Rutgers University, November 25th 2003

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# Supersymmetric Geometries of M-Theory

[DM, J. Sparks] hep-th/0306225

# Motivations

► M-Theory compactifications with fluxes

• It is interesting to study models with $\mathcal{N} = 1$ in $d = 3$ (e.g. in relation to the cosmological constant problem)

• String/M-Theory compactifications to four dimensions

► Holography

• AdS solutions dual to SCFT

• RG flow solutions

• Relation to interesting holographic flows in String Theory (e.g. Polchinski–Strassler, Klebanov–Strassler)

► Geometric conditions corresponding to M5-brane wrapping various cycles (especially 'exotic' ones, e.g. associative)

• The same conditions are satisfied by particular brane intersections

► Emphasize the role of $G$-structures and especially of generalized calibrations in the context of supersymmetric geometries with fluxes

## Killing spinor equations, special holonomy, and G-structures

Supersymmetric solutions of supergravity admit 'Killing spinors'

Killing spinor equations

$$\delta\psi_M = \nabla_M \epsilon + \Omega_M \epsilon = 0$$

(+ possible algebraic equations for $\epsilon$: $O\epsilon = 0$)

# independent solutions $\Longleftrightarrow$ unbroken supersymmetry

When all the fields are set to zero, except the metric, solutions to $\nabla_M \epsilon = 0$ are Ricci-flat, special holonomy manifolds:

| | |
|---|---|
| dim($M$)=$2n$ | $SU(n)$ (Calabi–Yau) |
| dim($M$)=$4n$ | $Sp(n)$ (hyper–Kähler) |
| dim($M$)=7 | $G_2$ |
| dim($M$)=8 | $Spin(7)$ |

- Including the form-fields (i.e. Fluxes) → $\Omega_M \neq 0$

→ the holonomy of the Levi–Civita (or spin-) connection is not a good principle for determining the number of solutions

▶ G-structures are the most appropriate mathematical framework to study geometries with fluxes [Gauntlett, DM, Pakis, Waldram]

A $G$-structure on a manifold $M$ is a (global) reduction of the frame bundle to a sub-bundle with fibre $G$

Key facts:

- equivalent to a set of (globally defined) $G$-invariant tensors, or spinors, on $M$
- tensors are decomposed into irreducible representations of $G$
- departure from special holonomy is measured by the Intrinsic Torsion

$$T \in \Lambda^1 \otimes g^\perp = \bigoplus_{i=1}^{n} \mathcal{W}_i \ , \qquad g \oplus g^\perp = so(n)$$

Each class is a characterized by calculable quantities, denoted $W_i$

- Why is this useful in supergravity?

Solutions to the Killing spinor equations define a particular $G$-structure, and its type ("class of intrinsic torsion") can be determined analyzing bosonic equations obeyed by the $G$-invariant forms

Flux = Intrinsic Torsion

## Example: $G_2$-structures in $d = 7$ [Fernandez,Gray]

- These are defined by the associative three-form $\phi$
- Under $SO(7) \to G_2$, $\mathbf{21} \to \mathbf{7} + \mathbf{14}$
- Classes of intrinsic torsion

$$T \in \Lambda^1 \otimes g_2^{\perp} = \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$$
$$\mathbf{7} \times \mathbf{7} \to \mathbf{1} + \mathbf{14} + \mathbf{27} + \mathbf{7}$$

$$\mathrm{d}\phi \in \Lambda^4 \cong \mathcal{W}_1 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4$$
$$\mathbf{35} \to \mathbf{1} + \mathbf{27} + \mathbf{7}$$
$$\mathrm{d} * \phi \in \Lambda^5 \cong \mathcal{W}_2 \oplus \mathcal{W}_4$$
$$\mathbf{21} \to \mathbf{14} + \mathbf{7}$$

E.g. $M$ has $G_2$-holonomy iff $\mathrm{d}\phi = 0 = \mathrm{d} * \phi$

# M-Theory on eight-manifolds with fluxes [DM,Sparks]

- Aim: characterize the most general supersymmetric M-Theory "compactifications" to three-dimensions

- 11-dim supergravity

Bosonic fields: $g_{MN}, G_{MNPQ}$

Killing spinor equation:

$$\delta\psi_M = \hat{\nabla}_M \eta - \frac{1}{288}\left(G_{NPQR}\hat{\Gamma}^{NPQR}{}_M - 8G_{MNPQ}\hat{\Gamma}^{NPQ}\right)\eta$$

Bianchi identity: $dG = 0$

$G$ eq. of motion: $d\hat{*}G + \frac{1}{2}G\wedge G = X_8 \rightarrow \chi(M_8) \sim \int G\wedge G$

- We are interested in compactifications to $d = 3$ which preserve (at least) $\mathcal{N} = 1$ supersymmetry.

warped metric $\quad d\hat{s}^2_{11} = e^{2\Delta(x)}(ds^2_3 + g_{mn}dx^m dx^n)$

$ds^2_3$ → $\mathbb{R}^{1,2}$ or $AdS_3$; $\quad g_{mn}dx^m dx^n$ → no ansatz

flux $\quad G_{mnpq}$ arbitrary, $\quad G_{\mu\nu\rho m} = \epsilon_{\mu\nu\rho} f_m$

spinor $\quad \eta = \psi \otimes \xi$

$\psi$ → 3 d; $\quad \xi$ → 8 d

Note: we have made no particular assumptions, apart from requiring the 3-dim external space to be Minkowski$_3$ or AdS$_3$

Reminder of known results [K. Becker], [Acharya,de la Ossa,Gukov]

• Assumption: $\xi$ is a $Spin(8)$ Majorana–Weyl spinor of definite chirality

$\Rightarrow$ It defines a $Spin(7)$-structure on $M_8$. Equivalently defined by the $Spin(7)$-invariant Cayley four-form

$$\Psi_{mnpq} = \xi^{\top}\gamma_{mnpq}\xi$$

• General solution

$$\begin{aligned} d\hat{s}^2_{11} &= H^{-2/3}ds^2(\mathbb{R}^{1,2}) + H^{1/3}d\tilde{s}^2(Spin(7)) \\ G &= \mathrm{vol}_3 \wedge \mathrm{d}(H^{-1}) + G_{27} \\ \tilde{*}\tilde{\Box}H + \frac{1}{2}G_{27}\wedge G_{27} &= X_8 \end{aligned}$$

$G_{27}$ is in the $\mathbf{27}$ representation of the $SO(8) \to Spin(7)$ decomposition of four-forms: $\mathbf{70} \to \mathbf{35} + \mathbf{27} + \mathbf{7} + \mathbf{1}$

• These are clearly M2-brane-type of solutions

• $\mathrm{AdS}_3$ compactifications are ruled out

• Additional Killing spinors of the same chirality reduce further the holonomy of $d\tilde{s}^2$. E.g. $\mathcal{N} = 2 \to SU(4)$ [K. Becker,M. Becker], $\mathcal{N} = 3 \to Sp(2)$, etc.

## Motivations for the existence of more general solutions

- More general Minkowski$_3$ vacua from wrapped M5-branes

M5 0 1 2 3 4 5 6 7 8 9 # ($G_2$) M5-branes wrapped on associative 3-cycles inside $G_2 \times S^1 \to \mathcal{N} = 1$

M5 0 1 2 3 4 5 6 7 8 9 # ($CY_3$) M5-branes wrapped on SLAG 3-cycles inside $CY_3 \times T^2 \to \mathcal{N} = 2$

- (Warped) AdS$_3$ vacua

M5-branes wrapped on 4-cycles or M2-branes wrapped on $S^1 \longrightarrow$ effective strings. In the 'near-horizon' limit should give AdS$_3$ vacua

Consider simply the Freund–Rubin solutions: $AdS_4 \times M_7$ with $M_7$ having weak $G_2$ holonomy, and write the metric as

$$d\hat{s}^2_{11} = \cosh^2(2mr)\, ds^2(\mathrm{AdS}_3) + dr^2 + d\tilde{s}^2_7$$

- M5/M2 bound states

M5-brane world volume action comprises a self-dual three-form $H$, inducing an M2-brane charge via a WZ coupling $\longrightarrow$ supersymmetric solutions corresponding to ('dyonic' or 'dielectric') M5/M2

- There were known supersymmetric solutions with $G$-flux, which were not of the type just reviewed

• Require the existence of a $G_2$-structure defined on $M_8$, or equivalently, of a non-chiral $Spin(8)$ Killing spinor

• We take $\eta = \mathrm{e}^{-\Delta/2}\psi \otimes (\xi_+ \oplus \xi_-)$ $\Gamma_9 \xi_\pm = \pm \xi_\pm$

▶ This is a completely generic spinor

▶ Note that we have $\mathcal{N} = 1$ in $d = 3$: $\nabla_\mu \psi + m\gamma_\mu \psi = 0$

The $G_2$-structure is defined by the following forms:

$$\begin{aligned} K_m &= \frac{1}{||\xi_+|| \cdot ||\xi_-||} \xi_+{}^{\mathsf{T}} \gamma_m \xi_- \\ \phi_{mnp} &= \frac{1}{||\xi_+|| \cdot ||\xi_-||} \xi_+{}^{\mathsf{T}} \gamma_{mnp} \xi_- \end{aligned}$$

or equivalently by two Cayley forms with opposite dualities

$$\Psi^{\pm}_{mnpr} = \frac{1}{||\xi_\pm||^2} \xi_\pm{}^{\mathsf{T}} \gamma_{mnpr} \xi_\pm$$

• The spinors $\xi_\pm$ cannot be normalized to unity! Rather, supersymmetry implies

$$\begin{aligned} \frac{1}{2}\left(||\xi_+||^2 + ||\xi_-||^2\right) &= 1 \\ \Rightarrow \frac{1}{2}\left(||\xi_+||^2 - ||\xi_-||^2\right) &\equiv \sin\zeta \end{aligned}$$

# The geometry on the eight-manifold

• We obtain a set of conditions on the forms, equivalent to the supersymmetry conditions

• The metric has the canonical form:

$$d\hat{s}_8^2 = e^{2\Delta}(g^7_{ij}(x,y)dx^i dx^j + e^{-6\Delta}\sec^2\zeta dy^2)$$

• At any fixed y, the $G_2$-structure has intrinsic torsion in $\mathcal{W}_3 \oplus \mathcal{W}_4$ **(27+7)**

• The fluxes are completely fixed:

$$f = e^{-3\Delta}d(e^{3\Delta}\sin\zeta) + 4mK\cos\zeta$$

$$F\sin\zeta - *F = e^{-6\Delta}d(e^{6\Delta}\phi\cos\zeta) - 4m(i_K*\phi - \phi\wedge K\sin\zeta)$$

• The $G$ equation of motion:

$$d(e^{6\Delta}*f) + \frac{1}{2}e^{6\Delta}F\wedge F \;=\; X_8$$

The total $G$-flux is defined as $G = e^{3\Delta}(F + \text{vol}_3 \wedge f)$

• Note that $m \neq 0 \to$ AdS$_3$ solutions are not ruled out

• $\sin\zeta = \pm 1 \to$ recover M2-brane $\perp$ $Spin(7)$ manifold (one spinor vanishes in this limit! And $m = 0$)

In the following I will illustrate some examples, for which the general equations simplify

## Example 1: $F$ is self-dual

• Imposing $F = *F$ (and $m = 0$) the general equations simplify considerably

• $e^{-3\Delta} = 1 - \sin\zeta$

• $d(e^{6\Delta}\Psi^-) = 0 \Rightarrow M_8$ is to conformal to $Spin(7)_-$-holonomy

• The fluxes are

$$\begin{aligned} f &= 3d\Delta \\ G_{\text{internal}} &= -d(e^{6\Delta}\cos\zeta\phi) \end{aligned}$$

• The Bianchi identity and $G$ equation of motion are satisfied automatically ✓

E.g. The "deformed M2-brane solutions" obtained in [Cvetic,Lü,Pope] are of this type (the case of $M_8 \propto \mathbb{R}^8$ has been checked explicitly in [K.Becker,M.Becker,Sriharsha])

$$G_{\text{internal}} = dx^{1234} \wedge dx^{5678}$$

$$e^{-3\Delta} = c - \mu\,\delta_{ij}\,x^i x^j$$

## Example 2: Vanishing internal flux $F$ (purely electric solutions)

- Imposing $F = 0$ the equations simplify again. Here we have:
- $e^{-\Delta} = \cos\zeta$
- $d(e^{3\Delta}\phi) = 4m e^{4\Delta} i_K * \phi \quad \Rightarrow$ (def: $\tilde{\phi} = e^{-3\Delta}\phi$)

$$d\hat{s}_{11}^2 = \sec^2\zeta\left(ds_3^2(\mathrm{AdS}_3) + \frac{1}{4m^2}d\zeta^2\right) + d\tilde{s}_7^2$$

► This is just $\mathrm{AdS}_4 \times M_7$. With $M_7$ having weak $G_2$-holonomy

## Wrapped or intersected M5-branes (and associated non-linear PDE's)

• In [Fayyazuddin,Smith] the supersymmetry conditions describing M5-branes wrapped on holomorphic cycles (or M5-branes intersections) were used to obtain non-linear PDS's. E.g.

M5 | 0 1 2 3 | 4 5 | 6 7 | 8 9 #  (4 5 6 7: $CY_2$; 8 9 #: $\mathbb{R}^3$)

$$\partial\bar{\partial}\sqrt{g_4} + \Box_{\mathbb{R}^3} J = \text{sources}$$

This is the Bianchi identity ($dG$ =sources) for the geometry – a type of $SU(2)$-structure in $d = 7$.

• In our approach these equations are reproduced as special cases, and generalizations to other cycles are straightforward. E.g.

Example 3a:

M5 wrapped on associative 3-cycles – $\mathcal{N} = 1$ in $d = 3$

M5 | 0 1 2 | 3 4 5 | 6 7 8 9 | #  (3 4 5 6 7 8 9: $G_2$; #: $\mathbb{R}$)

$$d_7\left[e^{-6\Delta} *_7 d_7(e^{6\Delta}\phi)\right] + \Box_{\mathbb{R}}(e^{6\Delta} *_7 \phi) = \text{sources}$$

Example 3b:

M5 wrapped on SLAG 3-cycles – $\mathcal{N} = 2$ in $d = 3$

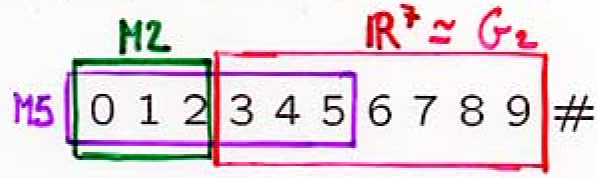
M5 | 0 1 2 | 3 4 5 | 6 7 8 | 9 #  (3 4 5 6 7 8: $CY_3$; 9 #: $\mathbb{R}^2$)

$$d_6\left[e^{-9\Delta} *_6 d_6(e^{6\Delta}\text{Im}\Omega)\right] + \Box_{\mathbb{R}^2}(e^{3\Delta}\text{Re}\Omega) = \text{sources}$$

## Example 4: A 'dyonic' solution (physical significance of $\sin\zeta$)

- M5/M2 bound state solution (in flat space)

[Izquierdo, Lambert, Papadopoulos, Townsend]




From the general conditions, inserting a simple ansatz, we obtain:

$$d\hat{s}^2_{11} = H^{-\frac{2}{3}}(\sin^2\alpha + H\cos^2\alpha)^{\frac{1}{3}}\Big[ds^2(\mathbb{R}^{1,2})$$

$$+\frac{H}{\sin^2\alpha + H\cos^2\alpha}d\mathbf{u}.du + Hd\mathbf{x}.dx\Big]$$

$\tan^2\zeta = \frac{1}{H}\tan^2\alpha$ $\quad$ $\alpha$ is a constant

Flux: $G = \frac{1}{2}\cos\alpha *_5 dH + \frac{1}{2}\sin\alpha dH^{-1}\wedge \mathrm{vol}(\mathbb{R}^{1,2}) + \sin 2\alpha(\dots)$

$G$ equation of motion $\quad\Rightarrow\quad \Box_{\mathbb{R}^5} H = 0$

► This solution interpolates between the flat M5-brane and M2-brane

- It is an instance of a class of 'dyonic' or 'dielectric' solutions, with interpolating supersymmetry

► $\sin\zeta$ measures the ratio of M5 to M2 brane charges.

E.g. $\sin\zeta = 0 \rightarrow$ the M2-brane charge vanishes

## The role of Generalized Calibrations

The set of supersymmetry conditions, constraining the type of $G$-structure, can be nicely reinterpreted in terms of generalized calibrations

• Special holonomy manifolds are characterized by (one or more) covariantly constant form(s) $\Xi$. These forms can be used to characterize special sub-manifolds, which are called 'calibrated'

A 'calibration' is a form $\Xi$, such that:

- $d\Xi = 0$

- $\iota_V^* \Xi \leq \text{vol}_V$ for any tangent plane V

A cycle $\Sigma$ is calibrated by $\Xi$ if the inequality is saturated for any plane tangent to $\Sigma$

▶ Important: a cycle $\Sigma$ is

calibrated $\Leftrightarrow$ volume minimizing $\Leftrightarrow$ supersymmetric

E.g. in a $G_2$-holonomy manifold there are the associative ($\phi$) and the co-associative ($*\phi$) calibrations

• In the presence of fluxes the notion of calibration should be extended $\rightarrow$ 'generalized calibrations'

The definition is changed replacing the requirement $\mathrm{d}\Xi = 0$ with

- $\mathrm{d}\Xi =$ appropriate fluxes

'Appropriate' means that the condition can be used to show that a probe brane has minimal energy when it wraps a calibrated cycle $\Sigma$:

$$E[\Sigma, \mathrm{flux}] \quad = \quad \mathrm{Mass}[\mathrm{vol}_\Sigma] + \mathrm{WZ}[\mathrm{flux}]$$

• The notion of calibration can also be extended in a different way, by switching on world-volume fields on a probe brane

• On an M5-brane there is a self-dual three-form $H$. Taking this into account, [Bärwald,Lambert,West] derived a bound for the M5-brane energy $E[\Sigma, H]$

$$\nu + \chi \wedge H \leq \mathrm{vol}_V\, E$$

$\nu$, $\chi$ are usual bi-linears constructed from parallel spinors

• Using $\mathrm{d}\nu = \mathrm{d}\chi = 0 = \mathrm{d}H \quad \Rightarrow \quad$ A calibrated pair $(H, \Sigma)$, that saturates the bound, gives minimal M5-brane energy in its equivalence class, and is supersymmetric

Using the Hamiltonian formulation of the M5-brane, we derived a generalized BPS bound in the presence of background $G$-flux:

$$\nu + \chi \wedge H + \mathcal{C}_0 \text{vol}_V \leq \text{vol}_V E$$

- $\mathcal{C}_0 \text{vol}_5 = i_k C_6 - \frac{1}{2} i_k C \wedge (C - 2H)$ is a new term

- $k$ is a time-like vector field $\to k = \frac{\partial}{\partial t}$

- the energy $E$ depends on the background $G$-flux

- $\text{d}H = \iota^* G$

▶ A pair $(\Sigma, H)$ is calibrated, i.e. it has minimal M5-brane energy and is supersymmetric iff, the forms $k, \chi, \nu$ satisfy

$$\begin{aligned} \text{d}k &= \tfrac{2}{3}\chi \lrcorner G - \tfrac{1}{3}\nu \lrcorner * G \\ \text{d}\chi &= i_k G \\ \text{d}\nu &= i_k * G - \chi \wedge G \end{aligned}$$

• These are the necessary and sufficient conditions for the existence of (at least) one Killing spinor in M-Theory [Gauntlett,Pakis]

• These conditions are equivalent to our set of conditions. E.g. $\chi \sim \sin\zeta \text{d}x^1 \wedge \text{d}x^2$

⇒ $\sin\zeta$ Measures the amount of M2-brane charge induced on the M5-brane by the WZ coupling with $H$

## M-Theory on six-manifolds [Gauntlett,DM,Sparks,Waldram]

(to appear)

Motivations:

• Study the most general (warped) supersymmetric $AdS_5$ solutions in M-Theory $\rightarrow$ dual to 4-dim SCFT

(• Characterize 5-dim Minkowski flux compactifications of M-Theory)

warped metric $\quad d\hat{s}^2_{11} = \mathrm{e}^{2\Delta(x)}(ds^2_5 + g_{mn}dx^m dx^n)$

$ds^2_5$ → $\mathbb{R}^{1,4}$ or $AdS_5$; $g_{mn}dx^m dx^n$ → generic $M_6$

flux $\quad G_{mnpq}$ arbitrary

spinor $\quad \eta = \psi \otimes \xi$

$\psi$ → 5d; $\xi$ → 6d

• if $\xi$ is a 6d chiral spinor

$\Rightarrow \quad G = 0$, $\Delta$ =constant, $M_6$ is a Calabi–Yau three-fold

(This follows also from the superpotential $W = \int J \wedge G$ [Behrndt,Gukov])

• Taking $\xi$ non-chiral, once again we can have non-trivial flux

• $\xi$ complex, non-chiral spinor $\Leftrightarrow SU(2)$-structure on $M_6$, characterized by

$$J, \Omega_{(2,0)}, K^1, K^2$$

# The geometry on $M_6$

- The metric has the canonical form:

$$d\hat{s}_6^2 = g^4_{ij}(x,y)dx^i dx^j + \sec^2\zeta dy^2 + e^{6\Delta}\cos^2\zeta(d\psi + \rho)^2$$

- $K^{2\#} = \sec\zeta \frac{\partial}{\partial\psi}$ is a Killing vector $\rightarrow$ $U(1)$ $R$-symmetry
- At any fixed y, $M_4$ is Kähler
- $\Omega = e^{i3m\psi}\Omega_0 \Rightarrow \Omega \in \mathcal{L}^k \otimes \Lambda^{2,0}$ ($\mathcal{L}$ is $U(1)$ bundle)
- The flux is completely fixed:

$$*_6 F = e^{-6\Delta} d(e^{6\Delta}\cos\zeta K^2) - 4m(J - K^1 \wedge K^2 \sin\zeta)$$

► The (2,0) form, and the spinors, are charged under the $U(1)$ isometry

- Analogous to the conifold (see [Klebanov,Witten]). The conifold is a cone over the $T^{1,1}$ space:

$$\begin{array}{ccc} U(1) & \rightarrow & T^{1,1} \\ & & \downarrow \\ & & S^2 \times S^2 \end{array}$$

$U(1)$ is an isometry $\rightarrow$ KK reducing gives back $S^2 \times S^2$

$S^2 \times S^2$ is Kähler, with (2,0)-form $\Omega_{(2,0)} = e^{i\varphi}\Omega_0$

$\varphi \in [0, 4\pi]$ $(k = 2)$

$\Rightarrow$ $\Omega_{(2,0)}$ has charge 2 $\rightarrow$ spinors have charge 1

# Example: $\mathcal{N}=1$ solution of [Maldacena,Nuñez]

- The solution was constructed in 7d gauged supergravity and uplifted to M-Theory is $\text{AdS}_5 \times_{\text{warped}} M_6$
- It corresponds to the near-horizon limit of M5-branes wrapped on $H^2$

## Geometry of $M_6$

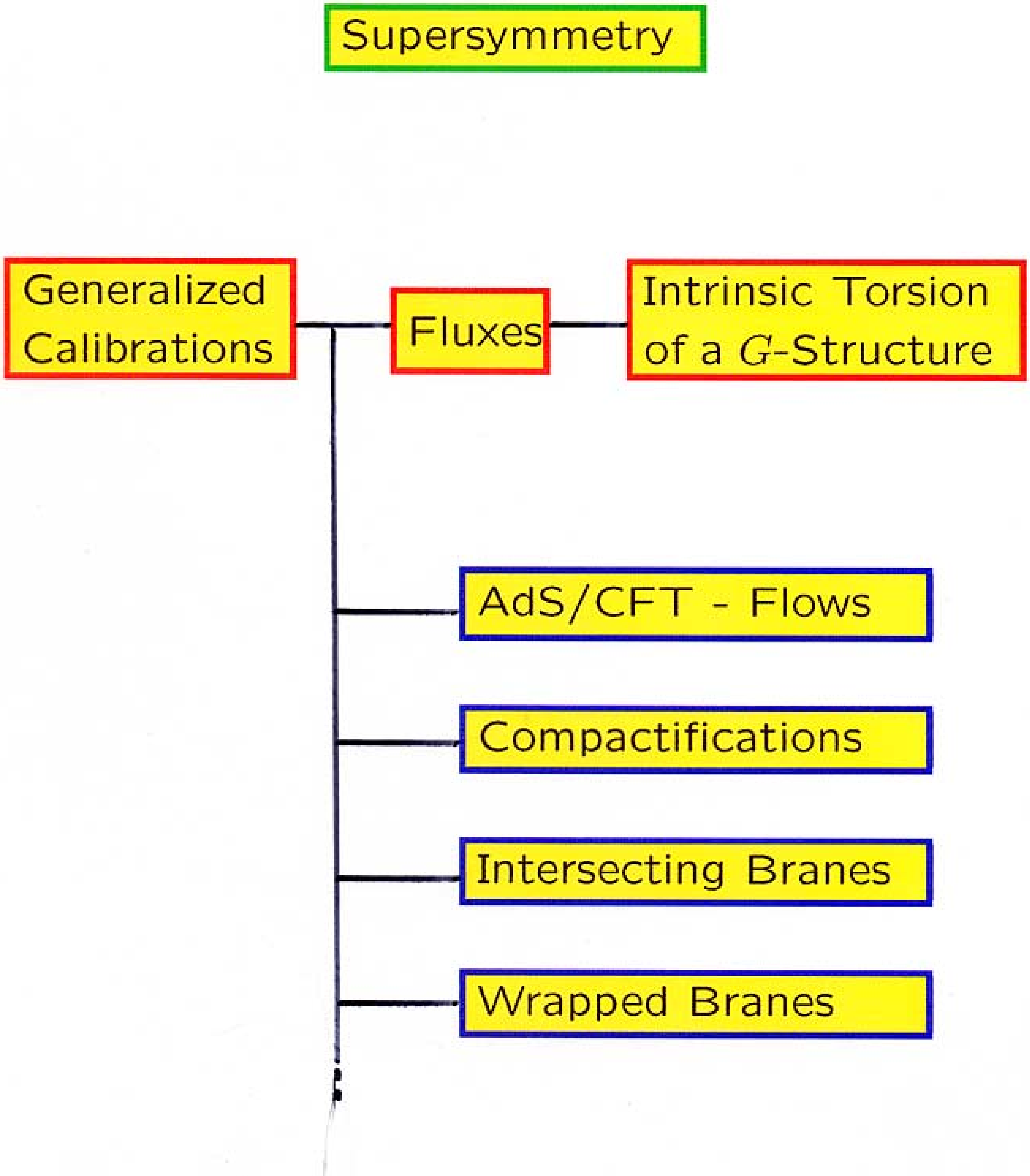
- The base is $H^2 \times S^2$ with non-Einstein metric

$$ds_4^2 \;=\; \frac{1}{Y^2}(dX^2 + dY^2) + f(y^2)(d\theta^2 + \sin^2\theta d\phi^2)$$

- $\Omega = e^{i3\psi} \quad \psi \in [0, \frac{4\pi}{3}] \quad \Rightarrow \quad \Omega$ has $R$-charge 2 ✓

▶ One can think of $M_6$ as a generalization of the conifold

| conifold | $M_6$ Maldacena–Nuñez |
|---|---|
| $r$ radial direction | cohomegeinity-one in $y$ |
| $T^{1,1}$ base of the cone | $M_5$ fixed-$y$ sections |
| $S^1 \rightarrow T^{1,1} \downarrow S^2 \times S^2$ | $S^1 \rightarrow M_5 \downarrow H^2 \times S^2$ |


Supersymmetry
Generalized Calibrations
Fluxes
Intrinsic Torsion of a $G$-Structure
AdS/CFT - Flows
Compactifications
Intersecting Branes
Wrapped Branes

## Possible directions for future work

▶ construct new examples of:

• AdS solutions (AdS/CFT or M-Theory vacua)

• 'dielectric solutions', i.e. solutions with interpolating supersymmetry

• compactifications to Minkowski space-time (hard, need to include corrections)

▶ Derive generalized superpotentials for flux compactifications. This is a general problem, i.e. arising also in String Theory compactifications, that can be addressed using generalized calibrations

# Generalized calibrations and superpotentials

Special holonomy $\longrightarrow$ Calibrations $\longrightarrow$ Superpotentials

[Gukov,Vafa,Witten] [Gukov]

$$W = \int (\text{calibration}) \wedge (\text{flux})$$

• M-Theory on $Spin(7)$-manifolds [Acharya,De la Ossa,Gukov] [M. Becker,Constantin]

$$W = \int \Psi \wedge G$$

$\Longrightarrow$

$$d\hat{s}^2_{11} = H^{-2/3} ds^2(\mathbb{R}^{1,2}) + H^{1/3} d\tilde{s}^2(Spin(7))$$

$$G = \text{vol}_3 \wedge \text{d}(H^{-1}) + G_{27}$$

$$\tilde{*}\tilde{\Box} H + \frac{1}{2} G_{27} \wedge G_{27} = X_8$$

• We have shown that there exist much more general solutions

$G$-structure $\longrightarrow$ Generalized Calibrations $\longrightarrow$ [?]

• There should be more general superpotentials related to generalized calibrations!

$$W = \int (\text{generalized calibration}) \wedge (\text{flux})$$