(Cherenkov) Radiation of spin excitations in the nonlinear transport regime and conductance peak in quantum wires

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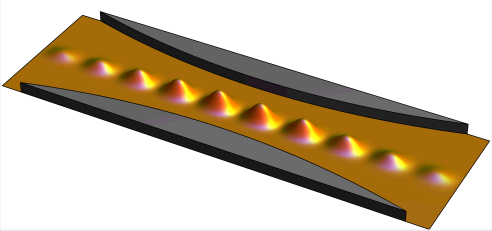
Rutgers University, 19 January 2010

Quantum wires

- (Quasi) one-dimensional electron systems, in which quantum physics plays a role.
- Conventionally, 2D semiconductor heterostructures with 1D constriction formed by gates

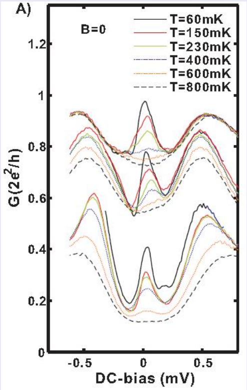


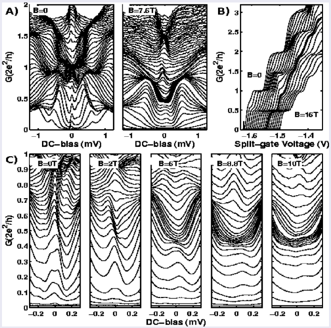
At low density, $na_B \ll 1$, $\Longleftrightarrow$ strong interactions

Wigner crystal correlations (near order)

Conductance peak in quantum wires

- $G(V) = \mathrm{d}I(V)/\mathrm{d}V$ exhibits a peak near zero bias.
- Common feature in experiments





from T.-M. Chen et al., Phys. Rev. B 79, 153303 (2009)

Conductance peak in quantum wires

Possible explanations?

Kondo peak

- requires magnetic impurity
- should be split in the magnetic field - not necessarily the case

This work

- peak arises in a inhomogeneous system of strongly interacting one-dimensional electrons
- No other ingredients required

Model and Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_{\mathrm{i}}$$

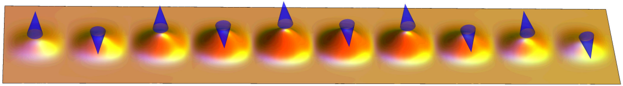
$$\hat{H}_0 = \int \mathrm{d}x\, \psi_\sigma^\dagger(x) \left[\frac{-\partial_x^2}{2m} + U(x) - \mu \right] \psi_\sigma(x)$$

$$\hat{H} = \frac{1}{2} \int \mathrm{d}x\, \mathrm{d}x'\, \psi_\sigma^\dagger(x) \psi_{\sigma'}^\dagger(x') \frac{e^2}{\kappa |x - x'|} \psi_{\sigma'}(x') \psi_\sigma(x)$$

$U(x)$ - gate potential

- Weak interactions in the leads, $r_s(x) = n(x) a_B \ll 1$
- Strong interactions in the wire, $r_s(x) = n(x) a_B \gg 1 \Rightarrow$ near Wigner crystal order.

Collective excitations in Wigner crystal regime



$$\hat{H} = \hat{H}_\rho + \hat{H}_\sigma$$

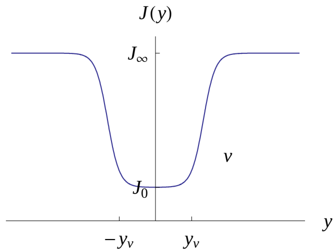
Charge:

$$\hat{H}_\rho = \frac{u_\rho}{2\pi} \int \mathrm{d}x \left[K_\rho (\partial_x \theta_\rho)^2 + \frac{1}{K_\rho} (\partial_x \varphi_\rho)^2 \right]$$

Spin: Heisenberg chain

$$\hat{H}_\sigma = \sum_l J(l + 1/2) \mathbf{S}_{l+1} \mathbf{S}_l$$

Strong interactions



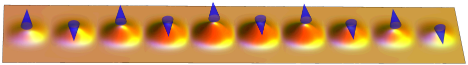
- exchange $J(x)$ – exponentially small compared to Coulomb (Fermi) energy $J(0) \sim 0 - 10$K
- $\Longrightarrow$ One can easily have $T, eV \gtrsim J(0)$

Assumption

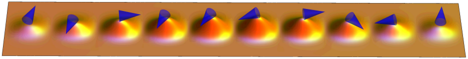
$J(x)$ varies slowly on lattice scale, $J'(x) \ll J(x)$
$\Longleftrightarrow$ $N \gg 1$ – number of sites in Wigner crystal, long wire.

Spin chain

Long (spin) wavelength limit: $T, eV \ll J(0)$ - bosonization OK



“spin incoherent” regime: $T, eV \gtrsim J(0)$



Previous results on spin-dependent transport

$I = GV$ linear transport regime $eV \ll J(0)$ [K. Matveev PRB (2004)]

- $G = \frac{2e^2}{2\pi\hbar}$ at $T \ll J(0)$ (recovers bosonization results)
- $G = \frac{e^2}{2\pi\hbar}$ at $T \gg J(0)$

Spin excitations suppress conductance!

– relevant to 0.7-structure physics - common for short wires, shifts to e^2/h in longer wires.

This work: nonlinear transport regime $eV \gtrsim J(0)$, $T = 0$

Can we expect similar in $\mathrm{d}I(V)/\mathrm{d}V$?

Transport in the regime of applied current I

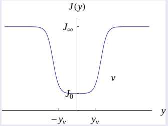
- Wigner crystal drifting through the wire with "velocity" $v = I/e$
- Calculate energy dissipation $W = W_\rho + W_\sigma$, $W = VI \Rightarrow V(I)$ curve.

Charge sector, $T, eV \ll \epsilon_F$

$W_\rho = R_\rho I^2$, $R_\rho = 2\pi\hbar/(2e^2)$.

Spin sector

$$\hat{H}_\sigma = \sum_l J(l - vt + 1/2)\mathbf{S}_{l+1}\mathbf{S}_l$$



Jordan-Wigner (JW) transformation

$$\hat{H}_\sigma = \frac{1}{2}\sum_l J(l - vt + 1/2)\left[a^\dagger_{l+1}a_l + a^\dagger_l a_{l+1} + 2\Delta\left(a^\dagger_{l+1}a_{l+1} + \frac{1}{2}\right)\left(a^\dagger_l a_l + \frac{1}{2}\right)\right]$$

$\Delta = 1$, system of strongly interacting fermions on a lattice

Find dissipation due to nonstationary inhomogeneous $J(l - vt + 1/2)$

XY model, $\Delta = 0$, noninteracting JW fermions

$$\hat{H}_\sigma = \frac{1}{2}\sum_l J(l - vt + 1/2)(a^\dagger_{l+1}a_l + a^\dagger_l a_{l+1})$$

Recipe

1. Solve single-particle scattering problem
2. Find distribution functions in the leads $\Longleftrightarrow$ Find dissipation

Classical limit, infinitely smooth $J(x)$

$$\dot{x} = \partial_p H(x,t,p),\ \dot{p} = -\partial_x H(x,t,p),\ H(x,t,p) = J(x - vt)\cos p$$

Galilean transformation $y = x - vt$, $\Longrightarrow$ stationary problem

$$H(y,p) = J(y)\cos p - pv$$

- x: Wigner crystal frame, constriction $J(x - vt)$ moving
- $y = x - vt$: laboratory frame, constriction $J(y)$ resting, but JW fermions acquire drift velocity v

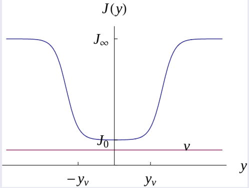
Dissipation W_σ may be determined by distribution functions in the leads

$$W_\sigma = \int_0^{2\pi} \frac{\mathrm{d}p}{2\pi} \left(\frac{\mathrm{d}\epsilon_\infty(p)}{\mathrm{d}p} - v \right) \epsilon_\infty(p)[f_+(p) - f_-(p)],\ \epsilon_\infty(p) = J(\infty)\cos p$$

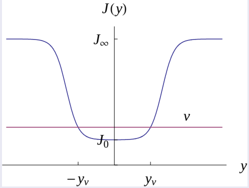
$$W_\sigma \neq 0 \Longleftrightarrow f_+(p) \neq f_-(p)$$

Two qualitatively different regimes, $\partial_p H(y,p) = -J(y)\sin p - v$

$v < J(0)$, $-\infty \to +\infty$ and $+\infty \to -\infty$

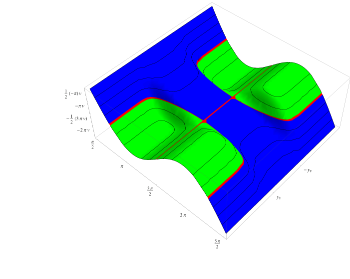


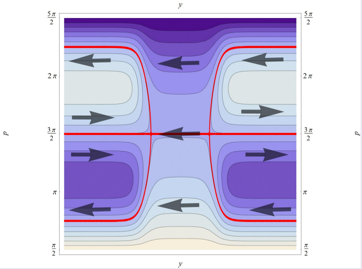
$v > J(0)$, No $-\infty \to +\infty$, only $+\infty \to -\infty$

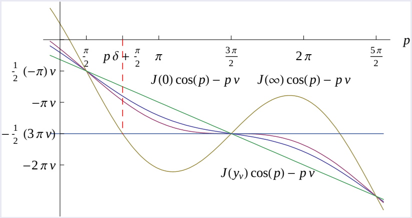


Turning points $\pm y_v$, $J(\pm y_v) = v$. In $-y_v < y < y_v$, only L-movers (from $y = +\infty$) exist.

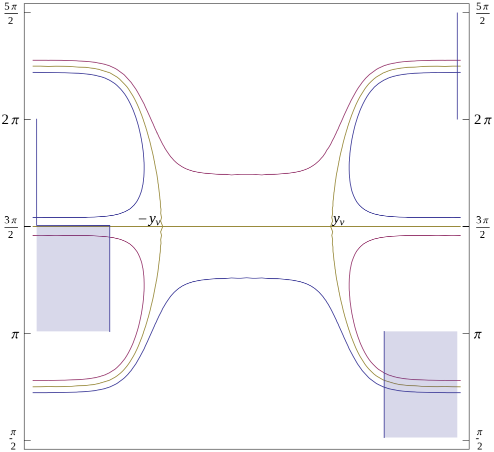
$v > J(0)$, $H(y,p) = J(y)\cos p - pv$



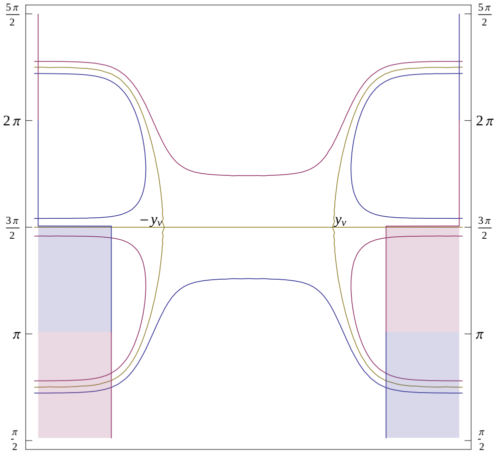




Distribution functions and dissipation

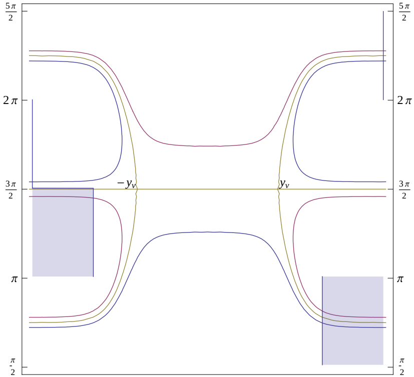


Distribution functions and dissipation



$f_+(p) = f_-(p) \Longrightarrow$ in the classical limit the dissipation is absent ($H = 0$, $T = 0$)

Quantum considerations.



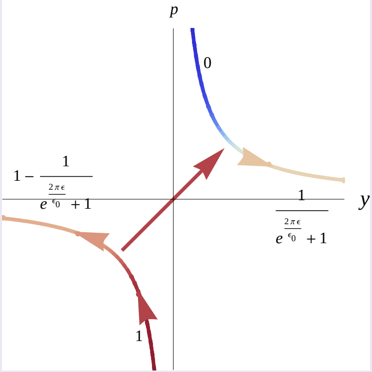
near $\pm y_v$ the same energy ϵ trajectories come arbitrarily close
$\Longrightarrow$ enhanced tunneling between trajectories.

$$H(y \approx \pm y_v, p) = -3\pi v/2 \pm J'(y_v)(y \mp y_v)p$$

knowledge of vicinity of $\pm y_v$ suffices to solve scattering problem.

Scattering problem for $\hat{H} = \epsilon_0 \hat{y}\hat{p}$

$$\hat{H}\chi(y) = \epsilon\chi(y),\ \hat{H} = \epsilon_0 \hat{y}\hat{p} = -\epsilon_0 \mathrm{i}(y\partial_y + 1/2) = \epsilon_0 \mathrm{i}(p\partial_p + 1/2)$$



$$1 - \frac{1}{e^{\frac{2\pi\epsilon}{\epsilon_0}} + 1}$$

$$\frac{1}{e^{\frac{2\pi\epsilon}{\epsilon_0}} + 1}$$

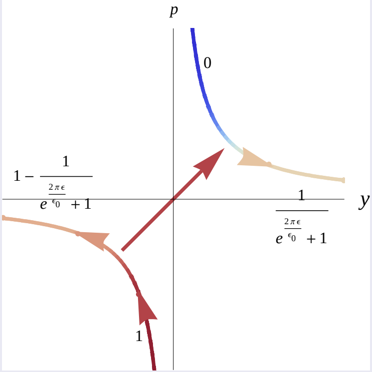
Rotate by $\pi/4$ and the problem equivalent to $H = -\frac{\partial_x^2}{2} - \frac{1}{2}\omega^2 x^2$, solution: L & L, vol. III.

$$\chi(y) = c_\pm^y \frac{1}{|y|^{1/2 - \mathrm{i}\epsilon/\epsilon_0}},\ y \gtrless 0$$

$$\chi(p) = c_\pm^p \frac{1}{|p|^{1/2 + \mathrm{i}\epsilon/\epsilon_0}},\ p \gtrless 0$$

Scattering problem for $\hat{H} = \epsilon_0 \hat{y}\hat{p}$

$$\hat{H}\chi(y) = \epsilon\chi(y), \;\; \hat{H} = \epsilon_0 \hat{y}\hat{p} = -\epsilon_0 \mathrm{i}(y\partial_y + 1/2) = \epsilon_0 \mathrm{i}(p\partial_p + 1/2)$$

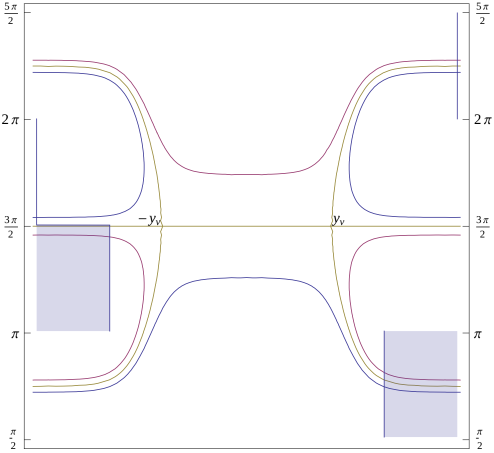


Wave incident from $p = -\infty$:
$c_+^p = 0$, $c_-^p = 1$
Scattered waves

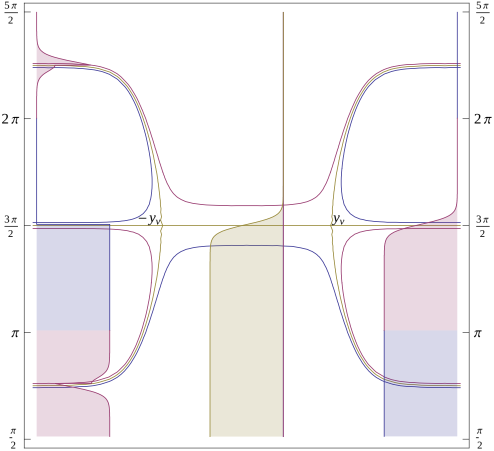
$$\chi(y) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p}{\sqrt{2\pi}} \mathrm{e}^{\mathrm{i}py}\chi(p)$$

Transmission and reflection coefficients are "Fermi functions":
$T(\epsilon) = |c_+^y|^2 = \frac{1}{\exp(2\pi\epsilon/\epsilon_0)+1}$,
$R(\epsilon) = |c_-^y|^2 = 1 - T(\epsilon)$

Distribution functions and dissipation



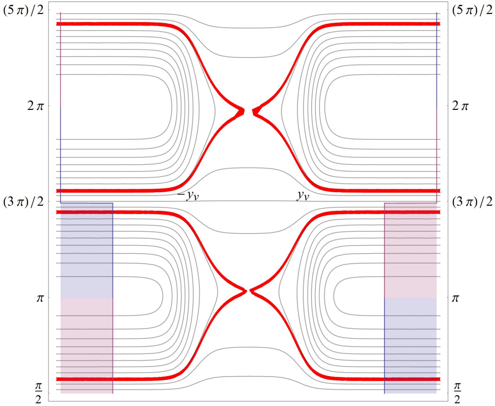
Distribution functions and dissipation



$f_+(p) \neq f_-(p) \Longrightarrow$ finite dissipation due to quantum scattering !!

$$W_\sigma = \frac{\ln(2e)}{2\pi} vJ'(y_v)$$

J(0)>v



$f_+(p) = f_-(p)$ even for quantum scattering, no dissipation, $W_\sigma = 0$

$$W_\sigma = 0,\ v > J(0)$$

$$W_\sigma = \frac{\ln(2e)}{2\pi} v J'(y_v),\ v > J(0)$$

spin *excitations* appear above threshold value $v = J(0)$ of “projectile” velocity
$\Longrightarrow$
“Cherenkov radiation” of spin excitations (?!)

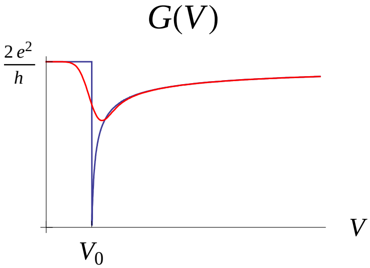
Conductance peak

$V(I) = R_\rho I + V_\sigma(I)$, $eV_\sigma(I)I = W_\sigma \propto J'(y_v)$

$$J(y) = J_0[1 + (y/y_J)^2]$$

$$G(V) = \frac{dI(V)}{dV} = \frac{1}{R_\rho}\left(1 - \frac{1}{\sqrt{1 + \frac{V - V_0}{V_0} N^2}}\right)$$

$V_0 = R_\rho I_0$, $I_0/e = J_0$- threshold value (for XY).



Conclusions

- Spin dynamics plays a key role in transport through quantum wires in Wigner crystal regime
- Experimental behavior well reproduced: conductance peak and T-dependence of G
- note: peak disappears in magnetic field as $G(V) \to \frac{e^2}{2\pi\hbar}$ for $\mu_B H \gg J(0)$
- nearest future: understand the role of interactions in XXZ model

THANK YOU
THE END