

New results on the String/Black hole correspondence

Based on :

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Plan:

- The problem
- Two dimensional black holes
- Higher dimensional black holes

For a four dimensional Schwarzschild black hole of mass M , the Hawking temperature is

$$T_{bh} = \frac{1}{8\pi M G_N}$$

As M decreases, T_{bh} increases (negative specific heat).

Question: What happens when T_{bh} reaches the string scale?

Susskind
Horowitz
Polchinski
⋮

String/Black hole correspondence

When the string frame curvature at the horizon of a black hole reaches the string scale, the typical black hole state becomes a typical string state with the same quantum numbers. The black hole and string entropies match in this regime up to factors of order one.

These ideas have been checked (qualitatively) to work for many kinds of charged and uncharged black holes.

But, they raise a number of questions:

* Why is curvature at the horizon $\approx$ string scale the right criterion?

Curvature is a local quantity, and the horizon is not a special place locally. Typically curvature varies between zero far from black hole and infinity at singularity.

There are examples where curvature peaks before one reaches the horizon. Transition still seems to be associated with curvature at horizon.

* What happens when curvature at horizon $\approx$ string scale

- Is black hole picture valid?

Naively, black hole should be described by a sigma model with $O(1)$ α' corrections, but no reason for g_s corrections to be large.

- Is free string picture valid?

Naively no, since $M \approx \frac{m_s}{g_s^2}$ so expect large mass corrections from loops.

- Is there a sharp transition between string and black hole phases, or smooth crossover from one description to other?
- Near transition, black hole is small ($R_{\text{schw}} \approx \ell_s$). The string it turns to is, at least naively, large. How can a small black hole turn into a large string?
- Can one do better than agreement up to $O(1)$ coefficients?

A nice playground where above questions can be answered is two dimensional black holes, for which the black hole sigma model is exactly solvable.

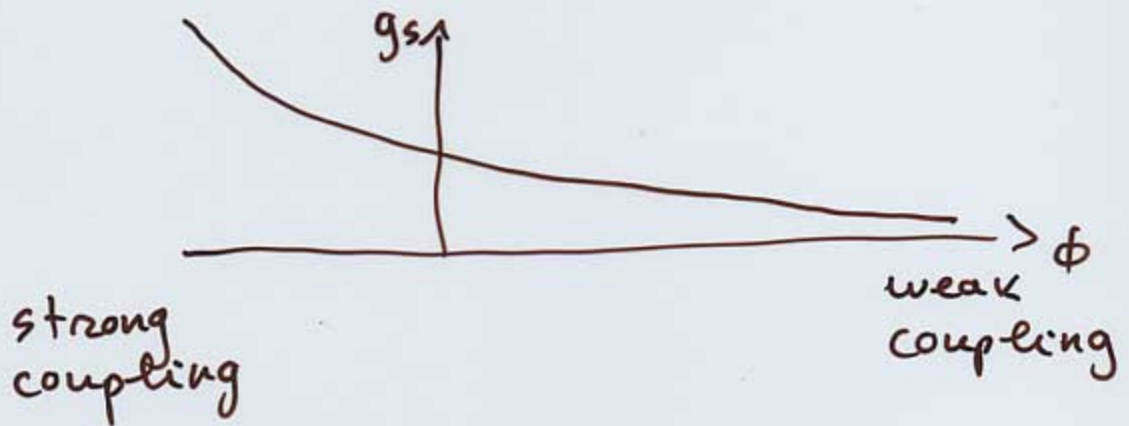
I will discuss this case first and then move on to higher dimensional ones.

Two dimensional black hole

We start with a two dimensional spacetime labeled by (ϕ, t) :

$$ds^2 = d\phi^2 - dt^2$$

$$\bar{\Phi} = -\frac{Q}{2}\phi, \quad g_s = e^{-\frac{Q}{2}\phi}$$



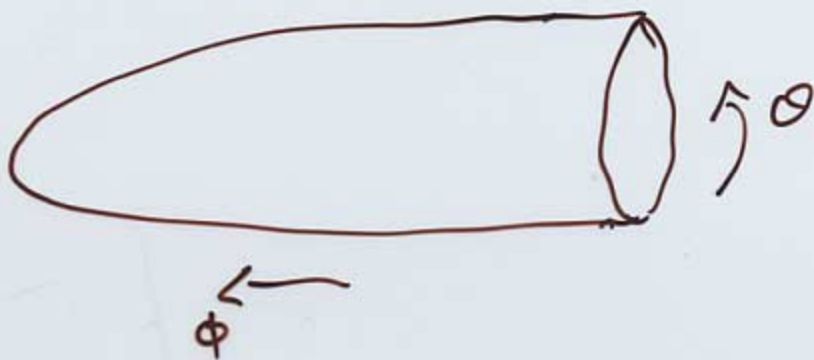
String theory in this spacetime has a black hole solution

$$ds^2 = d\phi^2 - (\tanh^2 \frac{\alpha}{2} \phi) dt^2$$

$$\bar{\Phi} = -\log \cosh \frac{\alpha}{2} \phi$$

It is exactly solvable, coset $\frac{SL(2, \mathbb{R})}{U(1)}$

To study thermodynamics, continue $t \rightarrow i\theta$. This gives cigar



Hawking temperature: $T_{bh} = \frac{3}{4\pi}$

Since temperature independent of energy,
entropy is Hagedorn: $S_{bh} = \frac{4\pi}{\underbrace{Q}_{\beta_{bh}}} E$

These results are exact (no α' corrections to metric and dilaton in fermionic string, due to $N=2$ superconformal symmetry).

To vary temperature, need to change Q (i.e. vary the model).

What happens when $T_{bh} \approx 1$?

Although the metric and dilaton are not corrected, there is an important effect that is missed: a condensate of a tachyon winding around θ :

FZZ
Giveon, DK



$$\langle T \rangle \sim \mu e^{-\frac{1}{3}\phi}$$

as $\phi \rightarrow \infty$

One way to understand this is to note that Euclidean black hole is defined in classical string theory as background that looks asymptotically like $R_\phi \times S^1$ and breaks winding symmetry around S^1 .

For small Q , tachyon goes rapidly to zero as $\phi \rightarrow \infty$ and influences physics only in a region of size $\delta\phi \approx 1$ around horizon.

As Q increases, the size of this region grows; it diverges as $Q^2 \rightarrow 2$:

$$\langle T \rangle \sim \mu e^{-\frac{1}{Q}\phi} = \underbrace{\psi(\phi)}_{\text{wavefn of tachyon}} g_s(\phi) \quad \underbrace{\sim e^{-\frac{Q}{2}\phi}}_{\text{metric factor}}$$

so: $\psi(\phi) \sim \mu e^{(\frac{Q}{2} - \frac{1}{Q})\phi}$

For $Q^2 > 2$, Euclidean black hole is non-normalizable due to tachyon condensate.

As $Q^2 \rightarrow 2$, the background looks more and more like infinite cylinders



with tachyon condensate

$$T = \mu e^{-\frac{4}{3}\phi}$$

The region near the tip of the cigar has been smeared by condensate.

What does condensate of winding tachyon mean?

Can think of it as a condensate of strings which are localized in ϕ and stretched in X .

$\Rightarrow$ We are computing $\text{Tr} e^{-\beta H}$ over multistring states in grand canonical ensemble.

Thus, expect black hole thermodynamics to approach that of fundamental strings as $Q^2 \rightarrow 2$.

Check:

We saw that

$$S_{bh} = \frac{4\pi}{Q} E$$

Fundamental string
entropy:

$$S_f = 4\pi \sqrt{1 - \frac{Q^2}{4}} E$$

As $Q^2 \rightarrow 2$, the two coincide, as expected.

Minkowski black hole

Continuation from Euclidean space
gives black hole in thermal bath
at Hawking temperature $T_{bh} = \frac{\mathcal{Q}}{4\pi}$.

An observer at fixed ϕ sees a
higher Hawking temperature

$$T_{bh}(\phi) = \frac{\mathcal{Q}}{4\pi} \frac{1}{\tanh \frac{\mathcal{Q}}{2} \phi}$$

Region in which

$$T_{bh}(\phi) > T_{hagedorn}$$

should receive large string corrections
!!

(susskind) Stretched horizon

Stretched horizon corresponds to

$$\frac{Q}{4\pi} \frac{1}{\tanh \frac{Q}{2}\phi} > \frac{1}{2\pi \sqrt{4-Q^2}}$$

This is the region in which the condensate of fundamental strings smears geometry.

As $Q^2 \rightarrow 2$, size of stretched horizon diverges. This happens when Hawking temperature approaches perturbative Hagedorn.

Summary of 2d black hole discussion

- sharp transition occurs when $T_{bh} = T_{str}$
- At that point we have a small black hole surrounded by a very large stretched horizon. Can't tell it from fundamental strings.
- Entropies of black holes and strings agree precisely at the transition.

Charged two dimensional black holes

The above discussion can be generalized to charged black holes. Find:

$$S_{\text{BH}}(m, q) = \frac{2\pi}{Q} (m + \sqrt{m^2 - q^2})$$

$$S_f(m, q) = 2\pi \sqrt{1 - \frac{Q^2}{4}} (m + \sqrt{m^2 - q^2})$$

Transition occurs again at $Q^2 = 2$.

At that point $\beta_{\text{BH}} = \beta_f = \frac{4\pi}{Q} \frac{1}{1-a^2}$

Chemical potential : $-\beta a = \left(\frac{\partial S}{\partial q} \right)_m$

$$a = \tan \frac{\alpha}{2}$$

$$\sin \alpha = \frac{q}{m}$$

Note that the transition can occur at a temperature that is arbitrarily low, if $\frac{q}{m} \approx 1$ or $\alpha \approx 1$.

$\frac{1}{\beta\phi}$ is the limiting temperature for a thermal ensemble with chemical potential α

Schwarzschild black hole in d dimensions:

$$ds^2 = - \left(1 - \left(\frac{r_0}{r} \right)^{d-3} \right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r} \right)^{d-3}} + r^2 d\Omega_{d-2}^2$$

$$r_0^{d-3} = \frac{16\pi G_d m}{(d-2) \Omega_{d-2}}$$

↑
area of unit $(d-2)$ -sphere

$$\beta_{\text{BH}} = \frac{4\pi r_0}{d-3}$$

Note: this is leading order in α' . Unlike $2d$, expect α' corrections to metric, dilaton... Will ignore them here.

Following two dimensional example,
expect that Euclidean Schwarzschild
has a condensate of tachyon wrapped
around S^1 ,

$$T(r) \sim \frac{1}{r^{d-3}} e^{-K_0 r^2}$$

as $r \rightarrow \infty$

Using leading α' solutions get:

$$K_0^2 = \left(\frac{r_0}{\alpha'} \right)^2 - \underbrace{1}_{\text{mass of closed string tachyon}}$$

contribution from winding around S^1 .

or

$$r_0 = d-3$$

At that point $\beta_{bh} = \beta_f$, i.e.

Hawking temperature approaches the limiting string temperature.

Plugging into entropy get:

$$\frac{S_{bh}}{S_f} = \frac{r_0}{d-2} = \frac{d-3}{d-2}$$

at transition

so black hole and fundamental string entropies do not quite agree, but this is to be expected since need to add α' corrections to gravity background.

black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2$$

$$f(r) = \left[1 - \left(\frac{r_-}{r} \right)^{d-3} \right] \left[1 - \left(\frac{r_+}{r} \right)^{d-3} \right]$$

$$r_{\pm}^{d-3} = \frac{8\pi Gd}{(d-2)\Omega_{d-2}} \left(m \pm \sqrt{m^2 - g^2} \right)$$

$$S_{bh} = \frac{\Omega_{d-2}}{4Gd} r_+^{d-2}$$

$$\beta_{bh} = \frac{4\pi r_+}{(d-3)(1-a^2)}$$

chemical
potential:

$$a = \left(\frac{r_-}{r_+} \right)^{\frac{d-3}{2}}$$

of Reissner-Nordstrom black hole is the same as that of fundamental strings, which have

$$S_f(m, q) = 2\pi (m + \sqrt{m^2 - q^2})$$

$$\beta_f = \frac{4\pi}{1-a^2}$$

Transition is again at

$$\beta_{\text{th}} = \beta_f \Rightarrow r_+ = d-3$$

where

$$\frac{S_{\text{th}}}{S_f} = \frac{r_+}{d-2} = \frac{d-3}{d-2}$$

independent of charge.

* Solve worldsheet sigma model of (Euclidean) Schwarzschild, RN black hole. Predictions: tachyon condensate, exact matching to fundamental strings at $\beta_{BH} = \beta_B$.

* Heterotic strings: a series of black hole solutions labelled by chemical potential, with no charge.