Supersymmetry and Moduli Stabilization in Heterotic M-theory

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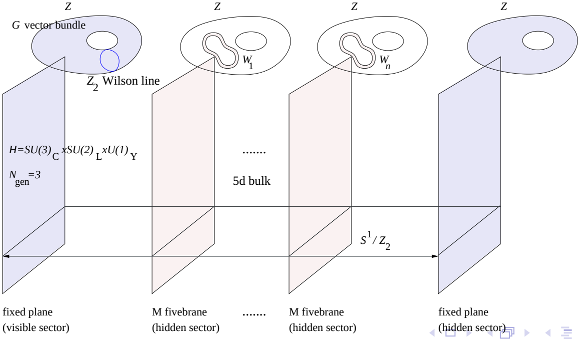
Outline

- $E_8 \times E_8$ Heterotic String and SUSY vacua
- Why we're interested (and the problems)
- Slope stability of a vector bundle
- Stability, Kähler moduli, and the 4d effective field theory
- Holomorphy and complex structure moduli
- Holomorphic bundles $\rightarrow$

$$\begin{cases} \text{Constraints on the complex structure moduli} \\ \text{A } 4d \text{ description} \\ \text{A hidden sector mechanism} \end{cases}$$

Review of Heterotic M-theory

- Horava-Witten Theory: The strongly coupled limit of the heterotic string
- Bulk is 11-dimsensional supergravity
- boundaries support 10-dim E_8 SYM theories
- M5 brane world volume actions for central branes



A heterotic model

One dimension out of 11 is already compact. So to produce a four-dimensional theory, consider the $E_8 \times E_8$ Heterotic string in 10-dimensions:

- One E_8 gives rise to the "Visible" sector, the other to the "Hidden" sector
- Compactify on a Calabi-Yau 3-fold, X - leads to $\mathcal{N} = 1$ SUSY in $4D$
- Also have a holomorphic vector bundle V on X (with structure group $G \subset E_8$)

  V breaks $E_8 \to G \times H$, where H is the Low Energy GUT group
  - $G = SU(n)$, $n = 3, 4, 5$ leads to $H = E_6, SO(10), SU(5)$
- Moduli and Matter
  - $X \Rightarrow h^{1,1}(X)$ - Kähler moduli and $h^{2,1}(X)$ -Complex structure moduli
  - $V \Rightarrow h^1(X, V \times V^\vee)$ Bundle moduli
  - and Matter $\Rightarrow$ Bundle valued cohomology groups, $H^1(V)$,$H^1(\wedge^2 V)$, etc.

Supersymmetric Vacua in Heterotic

The gaugino variation demands that a supersymmetric vacuum to the theory, must satsify the Hermitian-Yang-Mills Equations

- $\delta\chi = 0 \Rightarrow \begin{cases} F_{ab} = F_{\bar{a}\bar{b}} = 0 \\ g^{a\bar{b}} F_{a\bar{b}} = 0 \end{cases}$
- Solution depends on complex structure, Kähler and bundle moduli. Some regions of moduli space will provide a solution, some not.
- Question: Vary moduli such that SUSY is broken...what happens in EFT? Is there a four-dimensional description?

  Answer: There will be a new, positive definite contribution to the potential in the non-SUSY part of moduli space.

Dimensional Reduction

- $S_{partial} \sim \int_{M_{10}} Tr(F^{(1)})^2 + Tr(F^{(2)})^2 - Tr(R^2) + \ldots$
- Bianchi Identity:

  $dH \sim -(Tr(F^{(1)} \wedge F^{(1)}) + Tr(F^{(2)} \wedge F^{(2)}) - Tr(R \wedge R))$
- Wedge with a Kähler form, ω and integrate:

  $\int \omega \wedge (Tr(F^{(1)} \wedge F^{(1)}) + Tr(F^{(2)} \wedge F^{(2)}) - Tr(R \wedge R)) = 0$
- Using the fact that to lowest order X is Ricci-flat Kähler manifold.

  $\Rightarrow \int_{M_{10}} \sqrt{-g}(Tr(F^{(1)})^2 + Tr(F^{(2)})^2 - TrR^2 + 2(F^{(1)}{}_{a\bar{b}} g^{a\bar{b}})^2 + 2(F^{(2)}{}_{a\bar{b}} g^{a\bar{b}})^2 + 4(F^{(1)}{}_{ab} F^{(1)}{}_{\bar{a}\bar{b}} g^{a\bar{a}} g^{a\bar{a}}) + 4(F^{(2)}{}_{ab} F^{(2)}{}_{\bar{a}\bar{b}} g^{a\bar{a}} g^{a\bar{a}})) = 0$
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$$+(F^{(1)}{}_{ab}F^{(1)}{}_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}}) + (F^{(2)}{}_{ab}F^{(2)}{}_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}})\}$$

- So, $(F_{a\bar{b}}g^{a\bar{b}})^2$ and $(F_{ab}F_{\bar{a}\bar{b}}g^{a\bar{a}}g^{a\bar{a}})$ contribute positive semi-definite terms to the $4d$-potential and depend on the HYM equations!

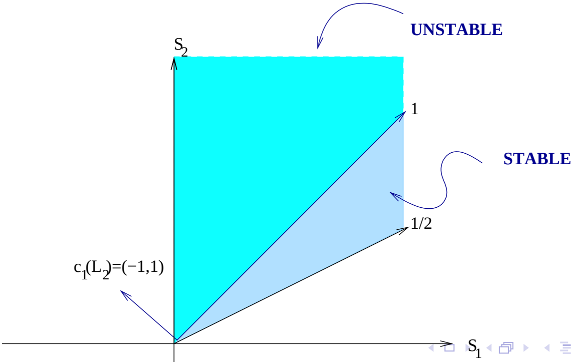
$$\begin{cases} \text{If moduli solve HYM} \rightarrow \text{Potential} = 0 \\ \text{If HYM not satisfied} \rightarrow \text{Potential} \neq 0 \end{cases}$$

- What is the explicit form of this potential?
- Don't know $F_{a\bar{b}}$, F_{ab} and $g^{a\bar{b}}$ except numerically.
- This potential is what we will derive...

Stability

- SUSY $\rightarrow$ Hermitian YM equations, a set of wickedly complicated PDE's
  $F_{ab} = F_{\bar{a}\bar{b}} = g^{a\bar{b}} F_{\bar{b}a} = 0$
- We are saved by the Donaldson-Uhlenbeck-Yau Theorem:
  On each poly-stable, holomorphic vector bundle V, there exists a Hermitian YM connection satisfying the HYM equations
- The slope, $\mu(V)$, of a vector bundle is
  $$\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge \omega \wedge \omega$$
  where $\omega = t^k \omega_k$ is the Kahler form on X (ω_k a basis for $H^{1,1}(X)$).
- V is Stable if for every sub-sheaf, $\mathcal{F} \subset V$, with $0 < rk(\mathcal{F}) < rk(V)$,
  $$\mu(\mathcal{F}) < \mu(V)$$
- V is Poly-stable if $V = \bigoplus_i V_i$, V_i stable such that $\mu(V) = \mu(V_i)\ \forall i$
- Conservation of Misery $\rightarrow$ Tough to find sub-sheaves.

- We will consider a bundle on the CY 3-fold, $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{array} \right]$, with $h^{1,1} = 2$.
- Where V is an $SU(3)$ bundle defined by
  $0 \to V \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(1,-1) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \xrightarrow{f} \mathcal{O}_X(2,1) \to 0$
  which is destabilized in part of the Kähler cone by the rank 2 sub-bundle
  $0 \to \mathcal{F} \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \to \mathcal{O}_X(2,1) \to 0$ with $c_1(\mathcal{F}) = -\omega_1 + \omega_2$.



Splitting a vector bundle

- On a line (in general a hyperplane) in Kähler moduli space, the sub-sheaf $\mathcal{F}$ becomes important
- Can describe V in terms of this sub-sheaf as $0 \to \mathcal{F} \to V \to V/\mathcal{F} \to 0$
- Space of such extensions given by $Ext^1((V/\mathcal{F}), \mathcal{F}) = H^1(X, \mathcal{F} \otimes (V/\mathcal{F})^\vee)$, where the origin of this group is a locus in the moduli space of V for which $V = \mathcal{F} \oplus V/\mathcal{F}$, with $c_1(\mathcal{F}) = -c_1(V/\mathcal{F})$
- On the line with $\mu(\mathcal{F}) = 0$, for SUSY to exist, need $V = \bigoplus_i V_i = \mathcal{F} \oplus V/\mathcal{F}$ to have a poly-stable bundle.
- This means the structure group changes!
  $SU(3) \to S[U(2) \times U(1)]$. Locally $S[U(2) \times U(1)] \approx SU(2) \times U(1)$
- Visible structure group changes to $E_6 \times U(1)$. New $U(1)$ gauge field in the visible $4d$ theory!

- E.g. $SU(3) \to S[U(2) \times U(1)]$.
- Visible structure group changes to $E_6 \times U(1)$. New $U(1)$ gauge field in the visible $4d$ theory!
- The enhanced $U(1)$ is "anomalous" (cancelled by Green-Schwarz Mechanism)
- Matter fields and "moduli" are now charged under this U(1).

  Locally, $E_8 \supset E_6 \times SU(2) \times U(1)$

  $248 \to (1,1)_0 + (1,2)_{-3/2} + (1,2)_{3/2} + (1,3)_0 + (78,1)_0 + (27,1)_1 + (27,2)_{-1/2} + (\bar{27},1)_{-1} + (\bar{27},2)_{1/2}$
- Bundle moduli decompose as

$$H^1(V \otimes V^\vee) \to \left\{ \begin{array}{ccccc} H^1(\mathcal{F} \otimes \mathcal{F}^\vee) & + & H^1(\mathcal{F} \otimes \mathcal{K}^\vee) & + & H^1(\mathcal{K} \otimes \mathcal{F}^\vee) \\ (1,3)_0 & + & (1,2)_{-3/2} & + & (1,2)_{3/2} \end{array} \right.$$

- E_6 Matter: $H^1(V) \to \left\{ \begin{array}{c} H^1(\mathcal{K}) + H^1(\mathcal{F}) \\ (27,1)_1 + (27,2)_{-1/2} \end{array} \right.$

- The complexified Kähler moduli, $T^k = t^k + 2i\chi^k$, transform with a shift symmetry through the axion, χ^k
  - The dilaton, S, and $M5$-brane position moduli also transform under this $U(1)$, but at higher order (we'll come back to this...)
- The $U(1)$-symmetry leads to a $U(1)$ D-term contribution to the $4d$ effective potential

$$D^{U(1)} \sim \frac{\mu(\mathcal{F})}{\mathcal{V}} - \sum_{M,\bar{N}} Q^M G_{M\bar{N}} C^M \bar{C}^{\bar{N}} \tag{1}$$

with a Fayet-Iliopolous (FI)-term $\sim \mu(\mathcal{F})$ -the slope of the relevant sub-bundle $\mathcal{F}$. Here $\mathcal{V}$ is the volume of the CY and C^M are $U(1)$ charged fields.

- This is the explicit form of the potential described earlier by dimensional reduction!
- We can now demonstrate how this EFT describes stability...

Spectrum and $U(1)$ charges

- At the "stability wall", $V \to \mathcal{F} + O(1,-1)$
- At a general point in the stable region: $h^1(V) = 2$, $h^1(V \otimes V^\vee) = 22$
- At the line of semi-stability:

Fields	$E_6 \times U(1)$ charges	number of fields
ϕ^α	1_0	7
f^I	$27_{-1/2}$	2
C^L	$1_{-3/2}$	16

- We can define our theory on the line and consider small perturbations.
- In general: $D^{U(1)} \sim \frac{\mu(\mathcal{F})}{\mathcal{V}} - \sum_{M,\bar{N}} Q^M G_{M\bar{N}} \mathcal{C}^M \bar{\mathcal{C}}^{\bar{N}}$

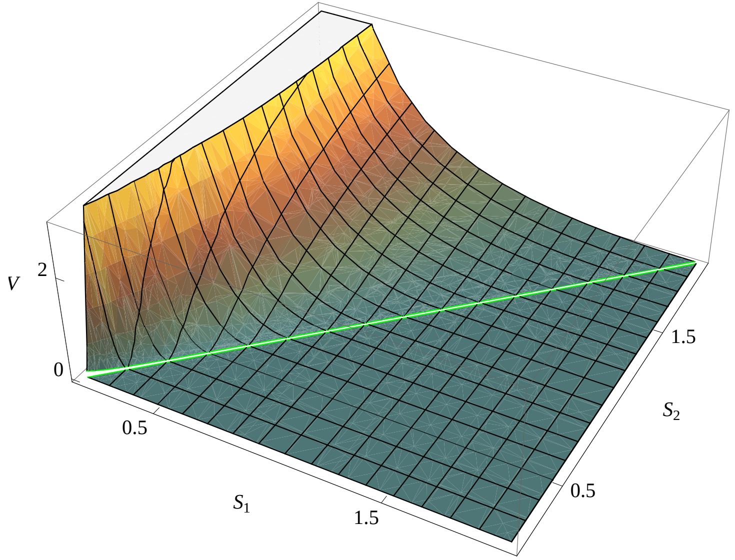
  Here: $D^{U(1)} \sim \frac{9}{4(4\pi)^{2/3}} \frac{-4t^1+t^2}{6t^1t^2+(t^2)^2} + \frac{3}{2} G_{L\bar{M}} C^L \bar{C}^{\bar{M}}$
- $D^{E_6} \Rightarrow \langle f^I \rangle = 0$

Stability in EFT

$$D^{U(1)} \sim \frac{\mu(\mathcal{F})}{\mathcal{V}} + \frac{3}{2} G_{L\bar{M}} C^L \bar{C}^{\bar{M}}$$

- Note that there exists only negatively charged matter!
- $\mu(\mathcal{F}) < 0 \;\Rightarrow < C^L > adjust \rightarrow D^{U(1)} = 0$, SUSY vacuum
- The vev $< C^L >$ describes motion in bundle moduli space away from the decomposable locus! (i.e. $Ext^1(\mathcal{K}, \mathcal{F}) \neq 0$)
- NO positively charged matter

  $\mu(\mathcal{F}) > 0 \;\Rightarrow D^{U(1)} \neq 0$ No SUSY vacuum
- $\mu(\mathcal{F}) = 0 \;\Rightarrow < C^L >= 0,\; D^{U(1)} = 0$, SUSY
- At the wall itself, $< C^L >= 0$ corresponds to the requirement that bundle be split, ($0 \in Ext^1(\mathcal{K}, \mathcal{F})$, as expected!)
- In the stable region: $1\; C^L$-field higgsed. That is, under 1 constraint ($D^{U(1)} = 0$), $16\; C^L \rightarrow 15\; C^L$ ($+7\; \phi^\alpha = 22$ bundle moduli, as expected!)



V
2
0
0.5
S_1
1.5
1.5
S_2
0.5

Conclusions –Kähler Moduli

- At such a 'Stability Wall', the vector bundle must decompose into a direct sum in order to preserve supersymmetry.
- This bundle decomposition $\Rightarrow$ an enhanced $U(1)$ in the visible theory
  - This $U(1)$ leads to a D-term potential that correctly models vector bundle slope-stability
- Observation: This D-term potential is independent of complex structure moduli for all anomaly free and $N = 1$ SUSY theories.
- Didn't have time to discuss...
  - 1 loop correction preserves notion of stability, but incorporates dilaton and 5-brane moduli
  - Stability walls can lead to transitions between bundles
  - Kähler cone substructure can lead to constraints on phenomenology: Yukawa textures, etc.

Holomorphic Vector bundles

- We have discussed the D-terms in detail... but what about the other contributions to the potential?
- Recall, a vector bundle is said to be holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle w.r.t a fixed complex structure. What happens as we vary the complex structure? Must a bundle stay holomorphic for any variation $\delta\mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No!
- In real coordinates we introduce the projectors

$$P_\mu^{\ \nu} = (\mathbb{1}_\mu^{\ \nu} + i\mathcal{J}_\mu^{\ \nu}) \quad \bar{P}_\mu^{\ \nu} = (\mathbb{1}_\mu^{\ \nu} - i\mathcal{J}_\mu^{\ \nu}) \tag{2}$$

Where $\mathcal{J}^2 = -\mathbb{1}$ is the complex structure tensor. Leads to

$$g^{\mu\nu} P_\mu^\gamma \bar{P}_\nu^\delta F_{\gamma\delta} = 0 \tag{3}$$

$$P_\mu^{\ \nu} P_\rho^{\ \sigma} F_{\nu\sigma} = 0 \quad , \quad \bar{P}_\mu^{\ \nu} \bar{P}_\rho^{\ \sigma} F_{\nu\sigma} = 0 \tag{4}$$

Varying the complex structure

- Consider change in $F_{ab} = 0$ under the perturbation

$$\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J} \qquad A = A^{(0)} + \delta A \tag{5}$$

$\delta\mathcal{J} \rightarrow \delta P$

- In terms of the original coords, $\delta\mathcal{J}_a{}^{\bar{b}} = -i\bar{v}^{\bar{b}}_{Ia}\delta\mathfrak{z}^I$ only non-vanishing component of $\delta\mathcal{J}$ (by integrability of C.S.)
- To first order this leads to

$$\delta\mathfrak{z}^I v^c_{I[\bar{a}]} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}}\delta A_{\bar{b}]} = 0 \tag{6}$$

- Rotation of $F^{1,1}$ into $F^{0,2}$ plus change in $F^{0,2}$ due to change in gauge connection.
- Question: For each $\delta\mathfrak{z}^I$ is there a δA which compensates?
- In general, the answer is not always.

Deformation Theory

There are three objects in deformation theory that we need

- $Def(X)$: Deformations of X as a complex manifold. Infinitesimal defs parameterized by the vector space $H^1(TX) = H^{2,1}(X)$. These are the *complex structure* deformations of X.
- $Def(V)$: The deformation space of V (changes in connection, δA) *for fixed* C.S. moduli. Infinitesimal defs measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$. These define the *bundle moduli* of V.
- $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$ where

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0 \qquad (7)$$

  If $\mathcal{P}$ is the total space of the bundle, $\mathcal{Q} = r_* T\mathcal{P}$.
- $H^1(X, \mathcal{Q})$ are the real moduli of a heterotic theory!

The Atiyah Sequence

- $0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0$ is known as the Atiyah sequence.
- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \xrightarrow{d\pi} H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^\vee) \to \dots \quad (8)$$

- If the map $d\pi$ is surjective then $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus H^1(TX)$
- But $d\pi$ not surjective in general! $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus Im(d\pi)$
- $d\pi$ difficult to define, but by exactness, $Im(d\pi) = Ker(\alpha)$ where

$$\alpha = [F^{1,1}] \in H^1(V \otimes V^\vee \otimes TX^\vee) \quad (9)$$

  is the Atiyah Class
- C.S. moduli allowed $\alpha(\delta_{\mathfrak{z}} v) = 0 \quad (0 \in H^2(V \times V^\vee))$. I.e. in $Ker(\alpha)$

$$\delta_{\mathfrak{z}}{}^I v^c_{I[\bar{a}} F_{|c|\bar{b}]} = D_{[\bar{a}} \Lambda_{\bar{b}]} \quad (= 0 \in H^2(V \times V^\vee)) \quad (10)$$

- Now, if we let $\Lambda = -2\delta A$ we recover

$$\delta \mathfrak{z}^I v^c_{I[\bar{a}]} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]} = 0 \tag{11}$$

- That is, the fluctuation of the $10d$ E.O.M. $F_{ab} = 0$ is implied by the Atiyah sequence.
- Note that the bundle moduli are unaffected (not fixed). I.e. an injection $0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q})$.
- We want to know:
  - $Ker(\alpha)$: Free C.S. moduli
  - $Im(\alpha)$: Stabilized C.S. moduli
- Why wasn't this done 20 years ago? $\Rightarrow$ General story not applied in heterotic string theory and tough to compute...
- Using algebraic geometry, this is just polynomial (Cech, etc) multiplication. Hard, but can be done!

Examples

- All good in principle... but what is $Im(\alpha)$? How many moduli fixed??
- Let's start simple...
- Line bundles?
  - For a line bundle on a $K3$, $Im(\alpha) = \mathbb{C}$
  - For a CY threefold, $\longrightarrow$ Line bundles do not constrain C.S. moduli.

    Always deform in the with X since $H^2(\mathcal{L} \otimes \mathcal{L}^\vee) = H^2(\mathcal{O}_X) = 0$
- However, what about simplest possible rank 2 bundle? $\rightarrow$ consider an an $SU(2)$ extension

$$0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0 \quad (12)$$

In principle, can stabilize arbitrarily many moduli!

A Threefold Example

- Let's consider an explicit extension: $0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0$
- For example on the Calabi-Yau threefold $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^2 & 3 \end{array} \right]^{3,75}$

$$0 \to \mathcal{O}(-2,-1,2) \to V \to \mathcal{O}(2,1,-2) \to 0 \quad (13)$$

- Why this one? Here $Ext^1(\mathcal{L}^\vee, \mathcal{L}) = H^1(X, \mathcal{O}(-4,-2,4)) = 0$ generically. Hence cannot define the bundle for general complex structure!
- Let $\mathcal{A} = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^2$. The Koszul sequence for X gives us

  $0 \to \mathcal{O}(-2,-2,-3) \otimes \mathcal{L}_{\mathcal{A}} \xrightarrow{p_o} \mathcal{L}_{\mathcal{A}} \to \mathcal{L}_X \to 0$

  $0 \to H^1(X, \mathcal{O}(-4-2,4)) \to H^2(\mathcal{A}, \mathcal{O}(-6,-4,1)) \xrightarrow{p_0} H^2(\mathcal{A}, \mathcal{O}(-4,-2,4))$
  $\to H^2(X,-4,-2,4) \to 0$
- For generic degree $\{2,2,3\}$ embedding polynomials, p, $Ext = 0$, but on a higher-codimensional locus, the cohomology can jump.

Jumping cohomology and the Atiyah class

- We can explicitly solve for when $ker(p) \neq 0$ and we find that on a 58-dimensional locus in C.S. moduli space, $h^1(X, \mathcal{O}(-4,-2,4)) = 18$.
- Begin at a point, p_0 for which $Ext \neq 0$, do Atiyah computation of linear deformations.
- Since this extension bundle cannot be defined away from this 58-dimensional locus we expect $Im(\alpha) \neq 0$
  - Note: Split bundle $\mathcal{L} \oplus \mathcal{L}^\vee$ is not supersymmetric for arbitrary Kähler moduli and not infinitesimally deformable to V.
  - $H^1(X, \mathcal{L}^{\otimes 2})$ does not disappear as we perturb the C.S., rather the one forms are simply no-longer $\{0,1\}$ w.r.t to the new C.S.

  As a result, we would expect that $im(\alpha) \geq 17$.
- Also since $im(\alpha) \leq h^2(V \otimes V^\vee) = dim(Ext^1(\mathcal{L}^\vee, \mathcal{L})) - 1 = 17$. Hence, $17 \leq im(\alpha) \leq 17$. So, we expect to stabilize exactly 17 C.S. moduli.

- What to do to compute $Im(\alpha)$?
- We need $\alpha = [F^{1,1}] \in H^1(End(V) \otimes TX^\vee)$ where

$$0 \to \mathcal{O}^{\oplus 3} \to \mathcal{O}(1,0,0)^{\oplus 2} \oplus \mathcal{O}(0,1,0)^{\oplus 2} \oplus \mathcal{O}(0,0,1)^{\oplus 3} \to TA \to 0$$

$$0 \to TX \to TA \to \mathcal{O}(2,2,3) \to 0$$

and we must determine the cohomology from

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & \mathcal{L}^{\otimes 2} \otimes TX^\vee & \to & V \otimes \mathcal{L} \times TX^\vee & \to & TX^\vee & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & \mathcal{L} \otimes V^\vee \otimes TX^\vee & \to & V \otimes V^\vee \otimes TX^\vee & \to & \mathcal{L}^\vee \otimes V^\vee \otimes TX^\vee & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & TX^\vee & \to & \mathcal{L}^\vee \otimes V \otimes TX^\vee & \to & \mathcal{L}^{\vee \otimes 2} \otimes TX^\vee & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

- Have explicitly generated polynomial basis of source, target and map for $H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^\vee)$
- Direct computation yields that $Im(\alpha) = 17$. No. of moduli stabilized!
- Interesting observation: The polynomial multiplication in the "jumping" cohomology locus $H^2(\mathcal{A}, \mathcal{O}(-6,-4,1)) \xrightarrow{p_0} H^2(\mathcal{A}, \mathcal{O}(-4,-2,4))$ is identical to the calculation of $H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^\vee)$, down to the exact monomials!
- For the $4d$ Theory: We have Gukov-Vafa-Witten superpotential $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}}\left(\omega^{3\mathrm{YM}} - \omega^{3\mathrm{L}}\right)$
- In Minkowski vacuum (with $W = 0$), F-terms: $F_{C_i} = \frac{\partial W}{\partial C_i} = -\frac{3\alpha'}{\sqrt{2}} \int_X \Omega \wedge \frac{\partial \omega^{3\mathrm{YM}}}{\partial C_i}$
- Dimensional Reduction Anzatz: $A_\mu = A_\mu^{(0)} + \delta A_\mu + \bar{\omega}^i_\mu \delta C_i + \omega^i_\mu \delta \bar{C}_i$

$$F_{C_i} = \int_X \epsilon^{\bar{a}\bar{c}\bar{b}} \epsilon^{abc} \Omega^{(0)}_{abc} 2\bar{\omega}^{xi}_{\bar{c}} \,\mathrm{tr}(T_x T_y) \left(\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2 D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]} \right)$$

$4d$ Effective Theory

- $0 \to \mathcal{L} \to V \to \mathcal{L}^\vee$ gives an $N = 1$ $4d$ theory with E_7 symmetry
- In general, $\mathfrak{z}$ is stabilized at the compactification scale. To explicitly describe F-terms F_{C_i}, we must find a region of moduli space for which $\mathfrak{z}$ is light.
- Here this happens near (but not on!) the Stability Wall. Extra $U(1)$ gives charges, $C_+ \in H^1(\mathcal{L}^{\otimes 2})$, $C_- \in H^1(\mathcal{L}^{\vee \otimes 2})$. E_7 singlets only in spectrum.
- Superpotential: $W = \lambda_{ia}(\mathfrak{z})\, C_+^i C_-^a + \Gamma_{ijab} C_+^i C_+^j C_-^a C_-^b$
- D-term: $D^{U(1)} = FI - G^+_{i\bar{j}} C_+^i \overline{C}_+^{\bar{j}} + G^-_{a\bar{b}} C_-^a \overline{C}_-^{\bar{b}}$
- Choose Vacuum: $< C_+ > \neq 0$ and $< C_- > = 0$. With C_+ chosen to cancel FI term.

- $\langle C_- \rangle = 0$ in vacuum $\Rightarrow W = 0$, $\partial W/\partial \mathfrak{z} = 0$, and $\partial W/\partial C_+ = 0$
- This leaves "The" F-term: $\frac{\partial W}{\partial C_-^a} = \lambda_{ia}(\mathfrak{z}) < C_+^i >= 0$
- Choose vacuum value of the C.S. so that $Ext \neq 0 \Rightarrow \lambda = 0$.

  Supersymmetric Minkowski vacuum!
- Now in fluctuation

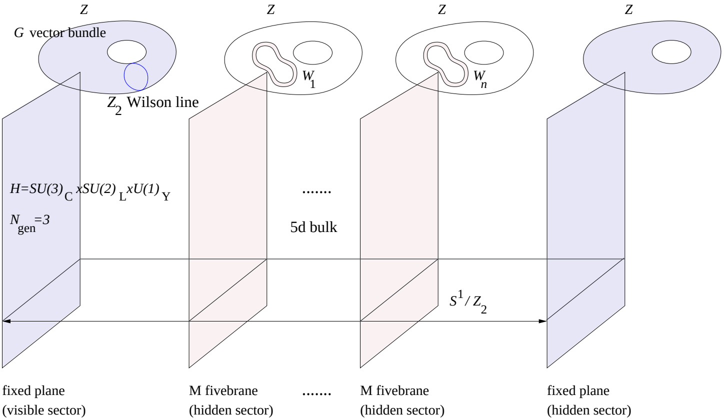
$$
\begin{aligned}
&\delta\left(W\right) = 0 \\
&\delta\left(\frac{\partial W}{\partial C_+^i}\right) = 0 \\
&\delta\left(\frac{\partial W}{\partial \mathfrak{z}^I}\right) = \frac{\partial \lambda_{ia}}{\partial \mathfrak{z}_\perp^I} < C_+^i > \delta C_-^a = 0 \\
&\delta\left(\frac{\partial W}{\partial C_-^b}\right) = \frac{\partial \lambda_{ib}}{\partial \mathfrak{z}_\perp^I} \delta \mathfrak{z}_\perp^I < C_+^i > + \Gamma_{ijab} < C_+^i >< C_+^j > \delta C_-^a = 0
\end{aligned}
$$

$\frac{\partial \lambda_{ia}}{\partial \mathfrak{z}^I}$ vanishes along the 58-dimensional locus. $\perp$ to locus, $\delta \mathfrak{z}_\perp^I$ gets a mass. δC_-^a also massive. Agrees with Atiyah Computation!

A Hidden sector mechanism

- Conclusion: A generic bundle perturbatively stabilizes some of the C.S. moduli
- We can find bundles that stabilize all or many of the complex structure moduli
- Such bundles probably not always well-suited for visible sector phenomenology (i.e. Three families, particle spectrum, etc).
- However, such bundles can *always* be added to the Hidden sector
  - For example, the $SU(2)$ extension $0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0$ can be defined on any CY with $h^{1,1} > 1$.
  - Slope-stable. I.e. D-terms vanish.
  - Generically satisfies anomaly cancellation: $c_2(TX) - c_2(V_1) - c_2(V_2) >= 0$
  - E_7 symmetry compatible with gaugino condensation, etc.

Stabilization in the Hidden sector



Conclusions – Complex Structure Moduli

- The presence of a *holomorphic* vector bundle constrains C.S. moduli
- The moduli of a heterotic compactification: $H^{1,1}(X)$, $H^1(V \otimes V^\vee)$, $Ker(\alpha)$
- $Im(\alpha)$ can be computed
- Leads to F-terms in 4-dimensions: $\frac{\partial W}{\partial C_I}$ where C_I are $4d$ matter fields
- The C.S. can be stabilized at the perturbative level without moving away from a CY manifold
    - Avoids problems of KKLT scenarios in heterotic
    - Allows us to keep heterotic model-building toolkit!
- Provides a general Hidden Sector mechanism for stabilizing the C.S. moduli in Heterotic (M-theory) compactifications.
- Work in progress – Add non-perturbative effects to remaining stabilize remaining moduli

The End