BLACK HOLE PRODUCTION, THE FROISSART BOUND, THE SOFT POMERON AND THE RHIC FIREBALL

FROM ADS-CFT.

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1. Introduction

High energy QCD

- fixed t, $s \to \infty$ $\sigma_{tot}(s) \sim s^{0.093(2)}$ (experim
  "soft Pomeron"
  - is not unitary at $s \to \infty$.

. Unitary behaviour: (Froissart bound)

$$\sigma_{tot}(s) \sim \frac{a}{M^2} \ln^2 s/s_0 \quad a \leq \pi.$$

either: $M_1 = \alpha \Lambda_{QCD}$ = mass of lightest gluebal
($\alpha = 1$ by def., but keep it)

or: M_π = pion mass, i.e. mass of almost Goldstone boson (for slightly brok χ symmetry)

. RHIC: $\sqrt{s} = 100 + 100$ GeV/nucleon in Au+A
→ fire ball of temp. $T = 176$ MeV.

. Try to derive these behaviours from gauge-gravity duality.

. Froissart for M_1 case, and simple arg. for M_π (exact description → future work).

$AdS_{d+1} \times X_{\bar{d}}$ dual : $\varepsilon = \frac{1}{D_{tot}-3} = \frac{1}{d+\bar{d}-2} = \frac{1}{7} \simeq 0.143$

from $\hat{M}_p = N_c^{1/4} M_{1,glueball} \sim 2Ge$

until $\hat{E}_R = N_c^2 M_{1,glueball} \sim 10Ge$

$\varepsilon = \frac{1}{2(d-1)+\bar{d}} = \frac{1}{11} \simeq 0.0909$ after that,

until $\hat{E}_p' = ?$

- Froissart behaviour after that.
- RHIC fireball = dual black hole,

of temperature $T = a \times 4 \frac{\langle m_\pi \rangle}{\pi} = a \times 175.76$ MeV

RHIC → in Froissart regime.

2. AdS-CFT and Polchinski-Strassle

- AdS-CFT (Maldacena, 1997)
  - a config. of (D-) branes curves space
- If we go near the brane ("near-horizon

→ duality:

Gauge th. on branes ⟶ gravity (string

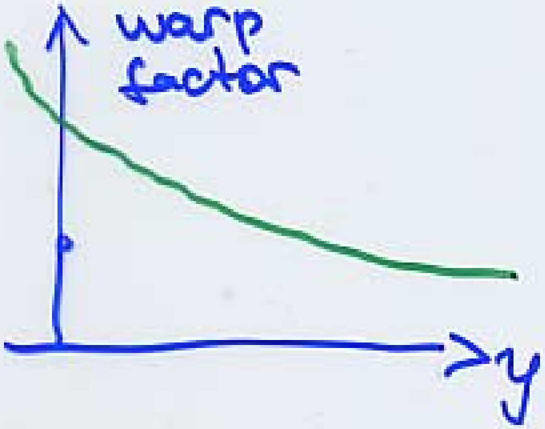
th. in limit curved

space

·CFT : $r \to 0$:

$$ds^2 = \underbrace{\frac{r^2}{R^2} d\vec{x}^2 + \frac{R^2}{r^2} dr^2}_{AdS_5} \underset{\times}{+} \underbrace{R^2 dS_X^2}_{X_5}$$

$$= e^{-2y/R} d\vec{x}^2 + dy^2$$



⇒ local AdS momentum: $\tilde{p}_\mu = \frac{R}{r} p_\mu$

- Non-conformal gauge th: gravity dual is modified in the IR = small r (large y

- 0-th order approx: put cut-off $r_{min} \sim R^2 \Lambda_{QC}$
  - simplest model → gives good results for many things.

- If we put also UV cut-off → RS 2-bran model.

Polchinski - Strassler (2001)

- scattering gauge th. glueballs (or ~~colo~~ colourless states in general) ⟶ scatter states in gravity dual.

$$e^{ip\cdot x} \longrightarrow e^{ip\cdot x}\,\psi(r,\underbrace{\Omega}_{\text{on } X})$$

$$\Rightarrow A_{gauge}(p) = \int dr\, d^5\Omega \sqrt{g}\, A_{string}(\tilde{p}) \prod_i \psi_i$$

α' = string tension $= R^2/(g_s N)^{1/2}$

$\hat{\alpha}'$ = gauge th. string tension $= \Lambda^{-2}_{QCD}/(g^2_{YM} N)^{1/}$

$g_s = g^2_{YM}$

$$\Rightarrow \underbrace{\sqrt{\alpha'}\,p}_{\text{string}} \leq \underbrace{\sqrt{\alpha'}\,p}_{\text{QCD}}$$

- If AdS scale large → use flat space scattering for $\mathcal{A}_{string}(\tilde{p})$

3. Giddings scenario – questions

(Giddings, 200

- Planck scale:

$$M_P = g_s^{-1/4}\alpha'^{-1/2} = \frac{N^{1/4}}{R} \gg \underset{\substack{\text{(string}\\ \text{scale)}}}{\alpha'^{-1/2}}$$

- Gauge th. "Planck scale"

$$\hat{M}_P = g_s^{-1/4}\hat{\alpha}'^{-1/2} = N^{1/4}\Lambda_{QCD}$$

- When $p_{QCD} \sim \hat{M}_P \rightarrow \tilde{p} \lesssim M_P \rightarrow$ we produce black holes in AdS.
- Black holes in D dimensions have horizon radius $r_H \sim E^{\frac{1}{D-3}}$

giving a geometrical cross section for B.H. formation $\sigma \sim \pi r_H^2 \sim \pi E^{\frac{2}{D-3}}$

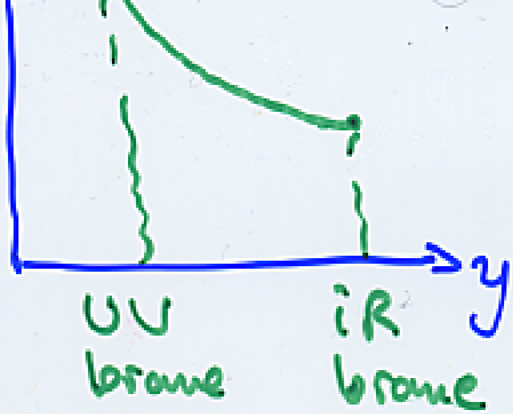
If $D = 10 \Rightarrow \sigma \sim s^{\frac{1}{7}}$

- When BH size r_H reaches AdS size R

$$\Rightarrow \hat{E}_{R,QCD} = N^2\Lambda_{QCD}$$

→ maximum behaviour = Froissart?

. Put point mass of $m = \sqrt{s}$ on iR brane



$\Rightarrow$ $h_{00,lin.} \sim G_4 \sqrt{s}\, \frac{e^{-M_1 r}}{r}$

$G_4^{-1} = M_P^3 R$

$M_1 = \frac{j_{1,1}}{R} =$ mass of lightes KK mode

→ mass of lightest glueball in QC

. Radion for the iR brane position (dist. between iR and UV brane) hass mass M_L.

→ mapped to pion in QCD, so $M_L \to M_\pi$. (Goldstone boson)

$M_L < M_1 \Rightarrow$ brane bending: $\frac{\delta L}{L}\Big|_{lin} \sim G_4 \sqrt{s} (M_L R) \frac{e^{-M_L r}}{r}$

So:

- If $M_1 < M_L$: $h_{00,lin} \sim 1 \Rightarrow$

$$\sigma_{QCD} = \sigma \sim \pi r_H^2 \sim \frac{\pi}{M_1^2} \ln^2(\sqrt{s}\, G_4 M_1)$$

- If $M_L < M_1$: $\frac{\delta L}{L}\Big|_{lin.} \sim 1 \Rightarrow$

$$\sigma_{QCD} = \sigma \sim \frac{\pi}{M_L^2} \ln^2(\sqrt{s}\, G_4 M_L^2 R)$$

- Why point mass on IR brane?

→ would like dynamical statemen
→ AdS scattering.

- Why $\sigma_{BH} \sim \pi r_H^2$?
- Why $h_{00,lin} \sim 1 / \frac{\delta L}{L}\Big|_{lin.} \sim 1$ gives good estimate?
- Do string corrections ($\Rightarrow N, g_{YM}$ finite in gauge th.) change resul

4. Aichelburg-Sexl waves and 't Hoo scattering

- Singular pp-waves (shockwaves) in flat space: $ds^2 = 2dx^+dx^- + (dx^+)^2 \Phi(x^i) \delta(x^+) + d$

where:

$$\Delta \Phi(x^i) = -16\pi G p \delta(x^i)$$

- Can put it in grav. backgrounds for gravity duals.

H.N. hep-th/ 0410124

then $\Delta = \Delta$ in grav. background.

⇒ for shockwaves, linearized sol. = exact sol.!

H.N. hep-th/041012

· 't Hooft : at $s \lesssim M_{Pl}^2$, $s \gg t$

· scattering of 2 photons → null geodesic scattering of one photon in A-S wave of 2nd photon.

$$\Rightarrow \frac{d\sigma}{d^2k} = \frac{4}{s}\frac{d\sigma}{d\Omega} = \frac{4(Gs)^2}{t^2}$$: Rutherford scattering.

$\alpha \to Gs$

· $s > M_{Pl}^2$, $\gg t$ → need both photons to create A-S waves → non linear gravi

BH formation?

5. Scattering of two A-S waves in $d \geq 4$

Eardley-Giddings (2002):

· analyze flat space $b=0$, $d>4$ scattering and $d=4$, $b\neq0$ → find $\sigma = \pi b_{max}^2(s)$ for formation of a trapped surface

is unknown, but ∃ "trapped surface" S : closed $d-2$ dim. surface of zero "convergence" $\theta=0$ at $(u = v = 0)$

$\xrightarrow[\text{theorem}]{GR}$ ∃ horizon outside it being formed.

- For Schwarzschild B.H., trapped surface = horizon = S^2 surface at $r=r_H$.

- Find max. impact parameter for formation of trapped surface (⇒ for B.H. formation)

K. Kang and H.N, hep-th/04090

$$b_{max}(s) = ct.(d) \times R_S(d) = a(d)\,(G_d\sqrt{s})^{\frac{}{d-3}}$$

$$R_{S(d)} = ct.(d)\,(G_d\sqrt{s})^{\frac{1}{d-3}}$$

$$\Rightarrow \sigma_{tot}(s) \geqslant ct.(d.) \times (G_d\sqrt{s})^{\frac{2}{d-3}}$$

String corrections

K. Kang and H.N. hep-th/040909

- Amati-Klimcik (1987): string-corrected shockwave $\Phi(x)$ (found by equating shockwave $\Phi(x^i)$ with string scatterin

$\rightarrow \Phi(x)$.)

- Scatter two A-K. waves in $D=4$:

$$\Rightarrow B_{max} = \frac{R_S}{\sqrt{2}}\left(1+e^{-\frac{R_S^2}{8\alpha' \log \alpha' s}}\right) \text{ if exponent is large } (s \to \infty)$$

$\Rightarrow$ exponentially small corr.

6. Scattering in gravity dual and Polchinski-Strassler map to QCD

K. Kang and H.N. hep-th 0410

AdS: A-S shockwave inside AdS - exact!

Clim. sol. = exact sol.!

$r \gg R$: $$\Phi \simeq 2R_{S(4)} \frac{R^6}{r^6 e^{-6y_0/R}}$$

B.H. formation: formalism generalized to curved space:

$$\Rightarrow e^{-y_0/R} b_{max}(s) = a \cdot R \left(R_S e^{-y_0/R}\right)^{1/6}$$

$a \simeq 1.351$ (number)

$R_S = 2G_4\sqrt{s}$

iR brane:

$r \gg R$: $\Phi \simeq R_S \sqrt{\frac{2\pi R}{r}}\, G\, e^{-M_1 r}$ exact?

(lin. sol = exact sol.)

B.H. formation:

$$b_{max} = \frac{\sqrt{2}}{M_1} \ln\left[\kappa R_S M_1\right] \quad \kappa \simeq 0.50$$

Polchinski-Strassler:

- $b_{max}(s)$: classical scattering.
- Need quantum amplitude to use P-S.
- Introduce black disk eikonal:

$$S = e^{i\delta(b,s)} \quad \begin{cases} \delta = 0 & b > b_{max}(s) \\ \Im m\, \delta = \infty \\ \mathrm{Re}\, \delta = 0 \end{cases} \Big\} \; b < b_{max}(s$$

such that

$$\frac{1}{s} \Im m\, \mathcal{A}_{\substack{elastic,\\ string}}(t=0) = \sigma_{tot} = \pi b_{max}^{2}$$

- Find $\mathcal{A}_{\substack{elastic,\\ string}}$ and use it in P-S formula

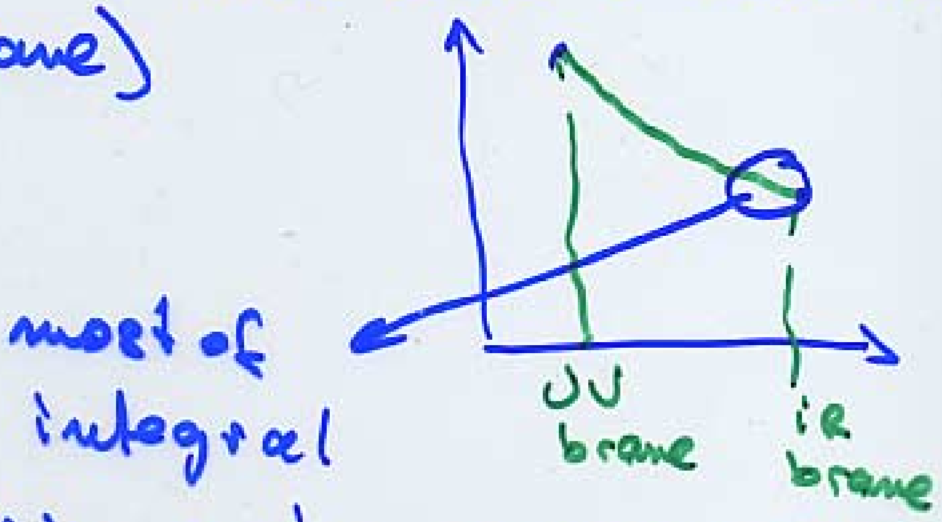
$\Rightarrow \mathcal{A}_{el., gauge} \Rightarrow \sigma_{gauge}$.

- $\sigma_{gauge} = K \, \sigma_{string}$

  ↳ model dependent const.

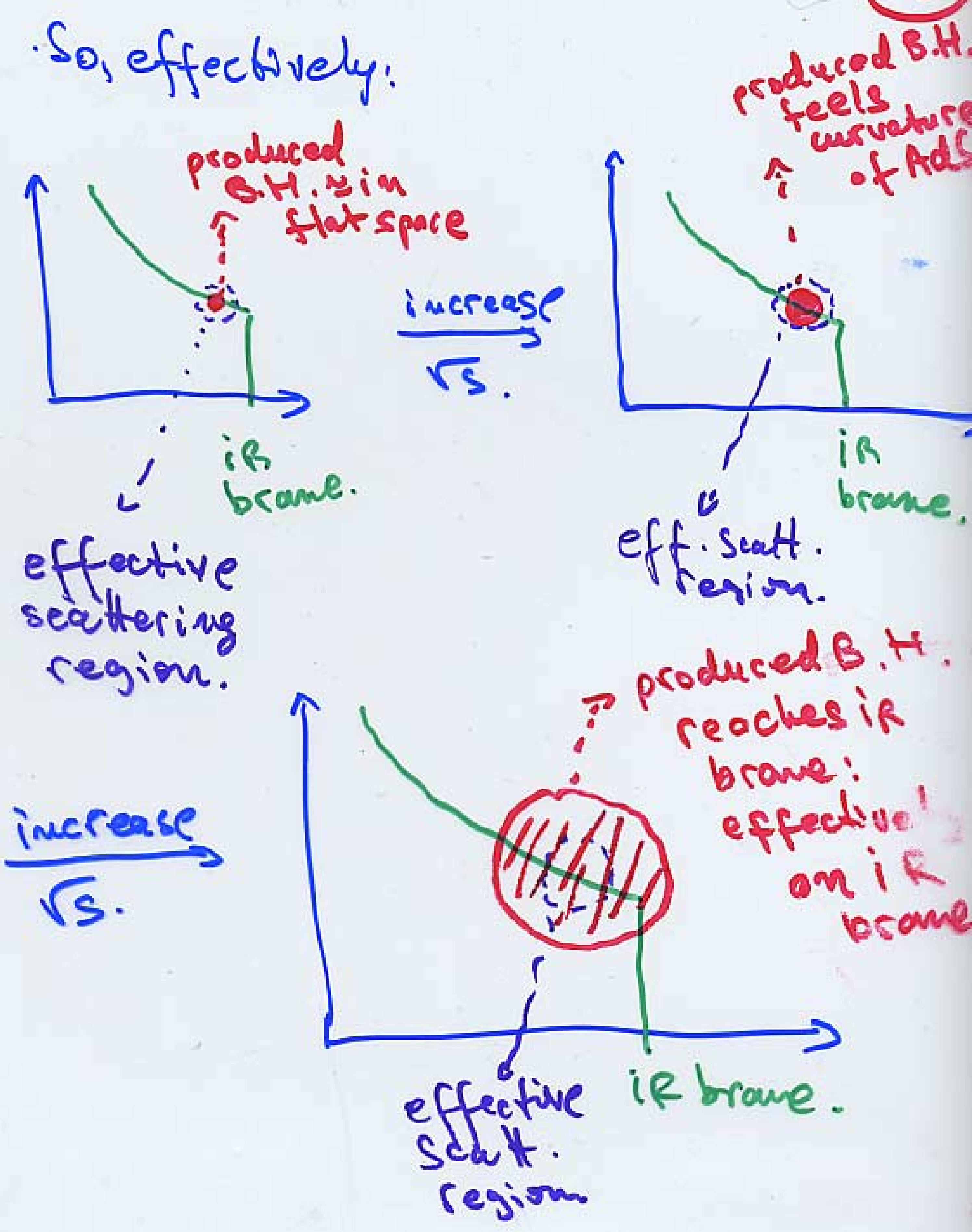
- subleading behaviour of σ_{string} modified.

- most of $\int$ comes from r close to r_{min} (iR → close to iR brane)

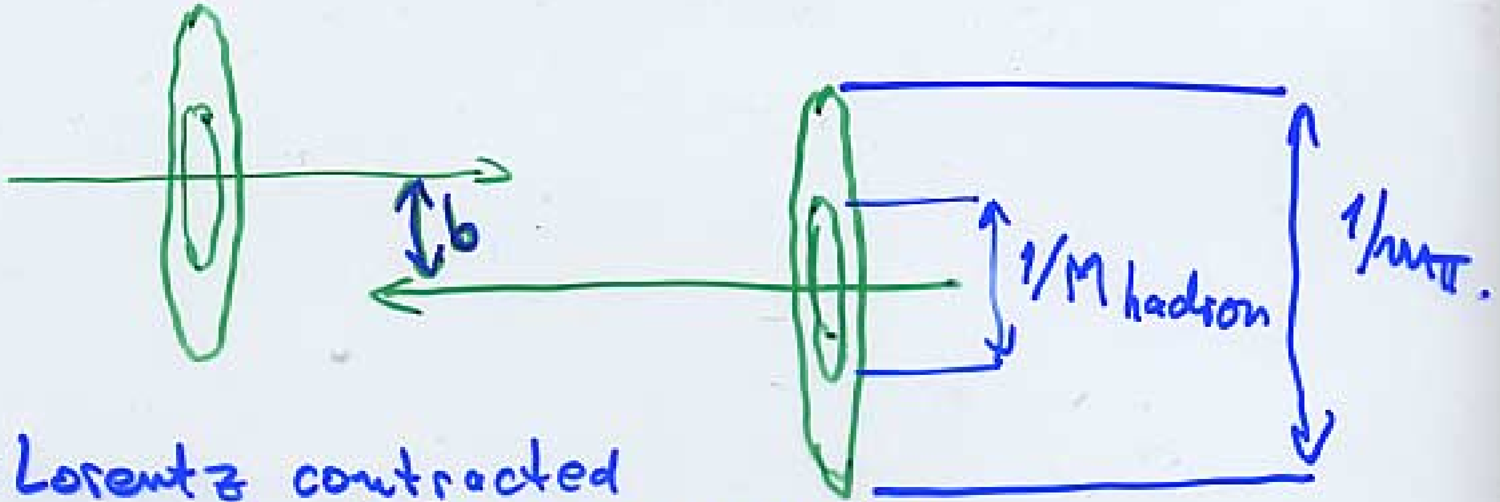


→ scattering can be considered to happen there.

So, effectively:



3. Heisenberg model (1952)



- Lorentz contracted hadron → pancake.
  Limit: shockwave: δ fct. distributed.
- Pion field around hadron: $r \sim 1/m_\pi$ also gets contracted
- Model: hadron "dissolves" in pion field.
  ⇒ collision of pion field shockwaves.
- If scalar pion free: $(\Box - m_\pi^2)\varphi = 0 \Rightarrow \frac{d\mathcal{E}}{dE_0} = \text{const.}$ – radiate energy – single pion energ

$$\Rightarrow \langle E_0 \rangle = \gamma m_\pi \frac{1}{\ln \gamma}$$

$$\gamma = \frac{1}{\sqrt{1-v^2}} = \sqrt{s}/M_{\text{hadron}}.$$

- Add $\lambda\varphi^4$ interaction → doesn't work.
- Instead, DBI action:

$$S = \ell^{-4}\int\sqrt{1+\ell^4\left[(\partial\varphi)^2 + m_\pi^2\varphi^2\right]}$$

$$\Rightarrow \langle E_0 \rangle \simeq m_\pi \ln \gamma$$

Pion field wavefct. should be $\sim e^{-r m_\pi}$

$\Rightarrow \quad \frac{\mathcal{E}}{\sqrt{s}} \equiv \alpha = e^{-b m_\pi}$ (prop. to wavefct. overlap)

$\Rightarrow \sqrt{s}\, e^{-b m_\pi} \cong \mathcal{E} = \langle E_0 \rangle$ at b_{max}.

$\Rightarrow \sigma_{tot} = \pi b_{max}^2 \simeq \frac{\pi}{m_\pi^2} \ln^2 \frac{\sqrt{s}}{\langle E_0 \rangle}$

→ higher deriv. DBI action needed s.t. $\langle E_0 \rangle \simeq m_\pi$.

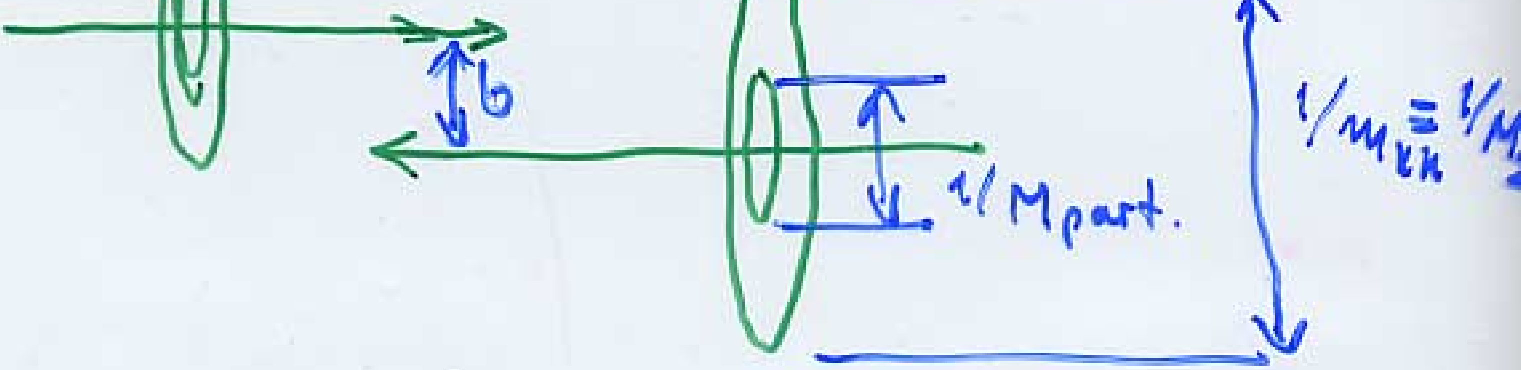
→ like real pion action (SU(2) nonlinear σ-model)

This should be similar for π → lightest glueball → pure Yang-Mills.

Comparison to dual theory

K. Kang and H.
hep-th/0501

Same picture! Analyze for gravity case → dua to pure YM case. ~~of~~ ~~lightest~~ Case of almos Goldstone boson → similar. π → radion: brane bending.



scatter grav. shockwaves in dual theory.

For Froissart saturation:

scattering on IR brane creates black holes.

shockwave profile:

(like pion wave fct.)

$$\Phi \simeq R_S \sqrt{\frac{2\pi R}{r}}\, c_1 e^{-M_1 r}$$

$$M_1 = \frac{j_{1,1}}{R} \simeq \frac{3.83}{R}$$ → ligh KK mode

→ lightest glueball

- Now, exact mechanism for calculating b_{max} from Φ: gravity → trapped surface calculation.
  → no need to postulate $\mathcal{E}/\sqrt{s} = e^{-b m_\pi}$.
- Black hole creation → dual soliton in pion field created.
  → then decays into free pions
- One needs nonlinear gravity to derive b_{max}
  ↔ need DBI nonlinear action, not free one, to have $\langle E_0 \rangle \simeq$ const., not $\langle E_0 \rangle \propto \sqrt{s}$.

And radion action = DBI.

$$\mathcal{L} = \ell_5^{-4}\sqrt{\det(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu})} =$$
$$= \ell_5^{-4}\sqrt{g}\,\sqrt{1+\ell_5^{-4}(\partial_a\phi)^2}$$

- Conjecture: maybe add m^2 in $\sqrt{\ }$ as Heisen did.
  → nonlinear generaliz. of radion stabilization:
  - flux stab. for Polchinski-Strassle
  - Goldberger-Wise for R-S.
- Also, conjecture: maybe 4d action for KK graviton M_1 is also DBI type.
- Arguments: 1st order DBI action is usual gravity + topological term.

$$\int\sqrt{\det(R^{ab}(\omega)+\ell^{-2}e^a\wedge e^b)} = \int \underset{\text{topological}}{R\wedge R} + 2\ell^{-2}\overset{\text{Einstein}}{R\wedge e} + \ell^{-4}\underset{\Lambda}{e^{\wedge 4}}$$

- Deser-Gibbons looked at action

$$\sqrt{\det(g_{\mu\nu}+aR_{\mu\nu}+bX_{\mu\nu})}$$

→ Maybe we have:

$$\sqrt{\det\big(g_{\mu\nu}+R_{\mu\nu}(g^{(1)}_{\mu\nu})+X_{\mu\nu}(g^{(1)}_{\mu\nu})\big)}$$

bilinear in KK graviton $g^{(1)}_{\mu\nu}$, rest is $g_{\mu\nu}$.

mass terms bilinear in $g^{(1)}_{\mu\nu}$.

H.N. hep-th/050103

- A-S shockwaves scattering in AdS_5:

  A-S shockwave: $\Phi(r \gg R) \simeq \bar{C} \frac{R^4}{r^6} e^{\frac{4y+2y}{R}}$

  $\Rightarrow b_{max} = \left(7^{-1/12}\sqrt{\tfrac{1}{7}}\right)\left(Re^{y_0/R}\right)\left[\frac{Rs}{Re^{y_0/R}}\right]^{1/6}$

  $\Rightarrow \sigma_{gauge} = \bar{K}\pi a^2 \left(\frac{\alpha' s}{\alpha'}\right)^{1/6} \sim s^{1/6}$.

- But, contradiction: flatspace BH $\Rightarrow \sigma_{gauge} \sim s^{\frac{1}{d-}}$

  $d=5: s^{1/2}$; $d=10: s^{1/7}$

  $s^{1/7} \to s^{1/6} \to \ln^2 s$ doesn't make sense in gauge th.

- If $d=5$: $s^{1/2} \to s^{1/6} \to \ln^2 s$ ok. but why X_5 so small as to be always neglected?

- If flat space first $\Rightarrow$ next, curvature of AdS and only after that, $\to$ IR brane.

- If X_5 size $\sim AdS_5$ size $\Rightarrow$ same $\sigma \sim s^{1/6} \to$ not good.

  $\Rightarrow X_5$ size $\bar{R} \gg AdS_5$ size R ?

  $\to$ AdS space modif. at small $\bar{r} \Rightarrow \bar{R}(\bar{r})$

- If $\bar{R}(\bar{r}) \uparrow \Rightarrow$ scattering happens away from IR brane.

$$A_{gauge} = \int \bar{R}^5(\bar{r}) \underbrace{\prod \psi_i(\bar{r})}_{\propto r^{-(\dots)}} A_{string}$$

and then also $\bar{R}(\bar{r}) \gg R$.

Solve for A-S shockwave on $AdS_{d+1} \times X_{\bar{d}}$

$$\underset{\text{large.}}{\downarrow}$$

$$\Delta \Phi = -16\pi G \rho\, \delta^{d-2}(x^i)\,\delta(y-y_0)\,\delta^{(\bar{d})}(z_\mu)$$

$$\Rightarrow \Phi = k_1 R_S \frac{R^n}{r^n} \sim \frac{1}{r^{2(d-1)+\bar{d}}} = \frac{1}{r^{11}}.$$

Find trapped surface:

$$\left(\frac{3\Phi}{2R}\Big|_{y=0}\right)^2 \left(1-\frac{b^2}{2r^2}\right) = 1$$

$$\Rightarrow b_{max} = \left(\sqrt{\tfrac{2n}{n+1}}\,(n+1)^{-1/n}\right) R \left(\frac{3k_1 R_S}{2R}\right)^{1/n}$$

$$R_S = G_4\sqrt{s}$$

$$\Rightarrow \sigma_{gauge} = \bar{K}\pi a^2 \left(\frac{\hat{\alpha}' s}{\alpha'}\right)^{1/n} \sim s^{1/n}.$$

Now $s^{1/7} \to s^{1/11} \to \ln^2 s \to$ OK.

Energy behaviours of gauge theories

Energy scales: $\frac{1}{R} \to \hat{E}_{AdS} = \Lambda_{QCD}$;

$\alpha'^{-1/2} \longrightarrow \hat{E}_S = \hat{\alpha}'^{-1/2} = \Lambda_{QCD}\,(g^2_{YM} N)^{1/4}$;

$M_P \to \hat{M}_P = N^{1/4}\Lambda_{QCD}$; correspondence princ. scal

$E_c \to \hat{E}_c = \Lambda_{QCD}\dfrac{N^2}{(g^2_{YM}N)^{7/4}}$;

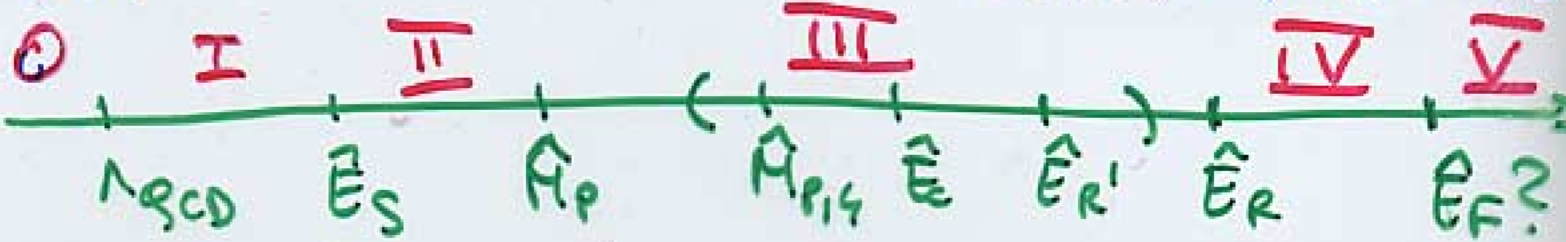
$E_R \Leftarrow (r_H \sim R) \longrightarrow \hat{E}_R = N^2 \Lambda_{QCD}.$

of gravity dual is 5d, we have also;

- scale of $G_4 s \sim 1 \longrightarrow \hat{M}_{P,4} = N^{3/8}\Lambda_{QCD}$.
- scale of $R_S = G_4\sqrt{s} \sim R_{AdS} \to \hat{E}_R{}' = N^{3/8}\Lambda_{QCD}$

($\hat{E}_R$ for pure AdS_5)

So:



I: single graviton exchange : 't Hooft scattering
(nonrenormalized Pomeron)

$$\frac{d\sigma}{d^2k_\perp} \sim (\hat{G}_4 s)^2$$

II: Regge behaviour $\mathcal{A}^{2\to2}_{gauge} \sim (\alpha' s)^{2+\alpha' t/2}$

III: black holes in flat space:

$$\sigma_{gauge} \sim s^{\frac{1}{D_{tot}-3}} = s^{1/7}$$

IV: Soft Pomeron:

$$\sigma_{gauge} \sim s^{\frac{1}{2(d-1)+d}} = s^{1/11}$$

starts at $\hat{E}_R = N_c^2 M_{1,glueball}$.

V: At unknown $\hat{E}_F \to$ Froissart:

$$\sigma_{gauge} \sim \frac{\pi}{M^2}\ln^2 s$$

If pion is lightest, $\hat{E}_F \to \hat{E}_F{}'(m_\pi) < \hat{E}_F$.

and $\sigma_{gauge} \sim \frac{\pi}{m_\pi^2}\ln^2 s$.

- String α' and g_s corrections small: scatter string corrected shockwaves (Amati-Klimcik) → contain α' and g_s corr.
- α' and g_s corr. small $\Leftrightarrow$ $\frac{1}{N}$ and $\frac{1}{g_{YM}^2 N}$ corr. small.

→ apply to real QCD?

- Then the new B_{max} for B.H. formation i

$$B_{max} = \frac{R_S}{\sqrt{2}}\left(1 + e^{-\frac{R_S^2}{8\alpha' \log \alpha' s}}\right)$$

⟵ uncorrected result. M_p^2/M_s^2

-scale : $E_0 \sim \Lambda$ [illegible]

⟵ can't be trusted though.

→ most likely at $E > \hat{E}_R$, corrections are small.

→ above $\hat{E}_R$, gauge th. analysis applies for real QCD as well.

→ only energy scales can get renormalized.

- $\Lambda_{QCD} \equiv \hat{M}_1 \sim 0.6 - 1.7$ GeV(exp), 50 ~ 1 Ge

⇒
- $M_p \sim \hat{E}_S \sim \Lambda_{QCD} \sim 1\text{-}2$ GeV
- $\hat{E}_R = N_c^2 M_1 \simeq 10$ GeV
- Unknown $\hat{E}_P'$, most likely $> \hat{E}_R$.

· Regimes $\underline{I}$, $\underline{II}$ nonexistent.

· $\underline{III}$: above 1-2 GeV : create black holes = nonlinear solitons of the lightest glueball field ; in flat space

- the behaviour $\sigma_{QCD} \sim s^{1/7} \simeq s^{0.143}$ could be changed by string corrections in this regime.
- for 2→2 scattering, one still finds Regge behaviour.

$\underline{IV}$: above 10 GeV : create black holes in $AdS_{d+1} \times X_{\bar{d}}$.

$$\sigma_{QCD} \simeq \underset{\substack{\nwarrow \\ \text{model dependent} \\ \text{const.}}}{\bar{k}} \, \pi \, a_n^2 \left(\frac{\alpha' s}{\alpha'}\right)^{1/n} \sim s^{1/n} = s^{1/11} \simeq s^{0.0909}$$

$\underline{V}$: above $\hat{E}_F'$ → create IR brane black holes

$$\sigma_{QCD} = \bar{k} \, \pi \, b_{max}(s)$$

$$b_{max}(s) = \frac{\sqrt{2}}{M_1} \ln[R_s M_1 k]$$

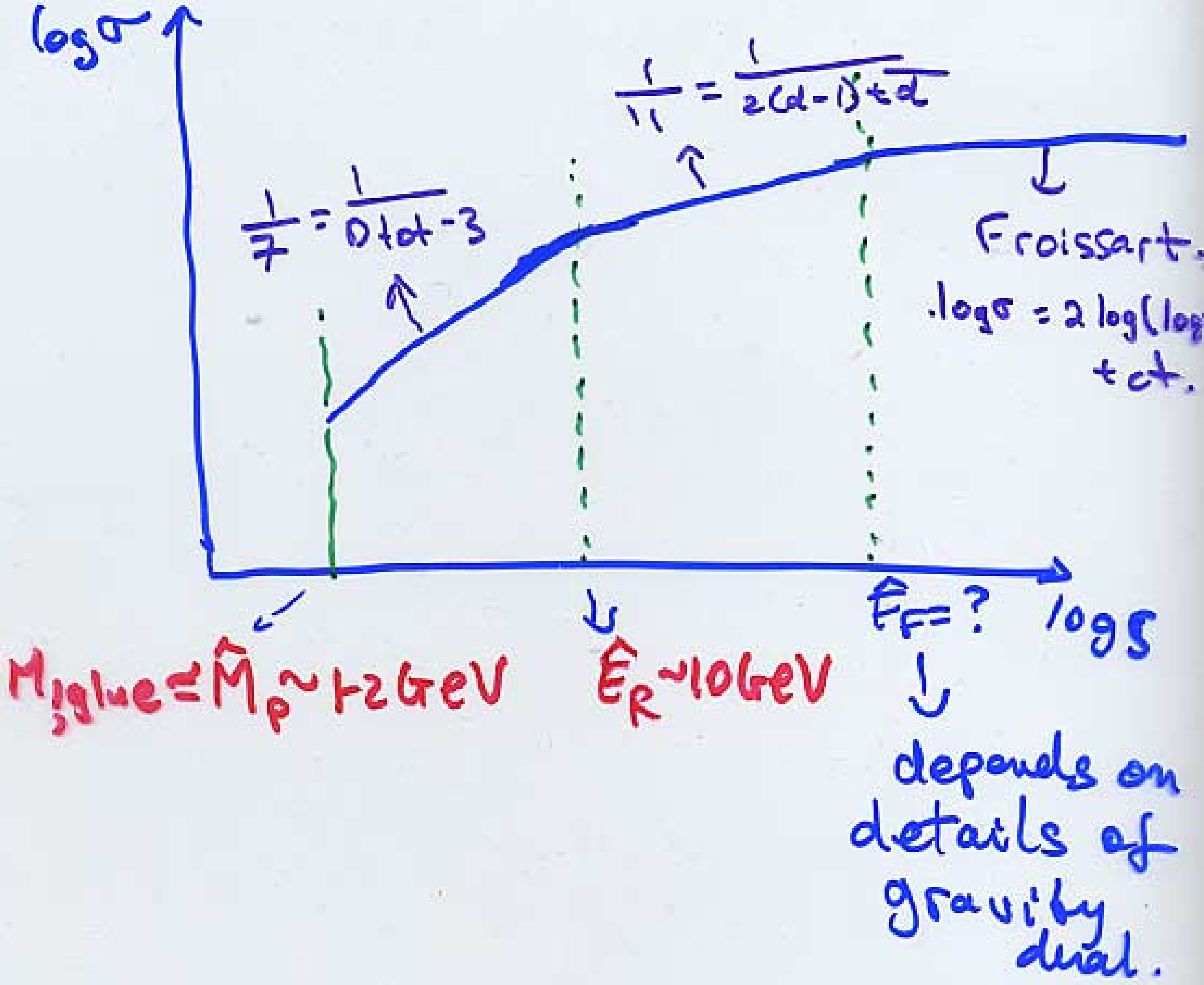
$R_s = G_4 \sqrt{s}$;

$M_1 = j_{1,1}/R$; $k \simeq 0.57$

$$\Rightarrow \sigma_{QCD} \sim \frac{1}{M^2} \ln^2 s$$

Froissart : coeff. $\frac{A}{M^2} \leq \frac{\pi}{m_\pi^2} = 60\,mb$.

So, expect.


log σ
$\frac{1}{7} = \frac{1}{D+d-3}$
$\frac{1}{11} = \frac{1}{2(d-1)+d}$
Froissart.
log σ = 2 log(log
+ ct.
log s
$M_{glue} \simeq \hat{M}_p \sim 1\text{-}2$ GeV
$\hat{E}_R \sim 10$ GeV
$\hat{E}_F = ?$
depends on details of gravity dual.

· Experim.:. First, found $\sigma_{tot,AB} = X_{AB}(s/s_0)^{\varepsilon}$

$X_{AB} \sim 10-35$ mb

$\varepsilon = 0.0933 \pm 0,0024$

For $\chi^2/d.o.f = 1$, we have

$\sqrt{s}_{min} = 9$ GeV

($\sqrt{s}_{min}$ lower ⇒ $\chi^2/d.o.f. \uparrow$)

· Later, argued that is replaced by

$\sigma_{tot} = Z_{AB} + B \ln^2 s/s_0$, down to $\sqrt{s} = 5$ GeV.

$\chi^2/d.o.f. = 0.971$

$Z_{AB} \sim 18-65$ mb.

and $B = 0.31$ mb $\ll 60$ mb.

· Either coincidence:
- why extend to lower $\sqrt{s}_{min}$?
- why the const. terms?
- 0.31 mb ≪ 60 mb?

but we expect from Heisenberg picture that coeff. is close to 60mb.

- expect s^{ε} terms into $\ln^2 s$ when $X_{AB}(s/s_0)^{\varepsilon} \sim 60$mb

· Or $\hat{E}_F' < \hat{E}_R$ and soft Pomeron and Froissart coexist

→ maybe $X_{AB}(s/s_0)^{\varepsilon} + B \ln^2(s/s_0)$

. Need more experiments.

. But where is the nonlinear soliton that decays?

- should have been observed by now.

It has! RHIC fireball = nonlinear soliton
- dual to black hole.

- Black hole has temperature, as does the fireball.

BH thermodynamics: $d\mathcal{M} = T dS$.

$$S = M_{P,4}^2 \frac{A}{4} \quad \text{(quantum gravity} \to \text{string th.)}$$

$$\Rightarrow d\mathcal{M} = \frac{\pi T}{4} M_{P,4}^2 \, dr_H^2$$

If $M_{P,4}^2 dr_H^2 = \frac{d\mathcal{M}}{a \cdot M_1} \Rightarrow T = a \cdot \frac{4 M_1}{\pi}$ (1)

Justify it:

- Linearized solution:

$$M_{P,4}^2 dr_H^2 = \frac{d\mathcal{M}}{M_1} \, 2\left(\frac{M_{P,4}^2}{M_1 \mathcal{M}}\right) \ln\Big(\frac{M_1 \mathcal{M}}{M_0}$$

- not quite.

- But! Hawking: 4d flat space BHs:

$T = \frac{k}{2\pi}$ k = surface gravity of horizon.

Under very mild assumptions, for us, $k \simeq M_1$ $\Rightarrow a = 1/8$. But need to redo the Hawking calculation for this solution. Then (1) is dimens. correct.

Then, If $a = 1 \Rightarrow T = \frac{4}{\pi} \langle m_\pi \rangle = 175{,}76$ MeV.

- Experim.: fireball: $T = 176$ MeV.

→ this could also be the χ
restoration (& deconfinement?)
phase transition temperature.

- Also, $\eta/s \sim 0.1 - 0.3$

η → shear viscosity
s → entropy.

but (theorem) black holes in gravity duals have $\eta/s = 1/4\pi$.

- At the core of ~~the~~ fireball → "Color Glass Condensate" (CGC) → absorbs hard scattered particles ("jet quenching")
- CGC = nonlinear pion field soliton = black hole interior
  - → jet quenching = information paradox
  - → information gets in B.H., radiation comes out.
- ⇒ very hard to probe soliton by extra part
  - → would probe black hole.
  - → maybe only formation and decay regio
- Interaction in CGC → Newtonian potentia

$$\underset{\text{gravity}}{U = -k\frac{G_4 M}{r}} \rightarrow \underset{\text{brane bending?}}{U = -k G_4(M_L R)\frac{M}{r}}$$

$$V(r) = -\frac{m_\pi M_1^{-1}}{N_c^{3/4} M_1^2} \, \kappa \frac{\mathcal{M}_1 \mathcal{M}_2}{r} = \cancel{-0.[illegible]^{2}}$$

$$= -0.06\,\text{GeV}^{-2} \, [M_1[\text{GeV}])^{-3} \, \kappa \frac{\mathcal{M}_1 \mathcal{M}_2}{r}$$

→ Coulomb-like ⇒ quarks and gluons deconfined.

- Also at lower energies (>10 GeV) create solitons = black holes ⇒ almost thermal decay → experiment?

- We used A-S shockwaves scattering to calculate black hole creation in the gravity dual, in Polchinski - Strassler scenario.
- We obtained:

I : $R^{-1}_{AdS} = \Lambda_{QCD} \longleftrightarrow \hat{E}_S = \Lambda_{QCD} (g^2_{YM} N)^{1/2}$:
single graviton exchange: $\frac{d^2\sigma}{d^2b} \sim (\hat{G}_4 s)^2$

II : $\hat{E}_S \longleftrightarrow \hat{M}_P = N^{1/4} \Lambda_{QCD}$ Regge behavior

III : $\hat{M}_P \longleftrightarrow \hat{E}_R = N^2 \Lambda_{QCD}$ $\sigma_{gauge} \sim s^{1/7}$
flat space B.H. prod.

IV : $\hat{E}_R \longleftrightarrow \hat{E}_F' = ?$ $\sigma_{gauge} \sim s^{\hbar} = s^{0.0009}$
Soft Pomeron

V : $\hat{E}_F' \rightarrow$ Froissart:
$\sigma_{gauge} \sim \frac{\pi}{M^2} \ln^2 s/s_0$.

- Real QCD: $\Lambda_{QCD} \sim \hat{E}_S \sim \hat{M}_P \sim 1\,\mathrm{GeV}$.
$\hat{E}_R \sim 10\,\mathrm{GeV}$
exper. : soft Pomeron $s^{0.093(2)}$
- Heisenberg model mapped exactly to dual model ⇒ nonlinear pion field soliton.

- Nonlinear soliton already observed at RHIC ! fireball.

- Temperature $T = a \cdot \frac{4 \langle m_\pi \rangle}{\pi}$

  $a = 1 \Rightarrow 175.76$ MeV

  exp.: 176 MeV.

- "Coulomb" potential inside soliton

$\leftrightarrow$ Newton potential inside black hole

- "Color Glass Condensate" = black hole interior.

- "Jet Quenching" (absence of hard scattering) = "information paradox".

- Thus RHIC is a string theory testing machine, analyzing B.H. creation and decay, and probing B.H. interior.