Dark matter direct detection with dielectrics

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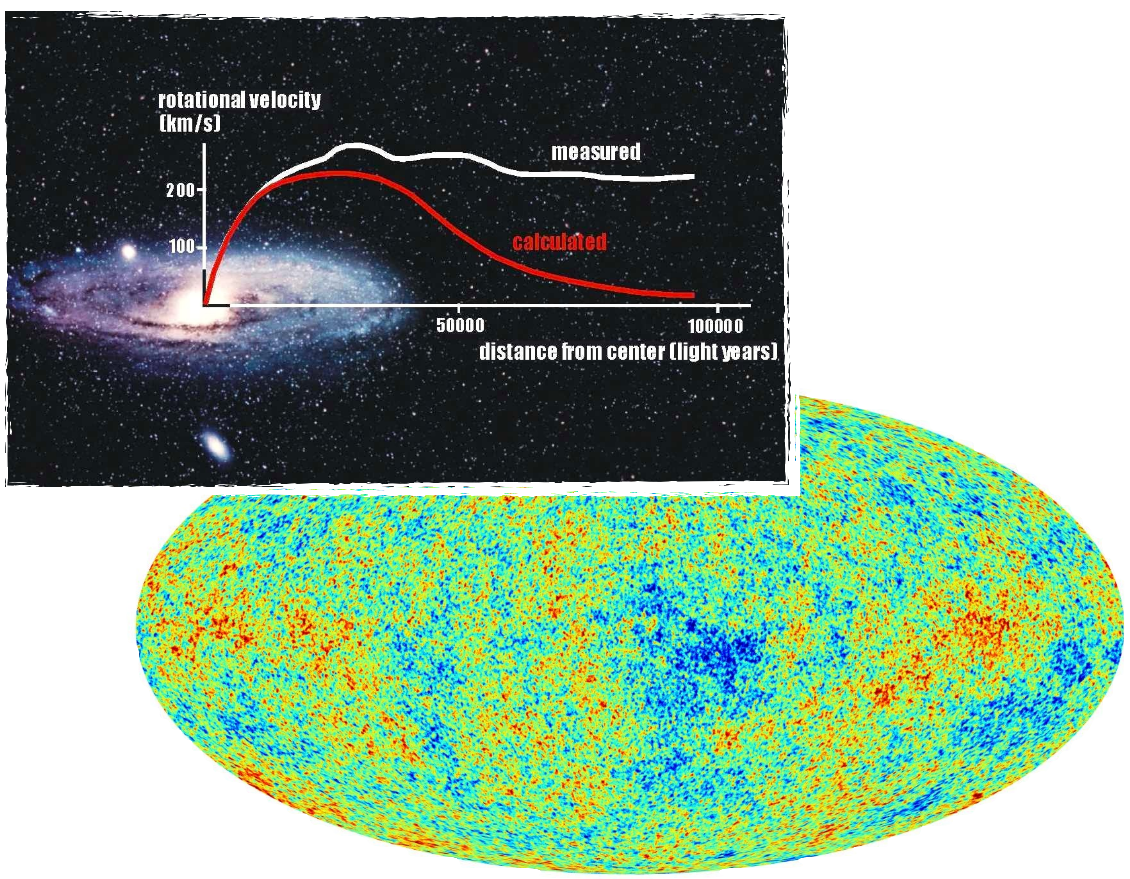
September 7, 2021
Rutgers NHETC Seminar

Based on work with Simon Knapen and Jonathan Kozaczuk
2003.12077, 2011.09496, 2101.08275, 2104.12786

Dark matter: evidence and searches

Gravitational evidence

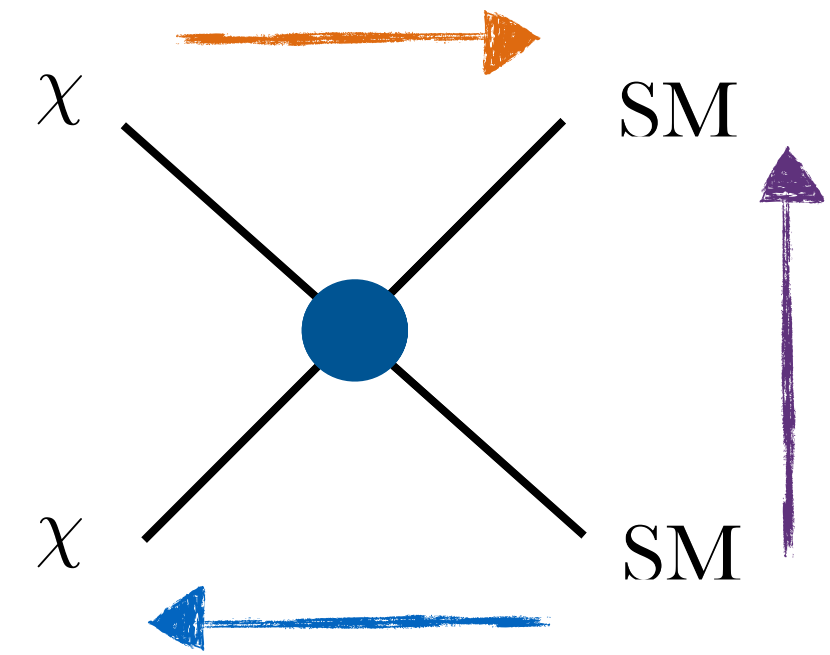


Cosmic Microwave Background

Particle searches

- ⭐ Self-interactions
- ⭐ Many cosmo/astro probes

Indirect Detection



Direct Detection

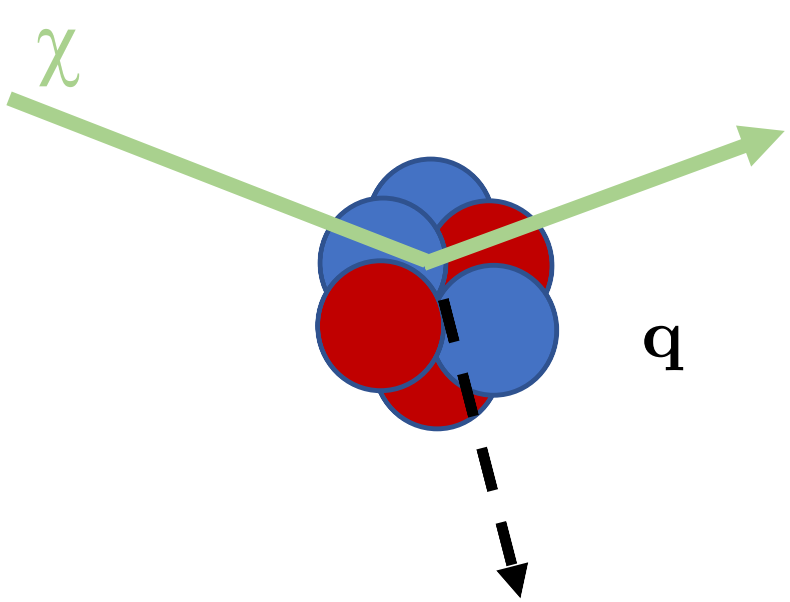
- ⭐ Electron scattering
- ⭐ Condensed mat systems

Collider

- ⭐ High Luminosity
- ⭐ Accelerator experiments

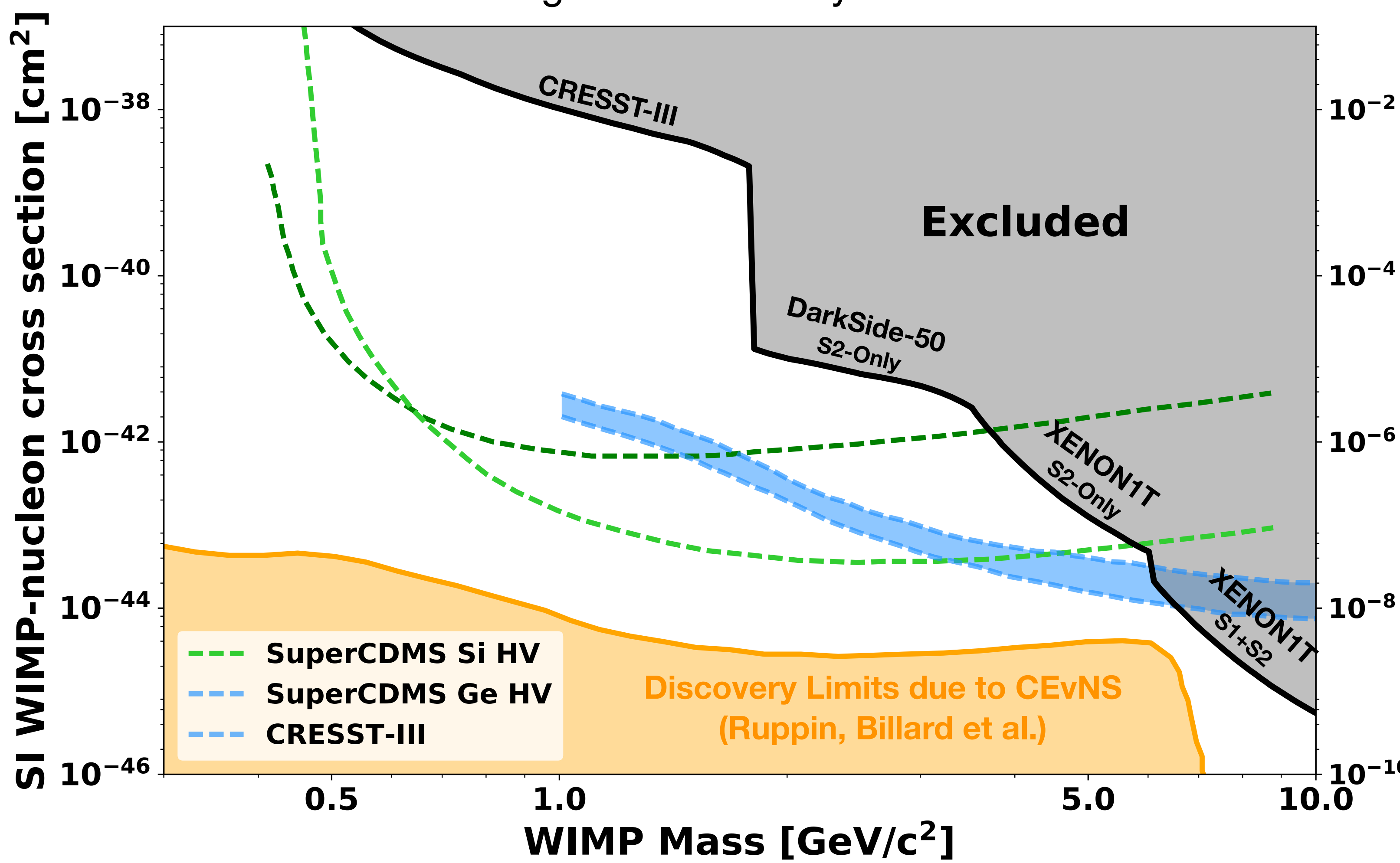
Detecting sub-GeV dark matter

Traditional approach to direct detection of dark matter: DM-nucleus scattering

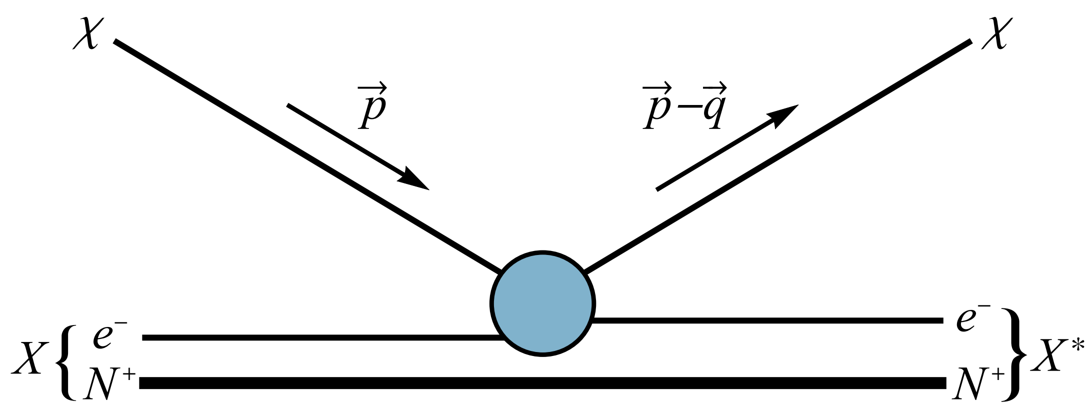


$$E_{NR} \leq \frac{2m_\chi^2 v^2}{m_N} \text{ for sub-GeV DM}$$

Figure from talk by Kaixuan Ni at DPF 2019



Electron recoils



e- in materials are not free or isolated particles

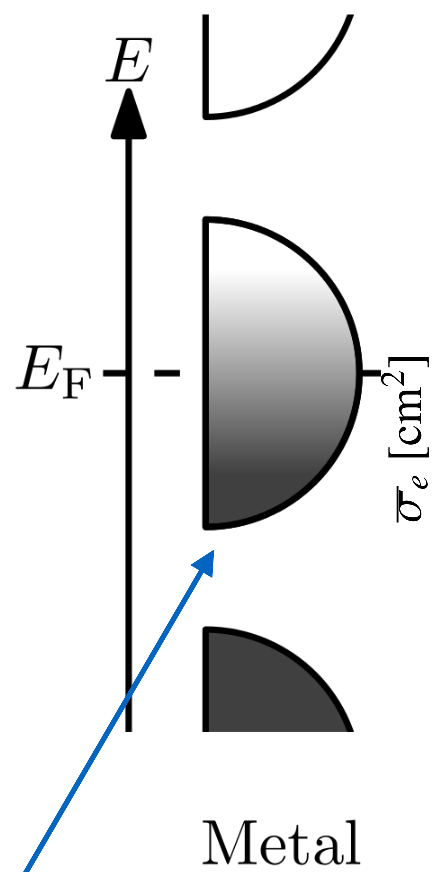
Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

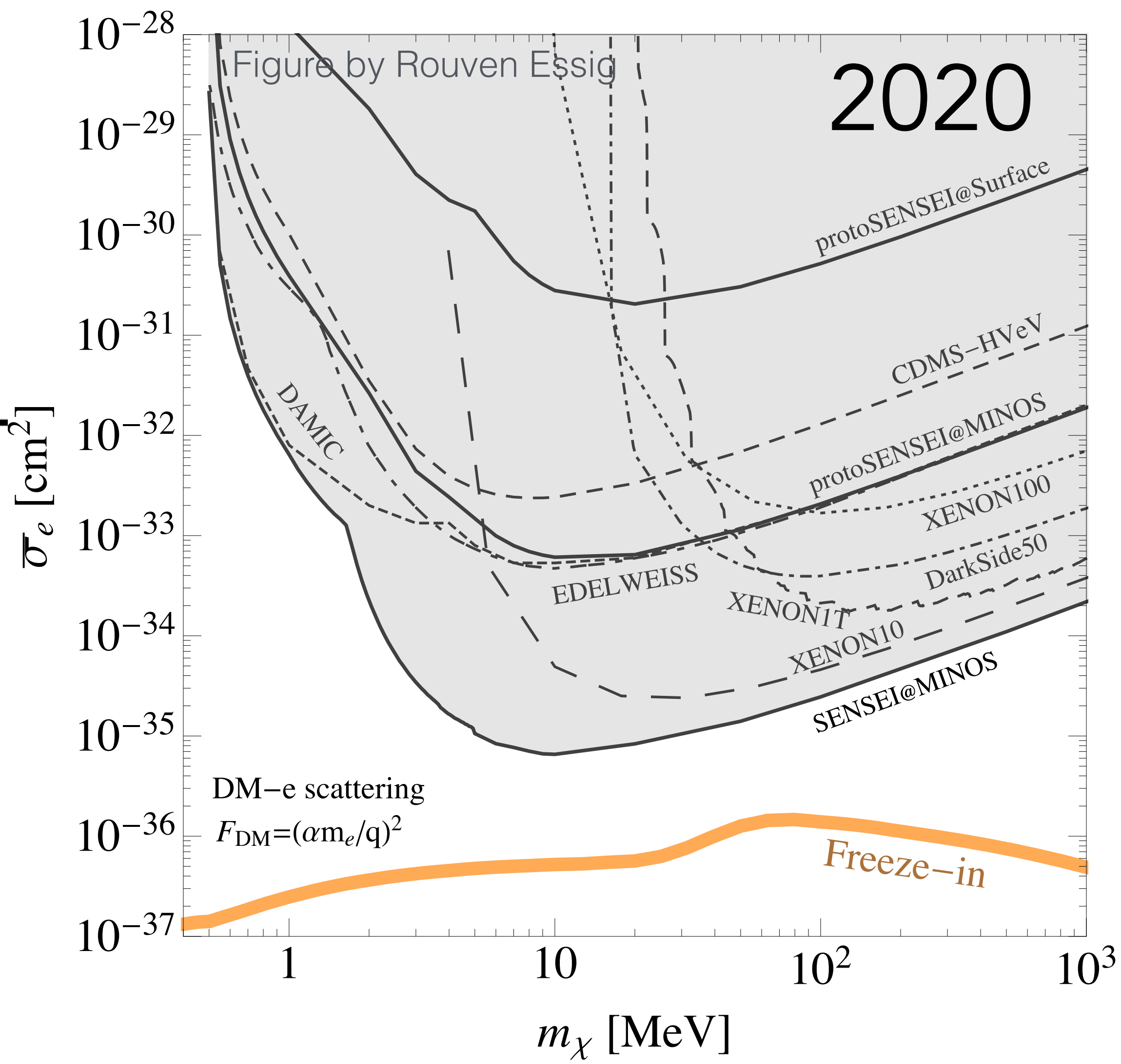
Complication: need to know eigenstates and wavefunctions in a many-body system.

Essig, Mardon, Volansky 2011;
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu 2015

Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

Electronic band structure





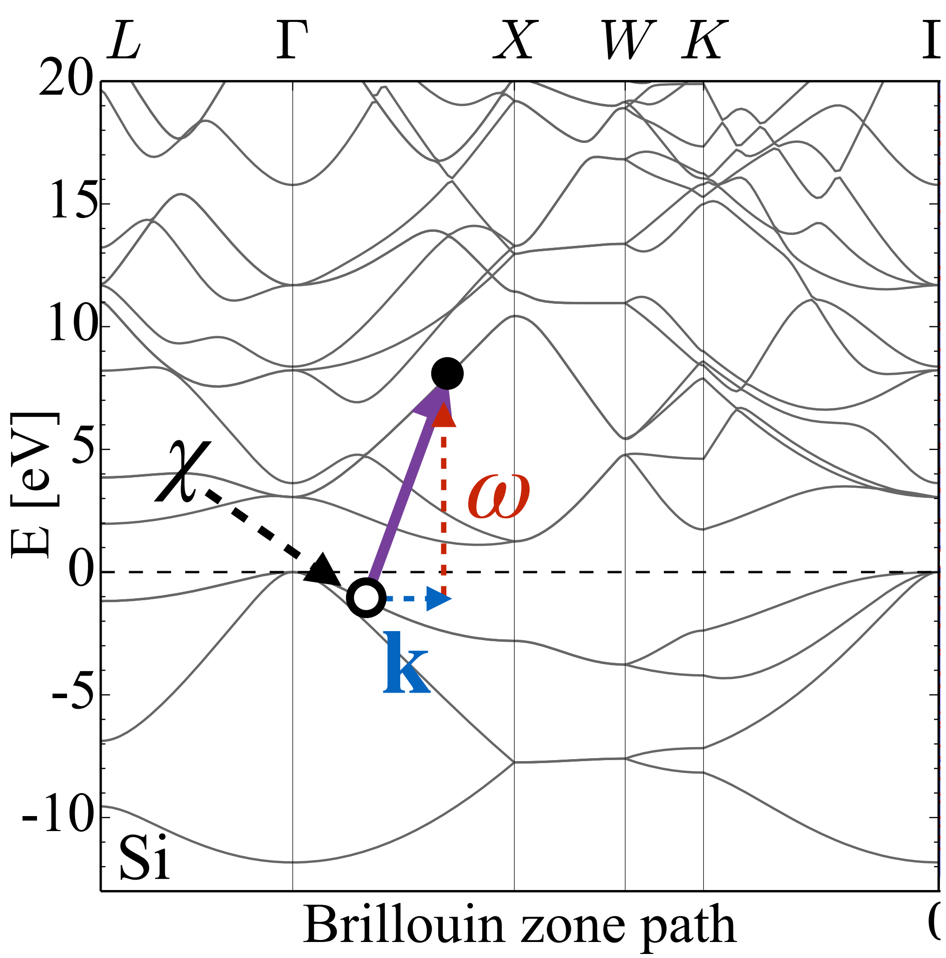
sub-eV gap in superconductor, Dirac material, etc

[Hochberg, Pyle, Zhao, Zurek 2015 Dirac: 1708.08929, 1910.02091, etc]

Single or few e- thresholds achieved in a number of experiments

Complication: need to know about excitations in a many-body system.

Semiconductor target



Independent particle approximation:

Wavefunction overlap

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(k) \sum_{\ell,\ell'} \sum_{\mathbf{p},\mathbf{p}'} |\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2$$

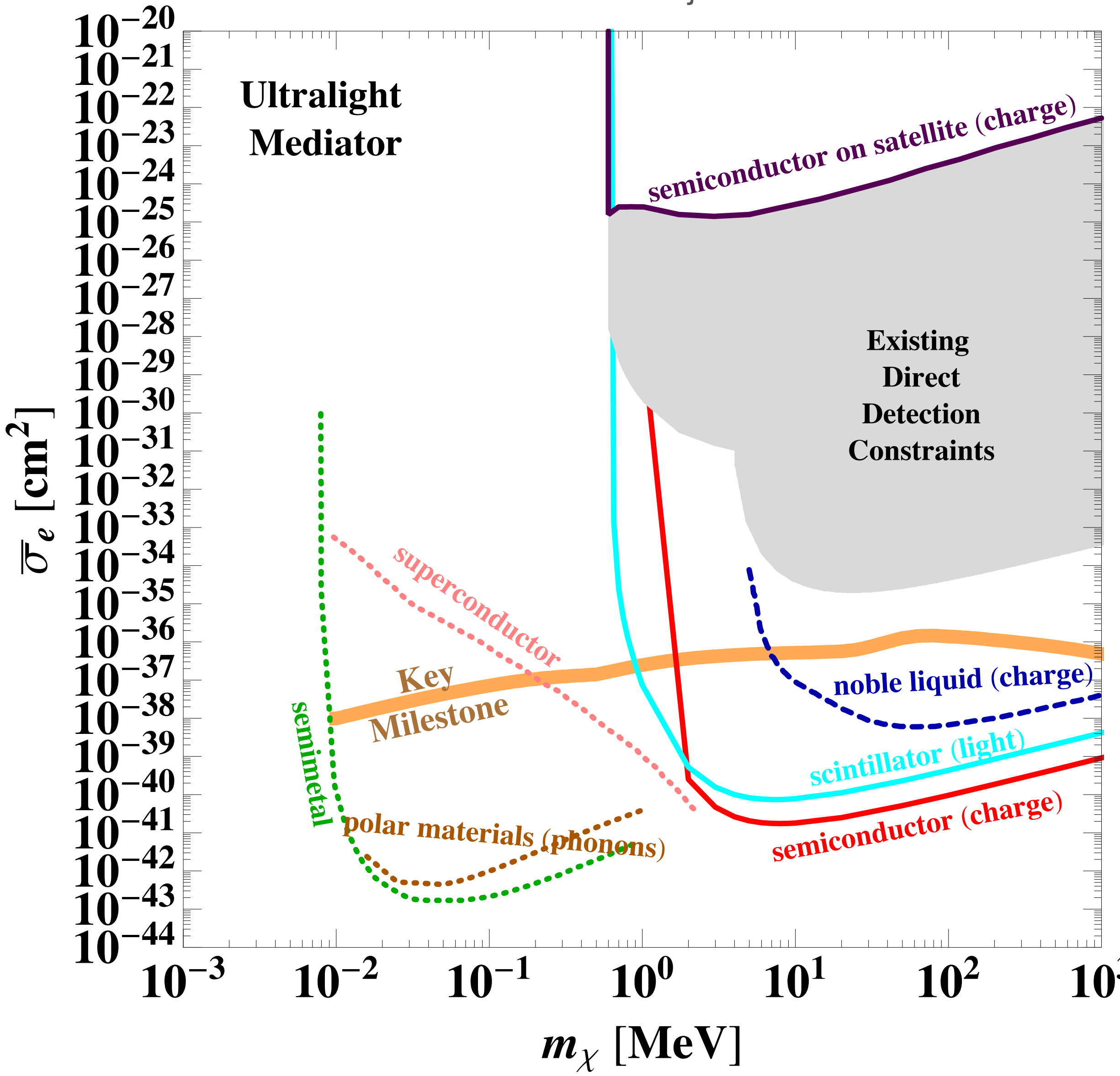
$$\times f^0(\omega_{\mathbf{p},\ell})\left(1 - f^0(\omega_{\mathbf{p}',\ell'})\right) \delta(\omega + \omega_{\mathbf{p},\ell} - \omega_{\mathbf{p}',\ell'})$$

Sum over occupied bands ℓ and Bloch momentum $\mathbf{p}$ to excited state $|\mathbf{p}', \ell'\rangle$

Does this capture all many-body effects?

Essig, Mardon, Volansky 2011;
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu 2015

From Basic Research Needs Report:
"Dark Matter Small Projects New Initiatives"



Now many papers studying different targets, proposed experiments, and new experiments in development.

All dielectrics

Today: how to describe DM scattering in all these materials in terms of dielectric response, and how we used this to identify and calculate new effects.

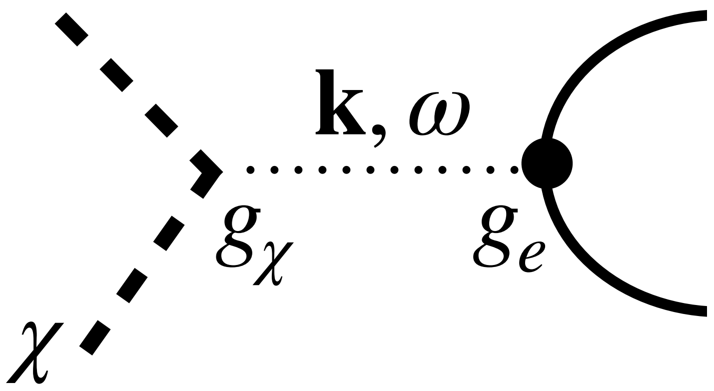
Outline

Energy loss function (ELF): $\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$

Implications for DM-electron scattering

Using the ELF to determine DM-nucleus scattering with the Migdal effect, DM-phonon scattering, and DM absorption

Linear response



$$H = -e \int d^3\mathbf{k}\; n_\mathbf{k} \frac{g_\chi g_e e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 + m_V^2}$$

↑ Electron number density

Dielectric response

$\epsilon^{-1}(\omega, \mathbf{k})$ — response of E fields*

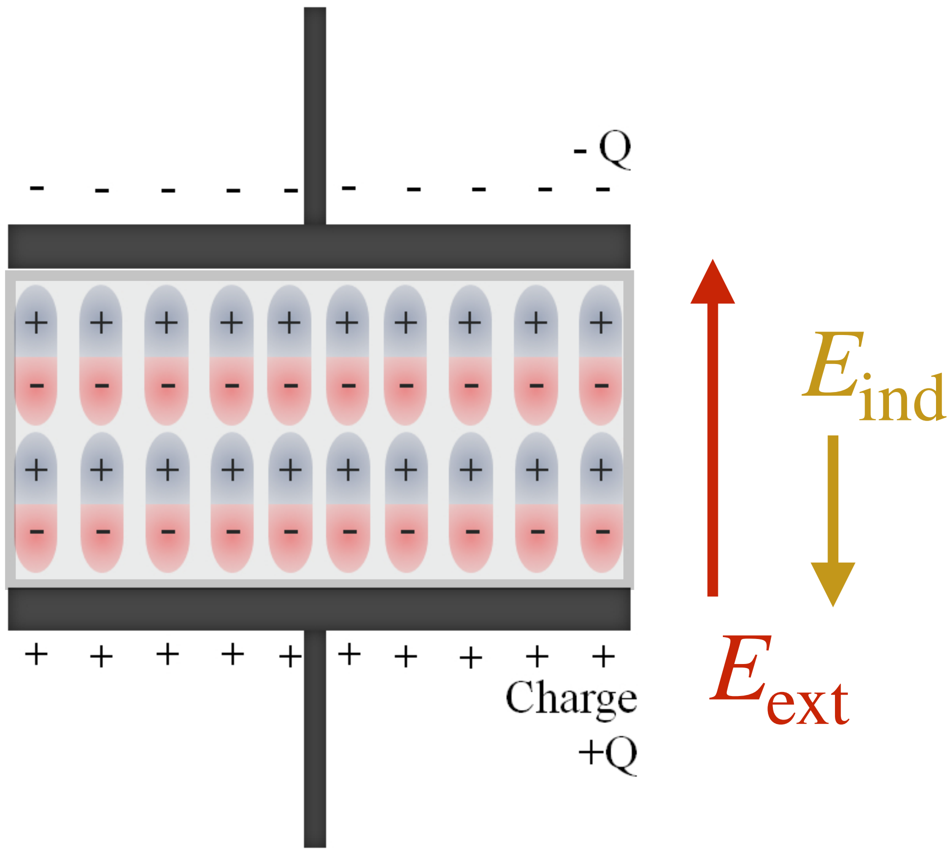
Susceptibility

$\chi(\omega, \mathbf{k})$ — response of electron number density

* Some technicalities: consider only longitudinal response; neglect crystal periodicity

Pines and Nozieres, Theory of Quantum Liquids;
Girvin and Yang, Modern Condensed Matter Physics

Dielectric response



$$\nabla \cdot \mathbf{E} = \frac{4\pi \rho_{\text{ext}}}{\epsilon}$$

$$\mathbf{E} = \frac{\mathbf{E}_{\text{ext}}}{\epsilon}$$

More generally:

$$\mathbf{E}(\mathbf{r}, \omega) = \int d^3\mathbf{r}' \, \epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \, \mathbf{E}_{\text{ext}}(\mathbf{r}', \omega)$$

$$\mathbf{E}(\omega, \mathbf{k}) = \epsilon^{-1}(\omega, \mathbf{k}) \, \mathbf{E}_{\text{ext}}(\omega, \mathbf{k})$$

Induced charge density*: $\rho_{\text{ind}} = \frac{\rho_{\text{ext}}}{\epsilon} - \rho_{\text{ext}}$

External perturbation

$$H = -en(\mathbf{r})\,\Phi_{\text{ext}}(\mathbf{r}) = -e\int d^3\mathbf{k}\ n_{-\mathbf{k}}\,\frac{4\pi\rho_{\text{ext}}(\mathbf{k})}{k^2}$$

Source

Linear response

$$\rho_{\text{ind}} = -en_{\text{ind}} = \chi\frac{4\pi e^2}{k^2}\rho_{\text{ext}}$$

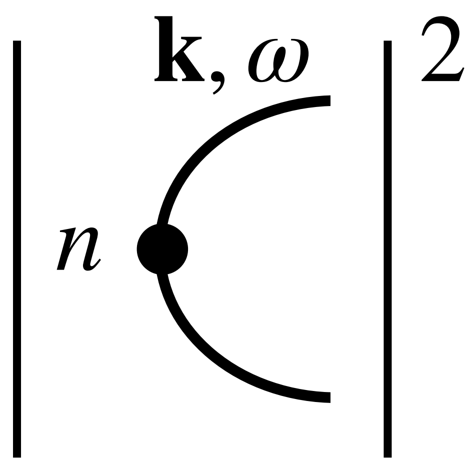
Susceptibility: $\chi(\omega,\mathbf{k}) = \frac{-i}{V}\int_0^\infty dt\ e^{i\omega t}\ \langle[n_{\mathbf{k}}(t), n_{-\mathbf{k}}(0)]\rangle$

* Assume dominated by electrons

Amount of screening is related to induced charge:

$$\frac{1}{\epsilon(\omega,\mathbf{k})} = 1 + \frac{4\pi e^2}{k^2}\chi(\omega,\mathbf{k})$$

$$H = -\,en(\mathbf{r})\,\Phi_{\text{ext}}(\mathbf{r})$$

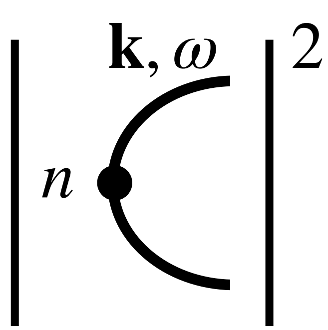


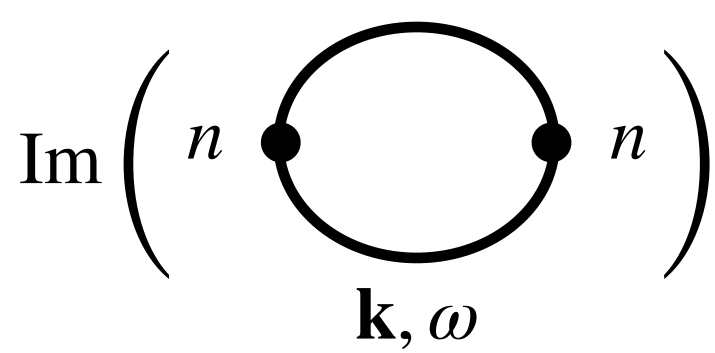
Fluctuation-dissipation theorem

Spectrum of fluctuations

$$S(\omega,\mathbf{k}) = \frac{2}{(1-e^{-\beta\omega})}\,\text{Im}\left(-\chi(\omega,\mathbf{k})\right)$$

Dissipation

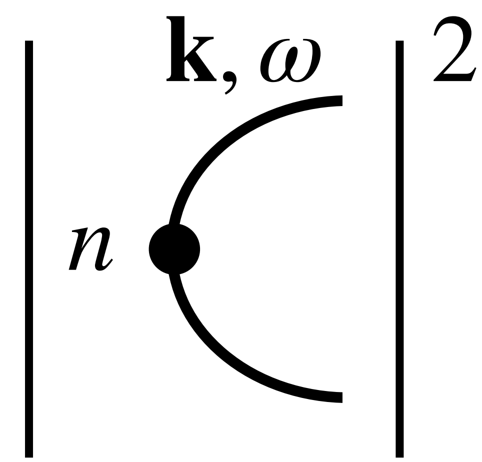




Amount of screening is related to induced charge:

$$\frac{1}{\epsilon(\omega, \mathbf{k})} = 1 + \frac{4\pi e^2}{k^2}\chi(\omega, \mathbf{k})$$

$$H = -\,en(\mathbf{r})\,\Phi_{\text{ext}}(\mathbf{r})$$



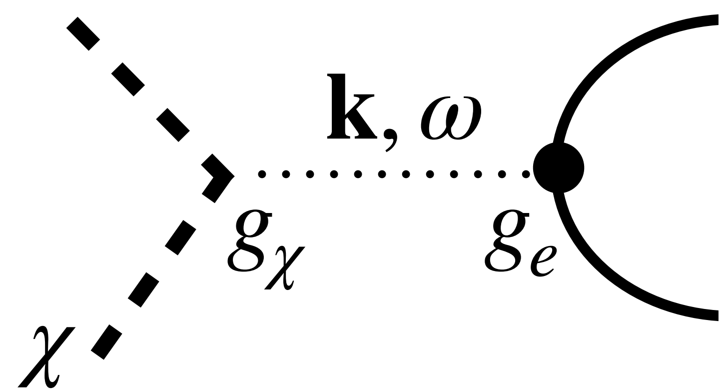
Fluctuation-dissipation theorem

$$S(\omega, \mathbf{k}) = \frac{2}{(1 - e^{-\beta\omega})}\,\mathrm{Im}\left(-\chi(\omega, \mathbf{k})\right)$$

$$= \frac{k^2}{2\pi\alpha_{em}(1 - e^{-\beta\omega})}\,\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Energy Loss Function (ELF)

DM-electron scattering



$$H = -e\int d^3\mathbf{k}\; n_{\mathbf{k}}\, \frac{g_\chi g_e e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 + m_V^2}$$

$$\frac{d\sigma}{d^3\mathbf{k}\,d\omega} \propto \bar{\sigma}_e\, F^2_{\rm med}(\mathbf{k})\, S(\omega,\mathbf{k}) \propto \bar{\sigma}_e\, F^2_{\rm med}(\mathbf{k})\, {\rm Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

Charge fluctuations

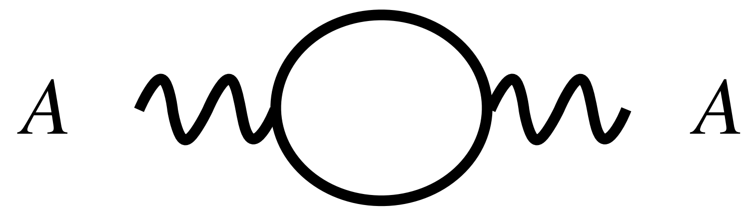
Energy Loss Function (ELF)

Implications {
1. Screening effects for vector and scalar mediators
2. Many approaches to calculate or measure ϵ

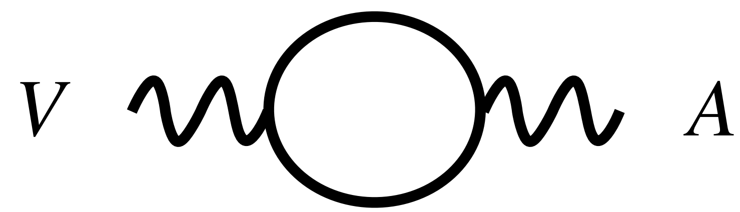
Vector-mediated scattering

Interaction basis: $g_e V_\mu \bar{e} \gamma^\mu e$

In-medium mass and mixing terms



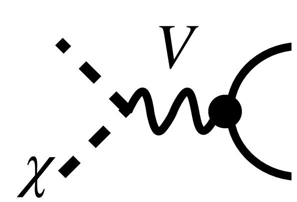
Π_{AA}



$$\Pi_{VA} = \frac{g_e}{e} \Pi_{AA}$$

$$\Pi_{AA}(\omega, \mathbf{k}) = k^2 (1 - \epsilon(\omega, \mathbf{k}))$$

In-medium (longitudinal) scattering amplitude:



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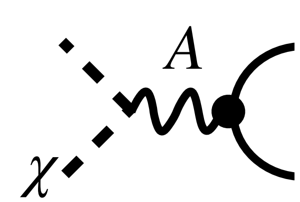


$$\sim \frac{1}{\epsilon(\omega, \mathbf{k})} \frac{g_\chi g_e}{k^2 + m_V^2}$$

An, Pospelov, Pradler 2013, 2014
Hochberg, Pyle, Zhao, Zurek 2015

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F^2_{\rm med}(\mathbf{k})\,\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

$$\propto \bar{\sigma}_e F^2_{\rm med}(\mathbf{k})\frac{\mathrm{Im}\,\epsilon(\omega,\mathbf{k})}{|\,\epsilon(\omega,\mathbf{k})\,|^2}$$

Proportional to DM-electron scattering form factor in the independent-electron approximation (RPA)

$$\mathrm{Im}\,\epsilon^{\rm RPA}(\omega,\mathbf{k}) = \frac{4\pi^2\alpha_{em}}{Vk^2}\sum_{\ell,\ell'}\sum_{\mathbf{p},\mathbf{p}'}|\,\langle\mathbf{p}',\ell'\,|\,e^{i\mathbf{k}\cdot\mathbf{r}}\,|\,\mathbf{p},\ell\rangle\,|^2$$
$$\times f^0(\omega_{\mathbf{p},\ell})\big(1-f^0(\omega_{\mathbf{p}',\ell'})\big)\;\delta(\omega+\omega_{\mathbf{p},\ell}-\omega_{\mathbf{p}',\ell'})$$

$|\,\epsilon(\omega,\mathbf{k})\,|^2$ screening for vector mediators considered in superconductors, Dirac materials.

Not previously included in signal rates for semiconductors.

Also not previously included for scalar mediators.

Vector and scalar mediators

$$-\mathcal{L} \supset g_\chi \phi \bar{\chi}\chi + g_e \phi \bar{e} e \qquad \to g_\chi \phi n_\chi + g_e \phi n$$

$$-\mathcal{L} \supset g_\chi V_\mu \bar{\chi}\gamma^\mu \chi + g_e V_\mu \bar{e}\gamma^\mu e \quad \to g_\chi V_0 n_\chi + g_e V_0 n$$

Non-relativistic scattering ($k \gg \omega$) is dominated by scattering through Yukawa potential

$$H = -e \int d^3\mathbf{k}\; n_\mathbf{k} \frac{g_\chi g_e e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2 + m_V^2}$$

DM-electron scattering via vector or scalar mediators is identical in the nonrelativistic limit

The energy loss function (ELF)

$$\text{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Theory

Many established approaches to ϵ

Include screening, local field effects

Include electron-electron interactions

Experiment

Optical measurements

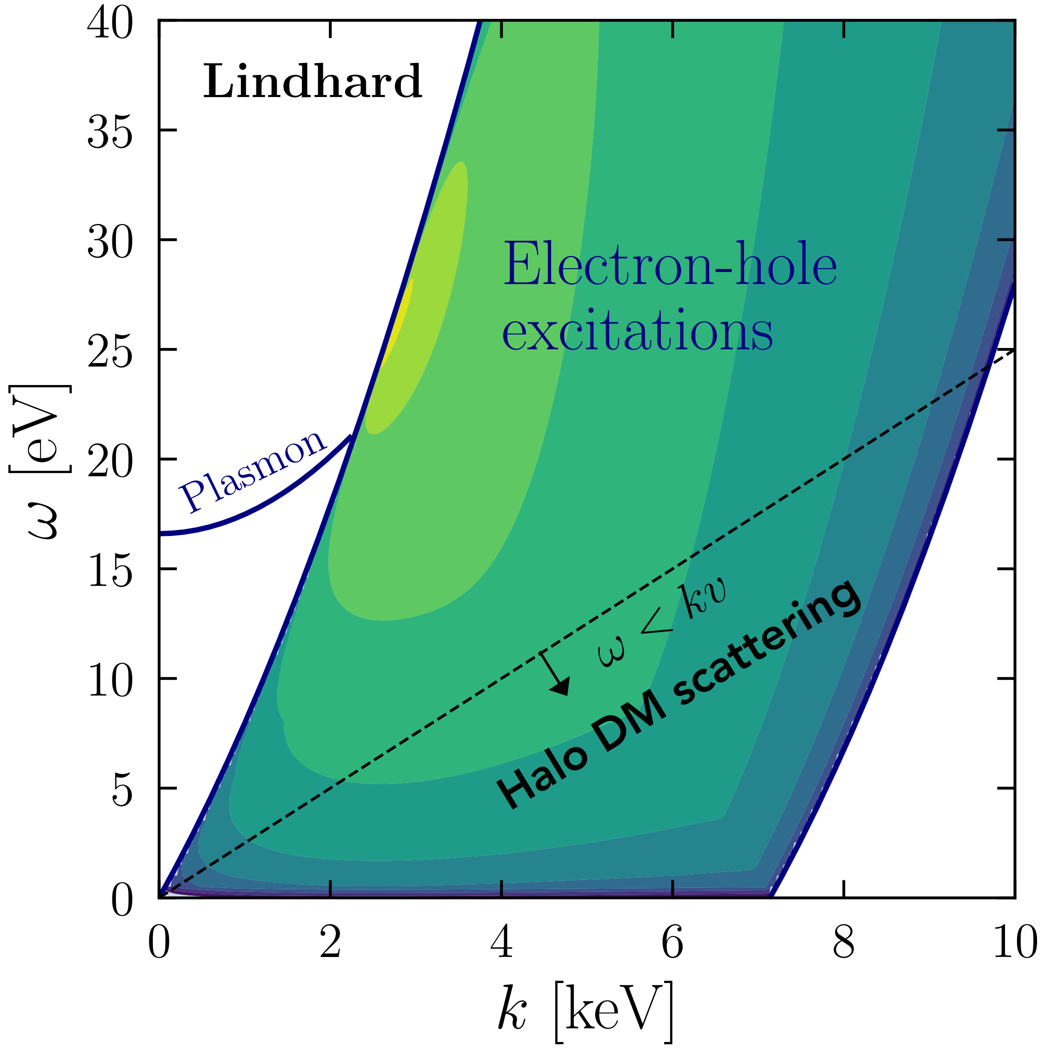
X-ray scattering

Fast electron scattering (EELS)

See Kurinsky, Baxter, Kahn, Krnjaic 2020 and Hochberg, Kahn, Kurinsky, Lehmann, Yu, and Berggren 2021 for complementary work and more emphasis on experimental calibration of dielectric function

Implications for DM-electron scattering

ELF in electron regime



Degenerate electron gas model

Missing dissipation

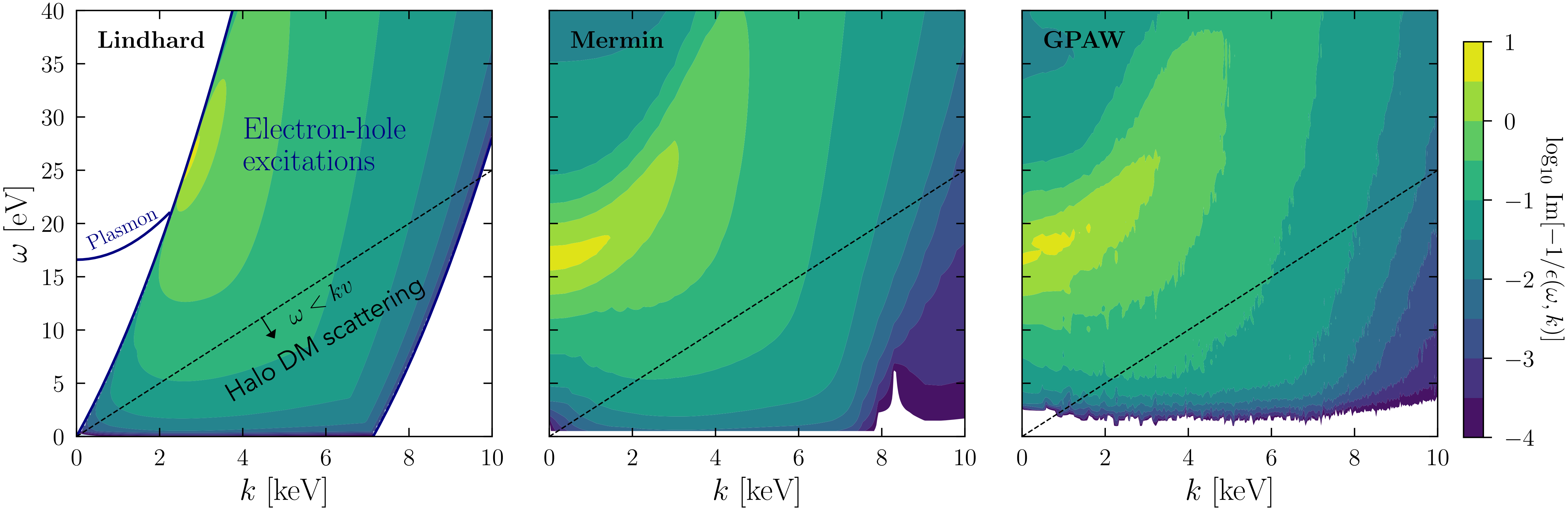
Random-phase approximation (RPA):

$$\epsilon^{\mathrm{RPA}}(\omega, \mathbf{k}) = 1 + \frac{4\pi\alpha_{em}}{Vk^2} \lim_{\eta\to 0} \sum_{\ell,\ell'} \sum_{\mathbf{p},\mathbf{p}'} |\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2 \frac{f^0(\omega_{\mathbf{p}',\ell'}) - f^0(\omega_{\mathbf{p},\ell})}{\omega + \omega_{\mathbf{p},\ell} - \omega_{\mathbf{p}',\ell'} + i\eta}$$

Emission – absorption

Can be calculated analytically in free degenerate electron gas with Fermi momentum p_F and plasma frequency $\omega_p = \sqrt{4\pi\alpha_{em} n_e / m_e}$

ELF in electron regime



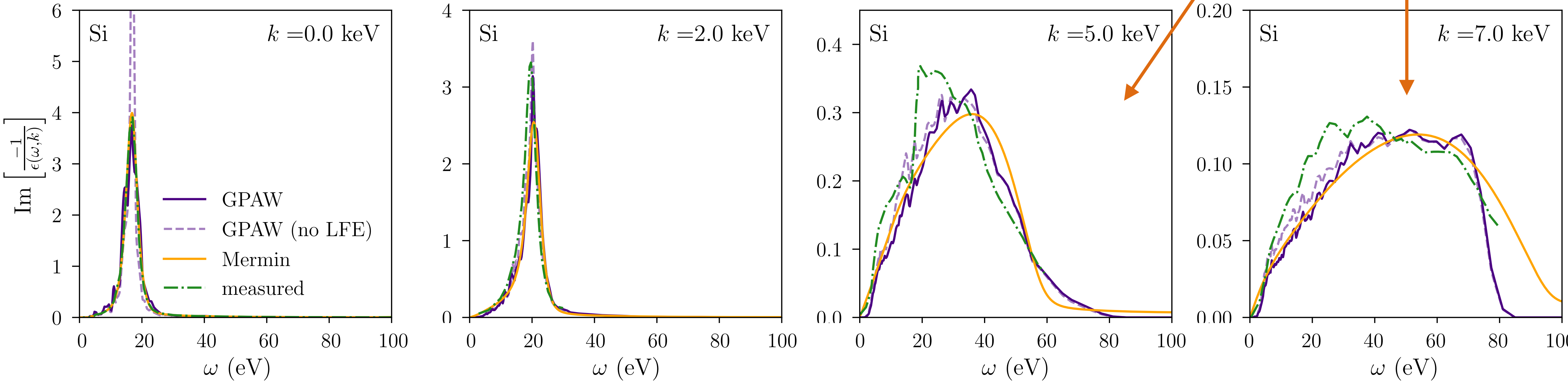
Degenerate electron gas model
Missing dissipation

Data-driven approach
fit optical/REELs data to sum of Mermin dielectrics (Lindhard with dissipation); doesn't work near band gap

From first principles
Time-dependent DFT calculation with GPAW

ELF in Silicon

DM scattering dominated by high k, need more data and/ or more involved DFT calculations



- GPAW
- GPAW (no LFE)

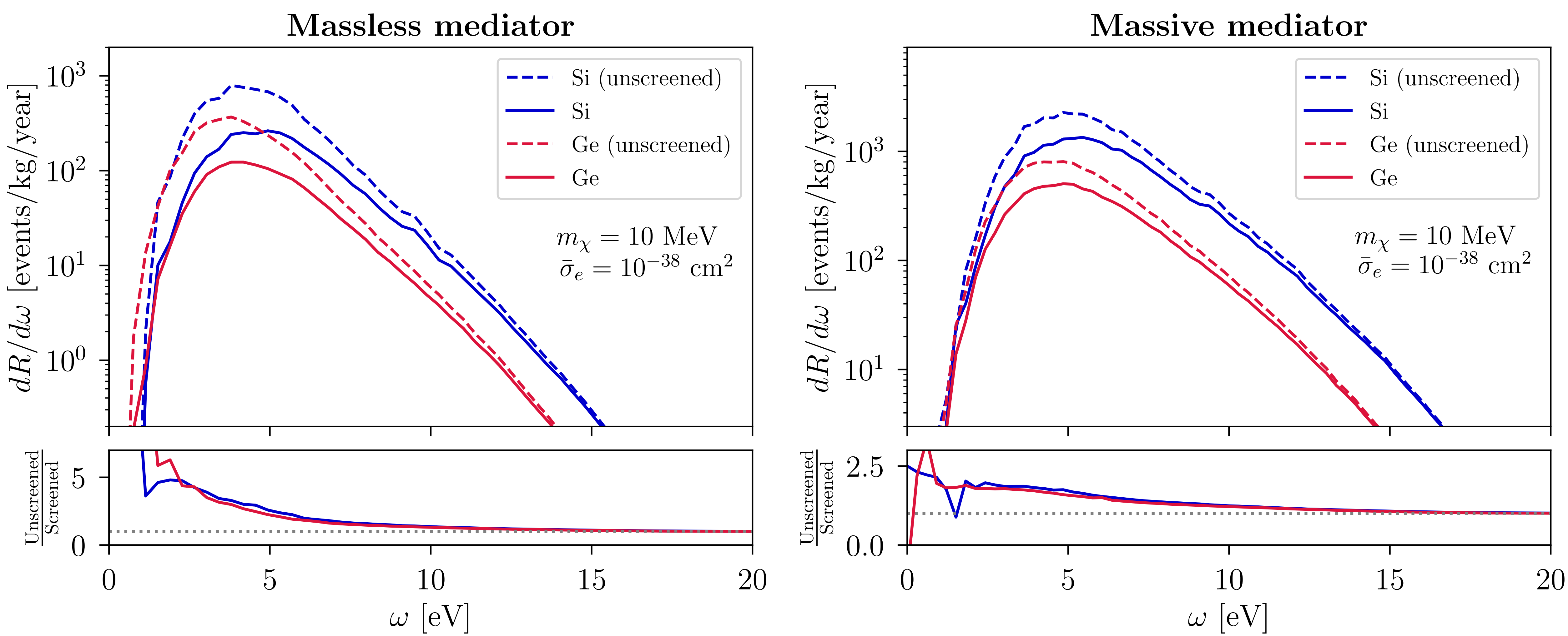
} DFT-based calculation

- Mermin — Data-driven approach
- measured — X-ray scattering

GPAW: Mortensen, Hansen, Jacobsen 2005;
Enkovaara, Rostgaard, Morstensen + 2010;
Mermin approach: Vos and Grande 2021
X-ray: Weissker et al. 2010

Knapen, TL, Kozaczuk 2101.08275

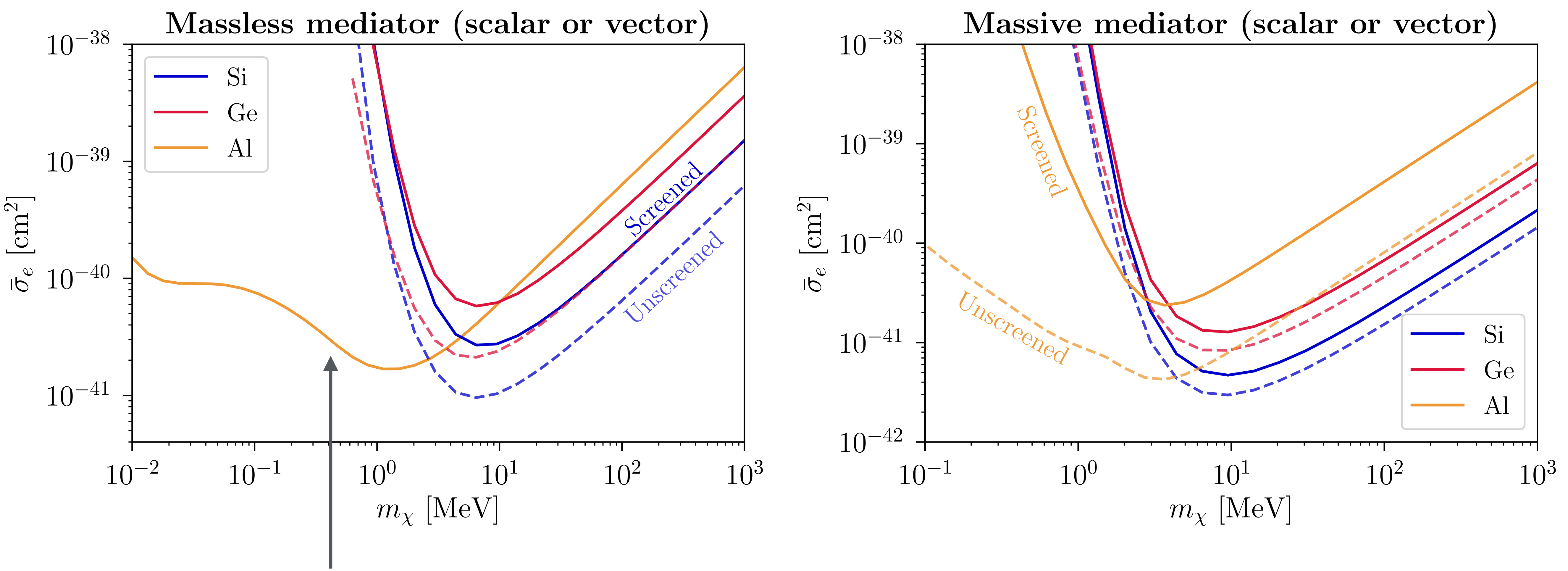
Implications for DM-electron scattering



Unscreened: $\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right) \to \mathrm{Im}\left(\epsilon(\omega,\mathbf{k})\right)$

Knapen, TL, Kozaczuk 2101.08275

Implications for DM-electron scattering



Metal/superconductor: large screening, but also massive gains in rate at low momentum

Knapen, TL, Kozaczuk 2101.08275

Summary, Part I

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F^2_{\rm med}(\mathbf{k})\,\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

DM-electron scattering is determined by rate to produce density fluctuations, which equivalently the energy loss function (ELF)

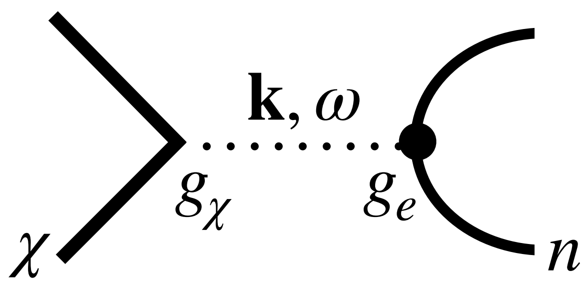
We calculated screening effects (scalar and vector mediators) and local field effects, which impacts sensitivity at O(1) level

More generally, can include many-body physics to desired accuracy in a variety of materials.

Using the ELF for DM-nucleus scattering with Migdal effect, DM-phonon scattering, DM-absorption

ELF for Dark Matter

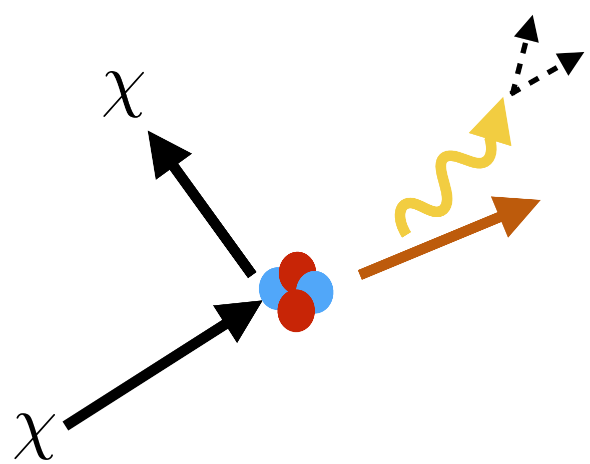
DM-electron scattering



$$\frac{d\sigma}{d^3\mathbf{k}\,d\omega} \propto \bar{\sigma}_e\, F^2_{\rm med}(\mathbf{k})\, \mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

Large momentum transfer (k ~ 1-10 keV) important for both processes

DM-nucleus scattering via Migdal effect



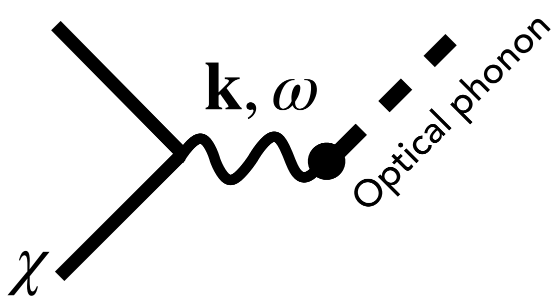
$$\frac{dP}{d^3\mathbf{k}\,d\omega} \propto \frac{4\pi\alpha_{em}Z^2_{\rm ion}}{\omega^4}\frac{|\mathbf{v_N}\cdot\mathbf{k}|^2}{k^2}\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

Based on work with Simon Knapen and Jonathan Kozaczuk
2003.12077+2011.09496 (Migdal), 2101.08275 (DM-electron), 2104.12786

ELF for Dark Matter

For kinetically-mixed dark photon mediators:

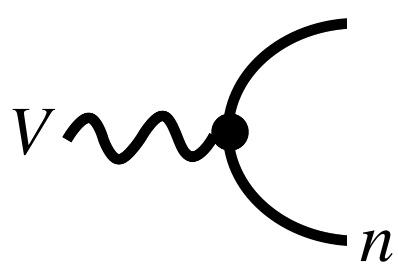
DM-phonon scattering

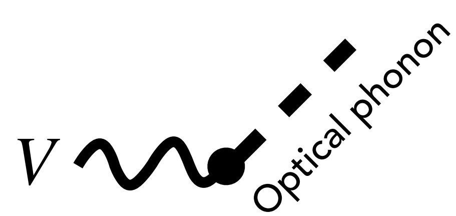


$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F^2_{\text{med}}(\mathbf{k})\, \text{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

Determined by ELF in the optical limit ($k \to 0$ *or* $k \ll \text{keV}$)

Bosonic DM absorption





$$R = \frac{1}{\rho_T}\frac{\rho_{\text{DM}}}{m_V}\kappa^2 m_V\, \text{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

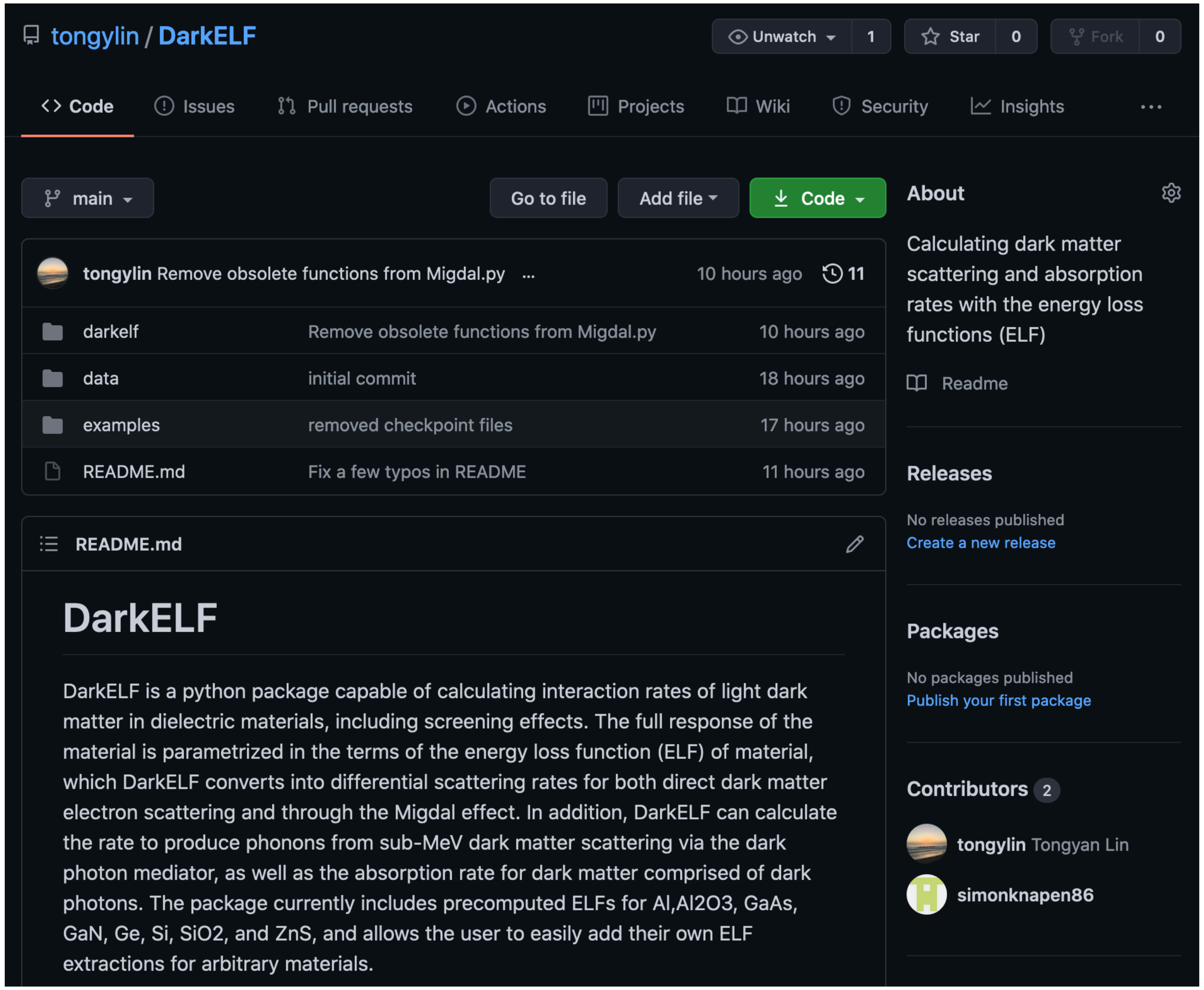
Based on work with Simon Knapen and Jonathan Kozaczuk
2003.12077+2011.09496 (Migdal), 2101.08275 (DM-electron), 2104.12786

ELF for Dark Matter

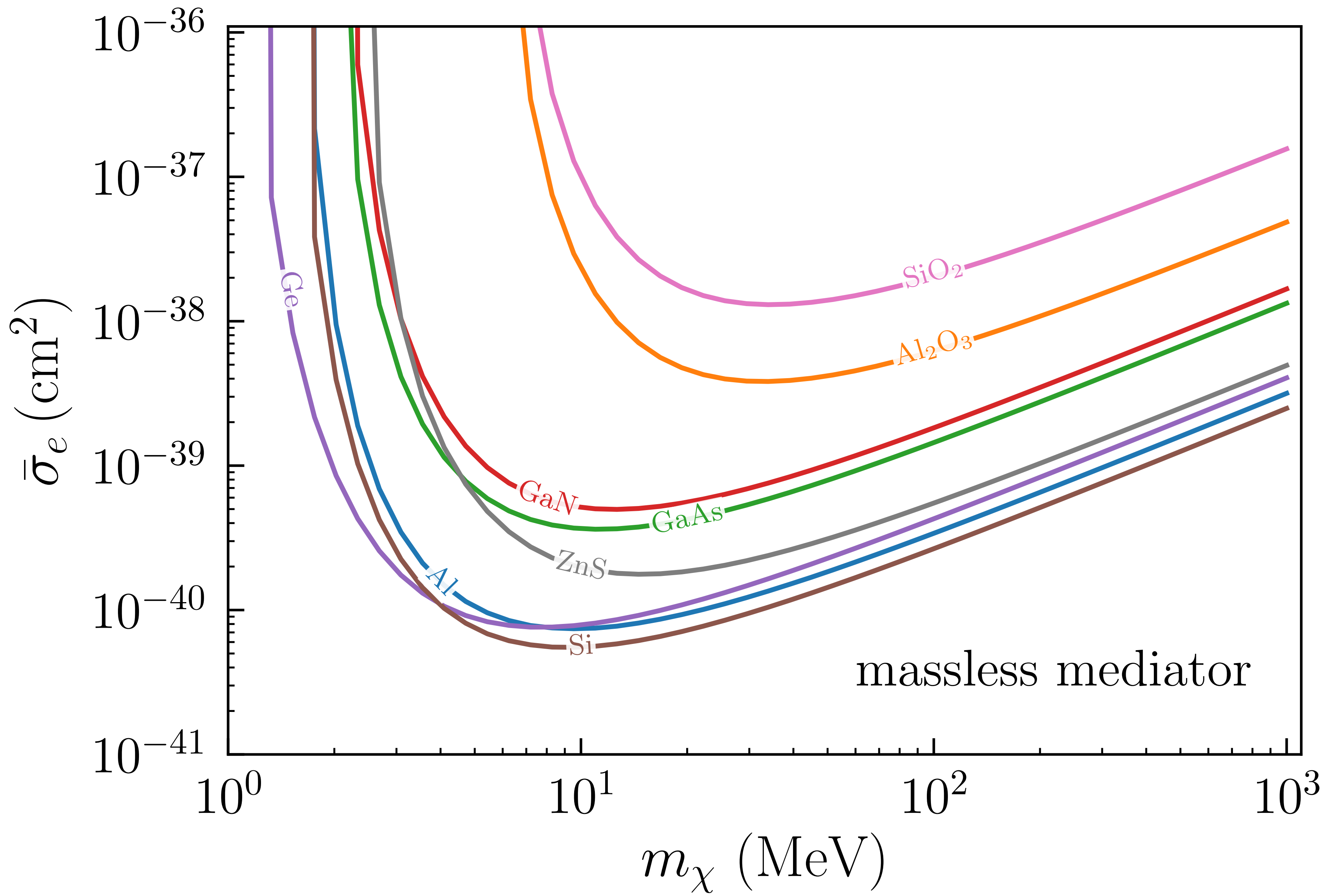
DarkELF: python package for dark matter energy loss processes with tabulated ELFs for a variety of materials (incl. Si, Ge, GaAs)

https://github.com/tongylin/DarkELF

Easy to add more materials/ELFs



Fast target material comparison

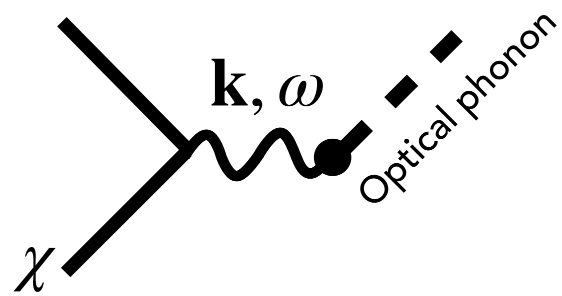


2e- threshold using data-driven Mermin method for ELF

Si, Ge particularly good due to lower thresholds

Knapen, TL, Kozaczuk, 2104.12786

Phonon excitations from sub-MeV dark matter



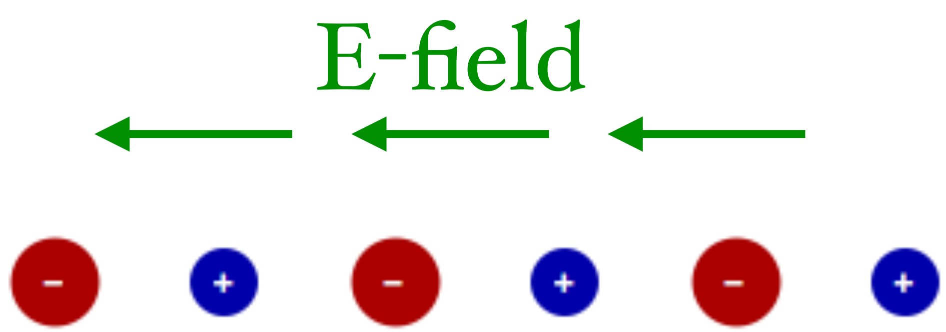
$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\mathrm{med}}^2(\mathbf{k})\,\mathrm{Im}\left(\frac{-1}{\epsilon(\omega,\mathbf{k})}\right)$$

For kinetically-mixed dark photon mediators, can **extend** DM-electron scattering rate to below E_{gap}

Same idea: dark photon couples to charge fluctuations, now includes ions

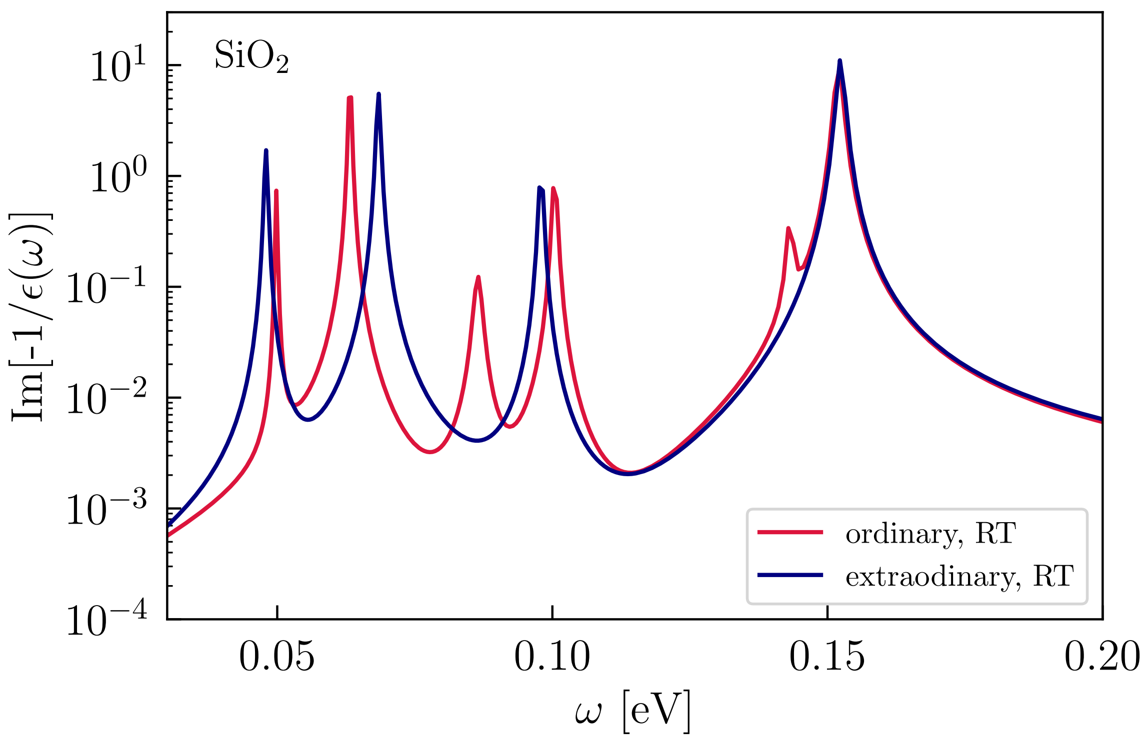
ELF in the phonon regime

Longitudinal optical phonons:



= Oppositely charged ions in crystal

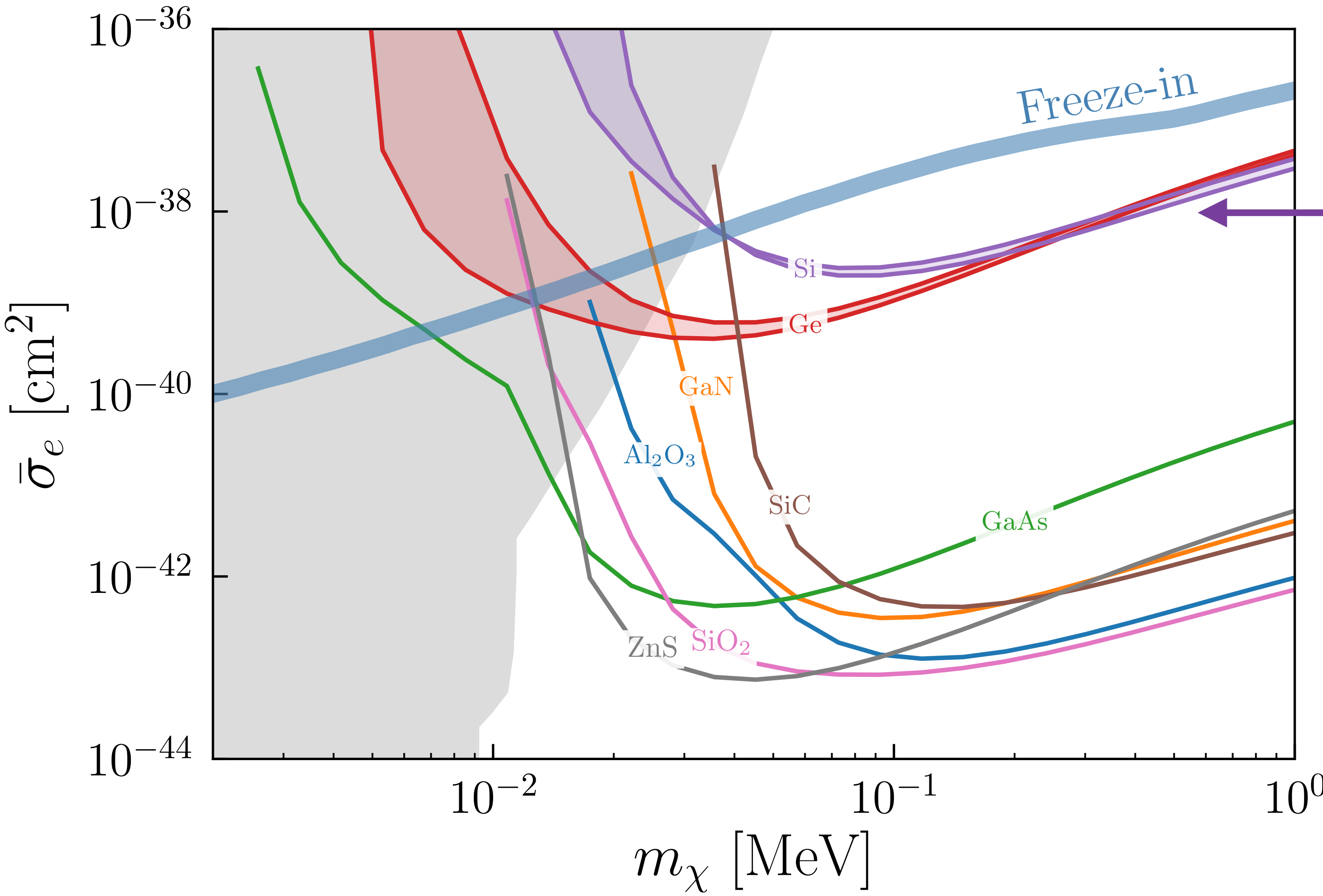
Dominated by longitudinal optical (LO) phonon resonances in polar materials



Use ELF determined from optical measurements ($k \ll keV$) since leading benchmark is scattering via (nearly) massless mediator

Knapen, TL, Kozaczuk, 2104.12786

Fast target material comparison with DM-phonon excitations



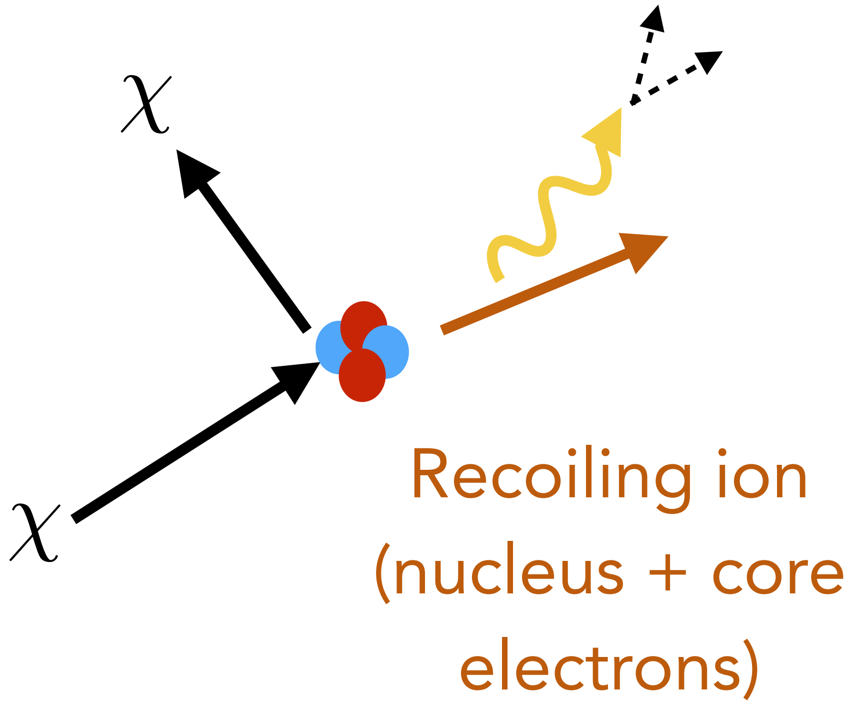
Estimating reach with multiphonon excitations in nonpolar materials

Using ELF very quickly reproduces rates from DFT-based calculations

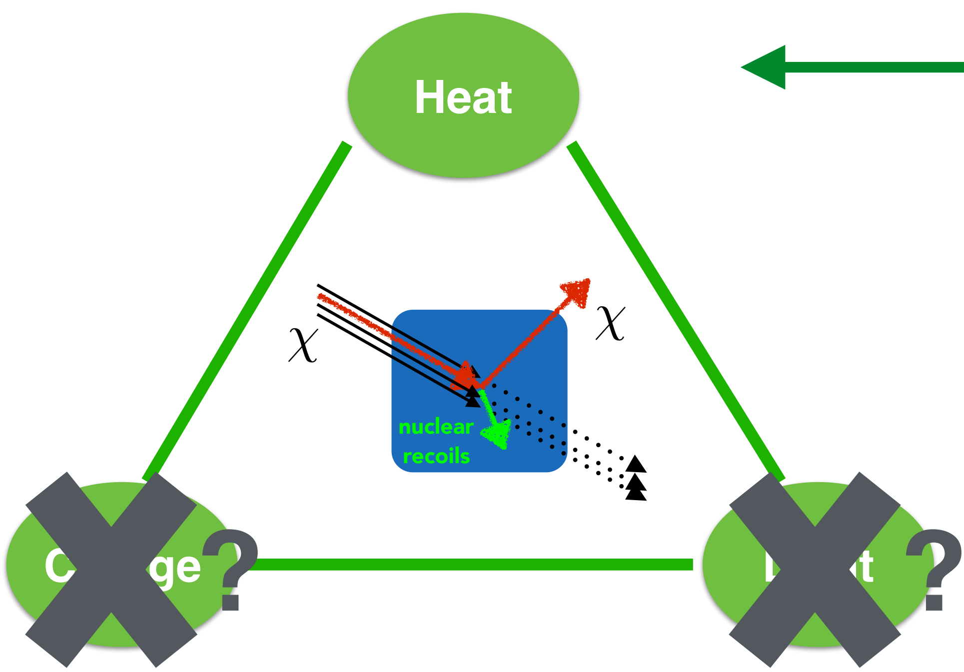
(e.g. Griffin, Knapen, TL, Zurek 2018)

Knapen, TL, Kozaczuk, 2104.12786

The Migdal effect in semiconductors



Challenges of low-energy nuclear recoils



Lower the heat threshold

- Detectors in development to reach ~eV scale thresholds and lower
- Search for single phonon excitations with sub-eV thresholds

Search for rare inelastic processes where electron recoil accompanies nuclear recoil

- Bremsstrahlung $\chi + N \rightarrow \chi + N + \gamma$
- Migdal effect $\chi + N \rightarrow \chi + N + e^-$

Graphic from talk by Kaixuan Ni

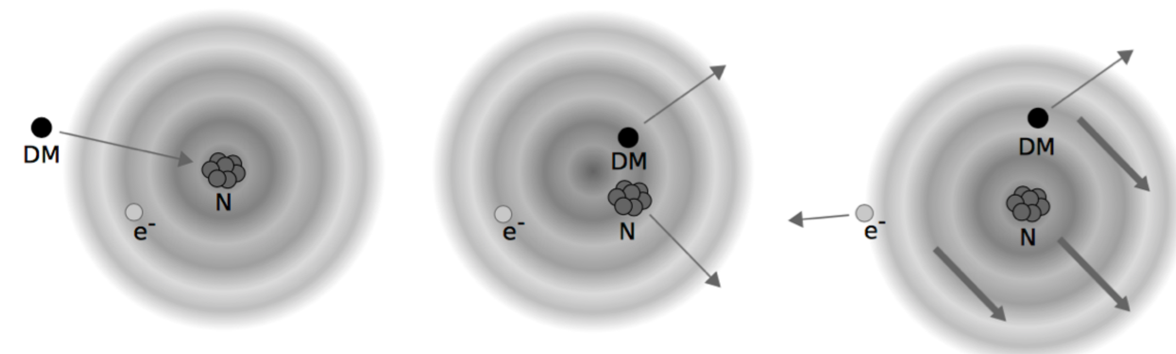
Atomic Migdal effect

Electrons have to 'catch up' to recoiling nucleus

Boost initial state to frame of moving nucleus:

$$|i\rangle \to e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle$$

From 1711.09906 (Dolan, Kahlhoefer, McCabe)



Nucleus recoils with velocity $\mathbf{v}_N$

Transition probability $|\mathscr{M}_{if}|^2$

$$\mathcal{M}_{if} = \langle f | e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta} |i\rangle \approx im_e \langle f | \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta |i\rangle$$

Small probability for "shake-off" electron, but allows low-energy nuclear recoil to be above the e- recoil threshold

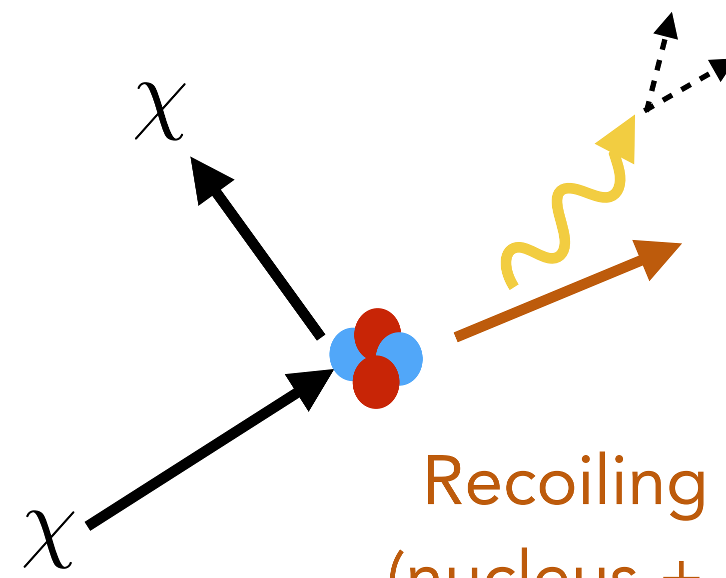
Ibe, Nakano, Shoji, Suzuki 2017
Dolan, Kahlhoefer, McCabe 2017
Bell, Dent, Newstead, Sabharwal, Weiler 2019

The Migdal effect as bremsstrahlung

Bremsstrahlung calculation

$$\chi + N \to \chi + N + e^-$$

treating N as nucleus with tightly bound core electrons. Valid for $10\ \mathrm{MeV} \lesssim m_\chi \lesssim 1\ \mathrm{GeV}$.



Recoiling ion (nucleus + core electrons)

Usual DM-nucleus scattering

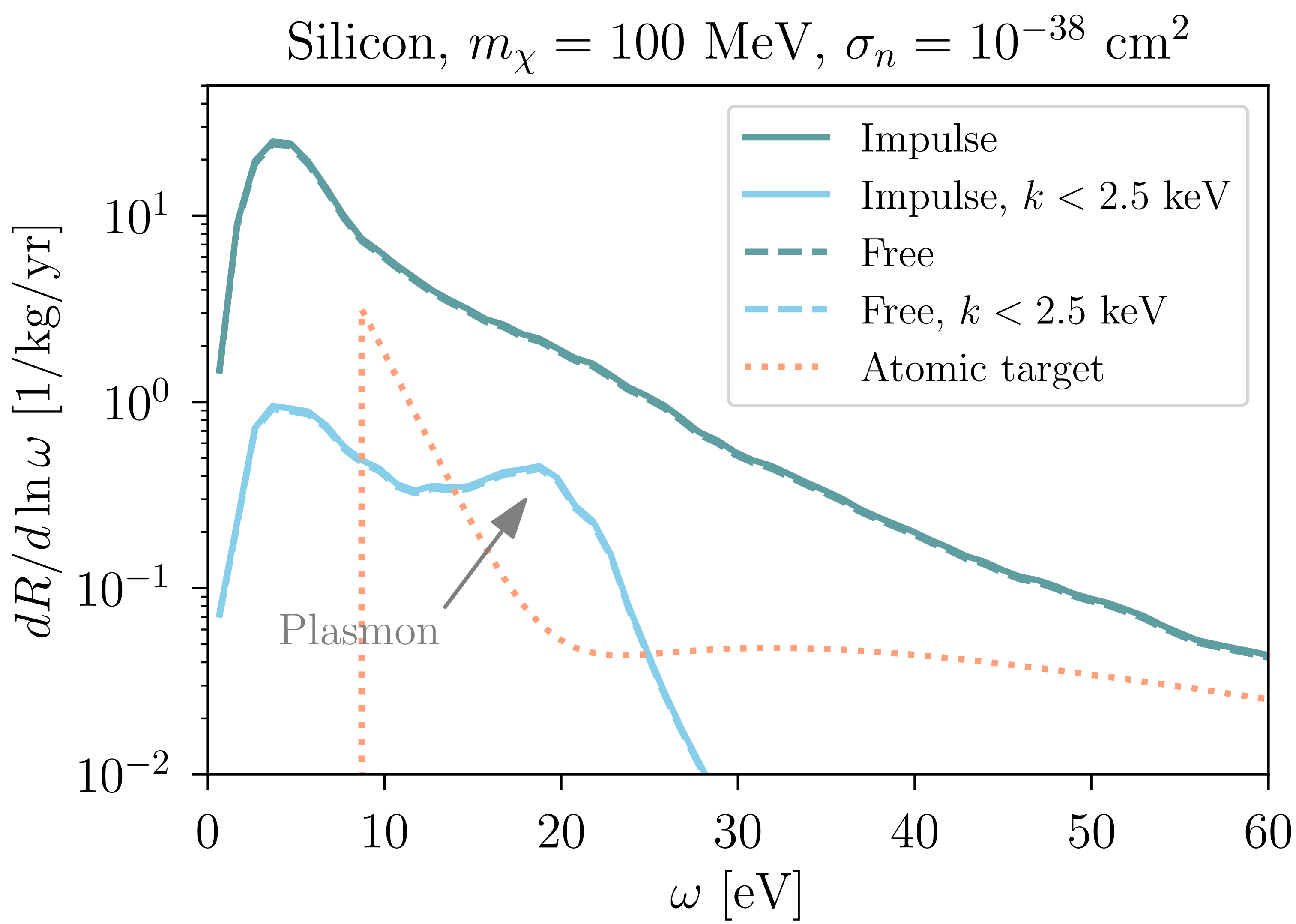
$$\frac{d\sigma}{d\omega} = \frac{2\pi^2 A^2 \sigma_n}{m_\chi^2 v_\chi} \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3} \delta(E_i - E_f - \omega - E_N) \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})^2$$

$$\times 4\alpha_{em} Z_{\mathrm{ion}}^2 \left[\frac{1}{\omega - \mathbf{q}_N \cdot \mathbf{k}/m_N} - \frac{1}{\omega} \right]^2 \mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

Form factor accounting for multiphonon response in a harmonic crystal

Differential probability of ion to excite an electron

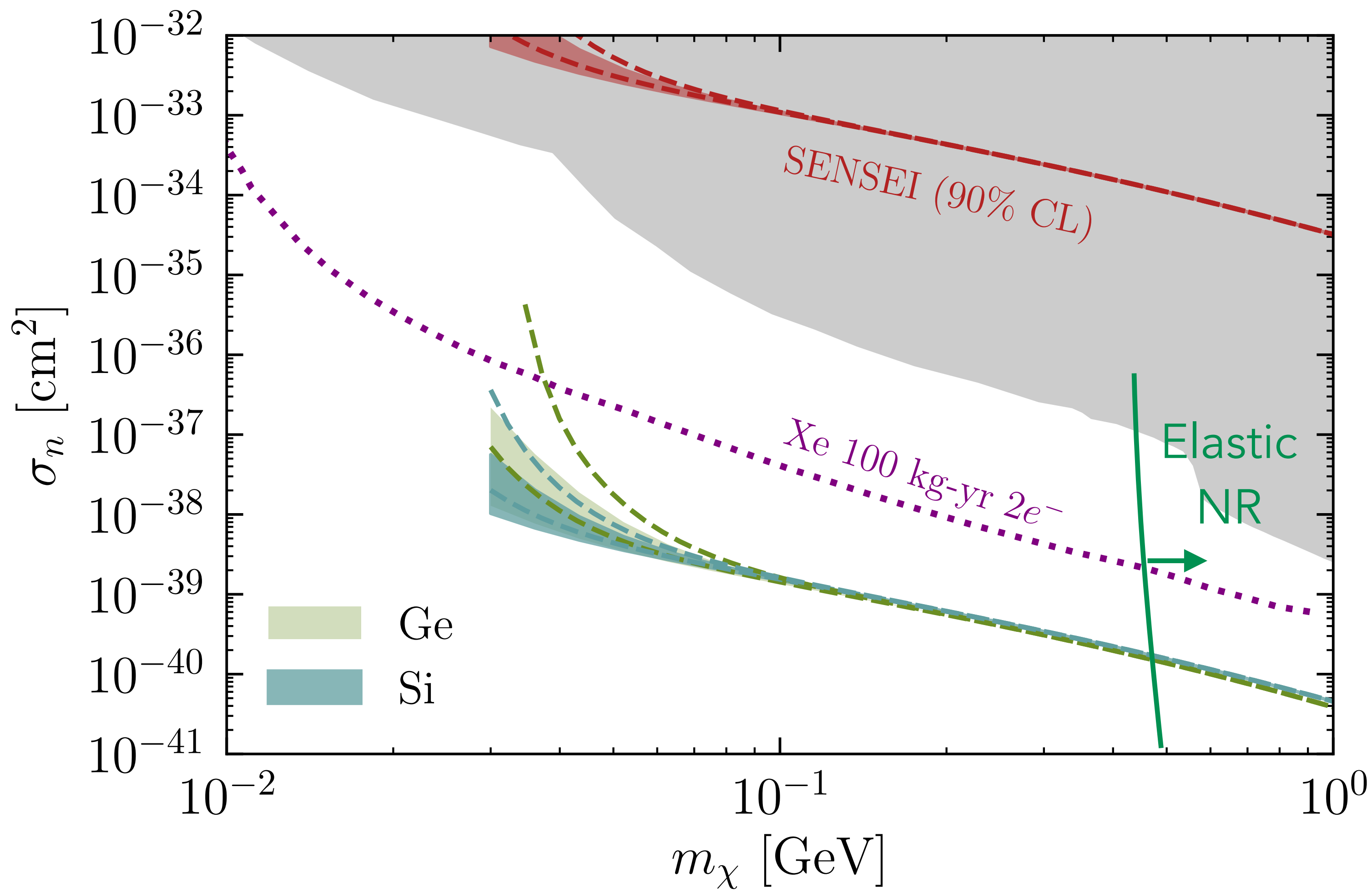
Full rate in semiconductors



Rate in semiconductors is much larger due to lower gap for excitations.

Sensitivity in semiconductors

1 kg-year exposure, with Q > 2 (similar to proposed experiments)



The Migdal effect in semiconductors can enhance sensitivity to nuclear recoils from sub-GeV dark matter

Essig, Pradler, Sholapurkar, Yu 2020
Barak et al. 2020 (SENSEI)
Elastic NR reach from Agnese et al. 2017

Conclusions

The energy loss function (ELF) in dielectric materials describes response to any electromagnetic probe (Standard Model or DM):

$$\mathrm{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

Unifies approach to multiple DM mediators, interactions and target materials

First principles calculations accounting for many-body effects

Data-driven and experimental calibration of ELF

We welcome use of DarkELF, a modular python package for DM interactions in terms of the ELF: https://github.com/tongylin/DarkELF