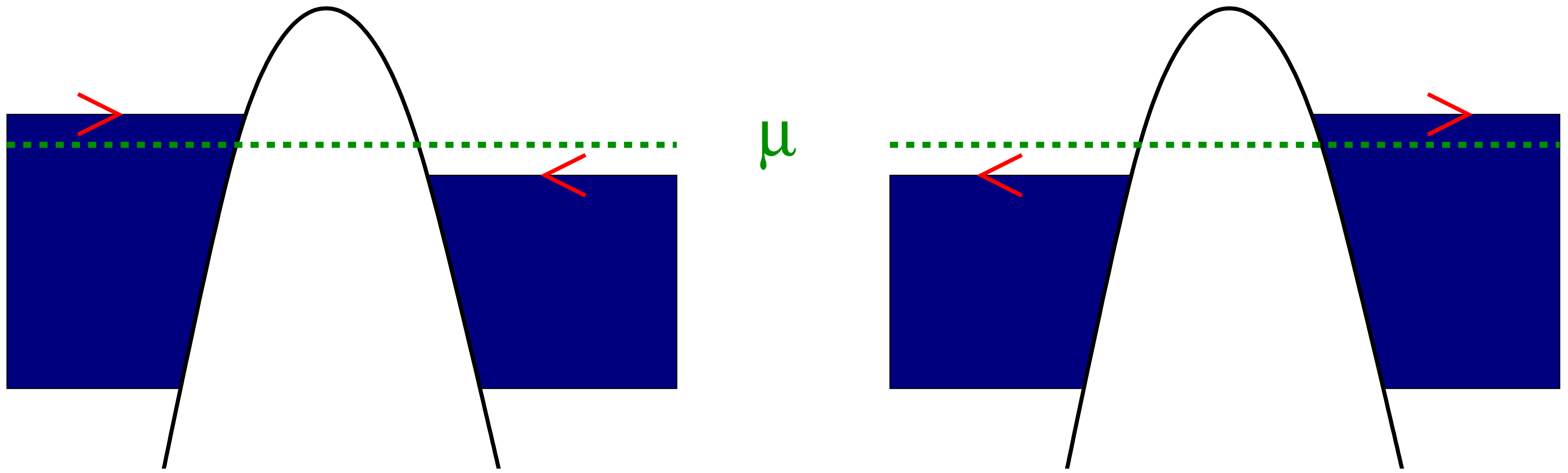
[DeWolfe, Roiban, Spradlin, Volovich, Walcher]



Here we scatter an incoming state with a soliton $q_{in} = 1$ to an outgoing state without a soliton $q_{out} = 0$.

RR-flux

A macroscopic number of solitons in the incoming and/or outgoing states correspond to unequal Fermi levels in the incoming state $\mu \pm \nu_{in}$ and/or the outgoing state $\mu \pm \nu_{out}$.



In spacetime this is RR-flux: $\langle \partial_\pm C \rangle = \nu_{in,out}$.

Particle number and energy conservation leads to
$\nu_{in} = \pm\nu_{out}$.

If this is not the case, we need to add tachyons in the past or the future to balance the energy. Otherwise the S-matrix element vanishes.

RR-flux in the 0A theory

The 0A theory has two gauge fields. Each of them has only one degree of freedom – its background electric field q, $\widetilde{q}$. These RR-fluxes are T-dual after compactification to the two 0B fluxes: q and $\widetilde{q}$ are dual to $q_{in} \pm q_{out}$.

As in 0B ($\nu_{in} = \pm\nu_{out}$), we need $q\widetilde{q} = 0$. If this is not the case, flux conservation forces us to add $q\widetilde{q}$ background fundamental strings stretched across the target space.

Type II

Orbifold the type 0 theories by leftmoving worldsheet fermion number. $T(p)$ and $C_-(p)$ are projected out. The twisted sectors have spacetime fermions.

IIA: Majorana fermion	$\Psi_-(p \leq 0)$	$\longleftarrow$
	$\Psi_+(p \geq 0)$	$\longrightarrow$
IIB: Weyl fermion	$\Psi_-(p \leq 0)$	$\longleftarrow$
	$\widetilde{\Psi}_-(p \leq 0)$	$\longleftarrow$
Chiral scalar	$C_+(p \geq 0)$	$\longrightarrow$

Comments

The projection in the twisted sectors is opposite to 10d.

IIA is worldsheet chiral; IIB is spacetime chiral.

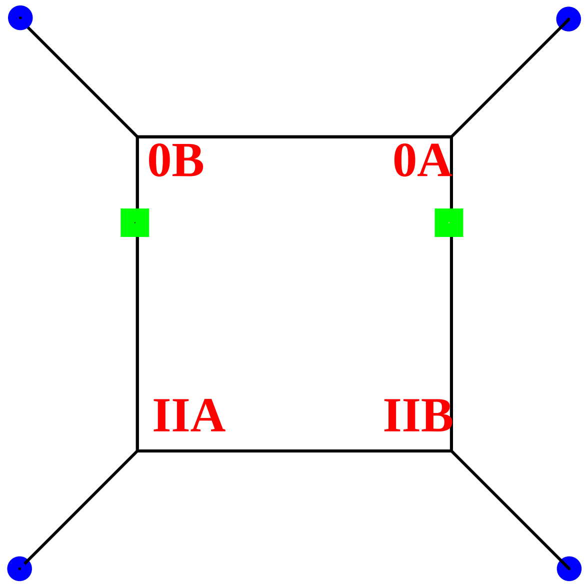
Finite number of particles – no Hagedorn density of states.

No unitary S-matrix of these excitations!
Expect: additional massless excitations – solitons made out of the chiral scalar C.

Compactifications

Compactify Euclidean time $x \sim x + 2\pi R$

Each of these theories has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry which we can twist by. The moduli space



Comments

All four theories are connected.

T duality relates different theories or a theory to itself.

The circles are selfdual points with continuous symmetry.

The squares were introduced in [Kutasov and NS].

Physical vertex operators – states in 1d – have either momentum ($w = 0$) or winding ($p = 0$). All of them except $p = w = 0$ are massive.

Torus Amplitudes

$$\Gamma = aR + \frac{b}{R}$$

$a =$ vacuum energy density – independent of the compactification. It is infinite in field theory but finite in string theory.

b is calculable in field theory as $\sum |p|$. For thermal circles ($R \sim \frac{1}{T}$) b measures the number of degrees of freedom.

T duality relates a and b of different compactifications. Therefore, a can be calculated as $\sum |w|$.

2d Heterotic String

First discussed in [McGuigan, Nappi and Yost].

HO: has $Spin(24)$ symmetry,

24 massless "tachyons" $T^I(p)$ $\leftrightarrows$

HE: has $Spin(8) \times E_8$ symmetry,

$\mathbf{8_v}$ massless "tachyons" $T^i(p)$ $\leftrightarrows$

$\mathbf{8_s}$ leftmoving fermions $\Psi^\alpha(p \geq 0)$ $\longleftarrow$

$\mathbf{8_c}$ rightmoving fermions $\widetilde{\Psi}^{\dot{\alpha}}(p \leq 0)$ $\longrightarrow$

Again, no Hagedorn density of states.

Twisted HO (THO)

Orbifold the HO theory by worldsheet fermion number

HO: has $Spin(24)$ symmetry,

24 massless "tachyons" $T^I(p)$ $\leftrightarrows$

THO: has $Spin(24)$ symmetry,

24 rightmoving fermions $\widetilde{\Psi}^I(p \leq 0)$ $\longrightarrow$

This theory is anomalous. However, the anomaly can be cancelled by adding a stretched fundamental heterotic string. Its 24 leftmoving fermions cancel the anomaly.

Equivalently, the 2d version of the Green-Schwarz mechanism involves adding the term $\int B$ to the action. The B tadpole is cancelled by the long string.

Compactifications

Depending on the radius and Wilson lines

$$\mathcal{M} = SO(13,1,\mathbb{Z})\backslash SO(13,1)/SO(13)$$

Infinite number of states with both p and w (not only pure p or w)!
No tachyons.

Examples: the thermal circles are selfdual

$$Spin(24) \times U(1) \to Spin(26)$$

$$Spin(8) \times E_8 \times U(1) \to Spin(10) \times E_8$$

Torus Amplitude

The torus amplitude depends on the moduli in $\mathcal{M}$.

Consider, for example, the HE theory on a thermal circle

$$\Gamma = \begin{cases} \frac{1}{R} & R \geq 1 \\ R & R \leq 1 \end{cases}$$

Note, the vacuum energy density (a) vanishes.
As expected, it is T-dual ($R \to \frac{1}{R}$), but it not smooth at the selfdual point $R = 1$!

Thermodynamics

The one loop approximation of $\mathrm{Tr}\, e^{-H/T}$ is smooth as a function of $T \sim 1/R$ (no Hagedorn), but it is not T-dual!

The standard proof that $\mathrm{Tr}\, e^{-H/T}$ = Euclidean circle amplitude is valid only for sufficiently small T (after Poisson resummation $[\int, \sum] \neq 0$ beyond some T).

The Euclidean time torus amplitude and $\mathrm{Tr}\, e^{-H/T}$ differ for small R!

$$\Gamma = \begin{cases} \dfrac{1}{R} & R \geq 1 \\ R & R \leq 1 \end{cases}$$

$$-\frac{F}{T} = \frac{1}{R} \qquad 0 < R < \infty$$

Physics of the Transition

The transition is driven by the $p = w/2 = \pm 1$ modes with $m(R) = \frac{1}{2}|R - \frac{1}{R}|$.
They extend the **8** of $Spin(8)$ tachyons to **10** of $Spin(10)$ at the selfdual point.
Their effective action

$$\mathcal{L}_\Phi = \frac{1}{2}|\partial_\phi \Phi|^2 + \frac{1}{2}m(R)^2|\Phi|^2$$

leads to

$$Z_\Phi = -\int \frac{dp}{2\pi} \log(p^2 + m(R)^2) = -|m(R)| + \text{const}$$

which is not analytic in R!

Euclidean Circle $\stackrel{?}{=}$ Temperature

If yes:

First order transition with negative latent heat

Lower entropy for higher T [Atick and Witten]

Standard thermodynamics inequalities are not satisfied

(is the system unstable? to what?)

If no:

Is T meaningful above the transition point T_c?

Long strings can explain the transition...

Conclusions

There are many interesting 2d string theories.

New phenomena: chiral spectrum, massless nonperturbative states, importance of added long stretched fundamental strings, new phase transitions, peculiar thermodynamics

It will be nice to have nonperturbative formulations (e.g. matrix models) of these theories.