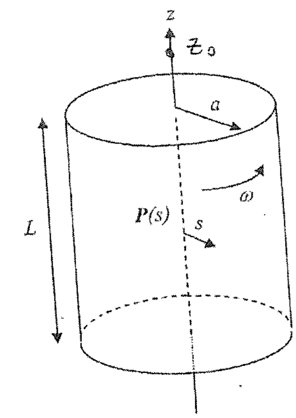
ED2

$\vec{P} = \frac{P_0}{a} s \hat{s}$

bound volume charge density $\rho_b = -\vec{\nabla}\cdot\vec{P}$

$\Rightarrow \rho_b = -\frac{1}{s}\frac{\partial}{\partial s}\left(\frac{P_0 s^2}{a}\right) = -2\frac{P_0}{a}$ = constant.



bound surface charge density? $\sigma_b = \vec{P}\cdot\hat{n}$

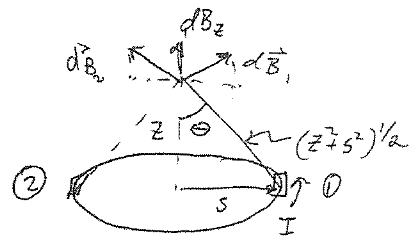
$\Rightarrow \sigma_b = \frac{P_0 a}{a}(\hat{s}\cdot\hat{s}) = P_0$

An element of bound charge moves with velocity $v = \omega s$

$\Rightarrow J = \rho v = \rho_b \omega s$

- Now, divide cyl. into rings $\Rightarrow$ disks $\Rightarrow$ cylinder. Consider contribution to B from ring of rotating charge $I = \rho_b \omega s \, ds \, dz$

Contribution from current element ①



$$d\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{(I\,d\vec{l}\times\hat{r})}{r^2} = \frac{\mu_0 I\,dl}{4\pi r^2}(\hat{\phi}\times\hat{r})$$

but, component parallel to plane of loop canceled by $d\vec{B}_2 \Rightarrow$ only z-component remains:

$$d\vec{B}_z = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\sin\theta\,\hat{z} = \frac{\mu_0}{4\pi} I\,dl \cdot \frac{s}{r^3}\hat{z} \qquad \text{where } r = (z^2+s^2)^{1/2}, \quad dl = s\,d\phi$$

$$\text{so } d\vec{B}_z = \frac{\mu_0}{4\pi}\frac{I\,s\,dl}{(z^2+s^2)^{3/2}}\hat{z} \Rightarrow \vec{B}_z = \frac{\mu_0}{2}\frac{I s^2}{(z^2+s^2)^{3/2}}\hat{z} = \frac{\mu_0}{2}\frac{\rho_b \omega s^3 ds\,dz}{(z^2+s^2)^{3/2}}$$

For a disk
$$\vec{B}_z = \frac{\mu_0 \rho \omega dz}{2}\int_{s=0}^{a}\frac{s^3 ds}{(z^2+s^2)^{3/2}}\hat{z} = \frac{\mu_0 \rho\omega dz}{2}\left(\frac{s^2+2z^2}{(s^2+z^2)^{1/2}}\right)\Bigg|_0^a \hat{z}$$

$$= \frac{\mu_0\rho\omega dz}{2}\left[\frac{a^2+2z^2}{(a^2+z^2)^{1/2}} - \frac{2z^2}{z}\right] = \frac{\mu_0\rho\omega dz}{2}\left[(a^2+z^2)^{1/2} + \frac{z^2}{(a^2+z^2)^{1/2}} - 2z\right]\hat{z}$$

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Finally, integrate over disks to find total contribution from cylinder

$$\vec{B}_{tot} = \frac{\mu_0 \rho \omega}{2} \int_{z_0}^{z_0+L} \left[(a^2+z^2)^{1/2} + \frac{z^2}{(a^2+z^2)^{1/2}} + 2z \right] dz \; \hat{z}$$

$$= \frac{\mu_0 \rho \omega}{2} \left[\frac{1}{2}\left(a^2 \ln\left(z+\sqrt{a^2+z^2}\right) + z\sqrt{a^2+z^2} \right) + \frac{1}{2}\left(z\sqrt{a^2+z^2} - a^2 \ln\left(z+\sqrt{a^2+z^2}\right) \right) + z^2 \right]_{z_0}^{z_0+L} \hat{z} = \frac{\mu_0 \rho \omega}{2} \left[z\sqrt{a^2+z^2} + z^2 \right]_{z_0}^{z_0+L} \hat{z}$$

$$= \frac{\mu_0 \rho \omega}{2} \left\{ (z_0+L)\sqrt{a^2+(z_0+L)^2} + (z_0+L)^2 - z_0\sqrt{a^2+z_0^2} - z_0^2 \right\} \hat{z}$$

$$\boxed{\vec{B}^{bulk}_{TOT} = -\left(\frac{\mu_0 P_0 \omega}{a}\right) \left\{ (z_0+L)\sqrt{a^2+(z_0+L)^2} + (z_0+L)^2 - z_0\sqrt{a^2+z_0^2} - z_0^2 \right\} \hat{z}}$$

Now surface current density gives

$$I^{surf} = \sigma \omega a \, dz \; , \quad d\vec{B}_z = \frac{\mu_0}{4\pi} \frac{I \, dl}{r^2} \sin\theta \; \hat{z}$$

where $dl = a\,d\phi$; $\sin\theta = \frac{a}{(a^2+z^2)^{1/2}}$, $r^2 = (a^2+z^2)$

So, integrating over ϕ:

$$\vec{B}_z = \left(\frac{\mu_0}{4\pi}\right)(2\pi)\frac{(\sigma \omega a)}{(a^2+z^2)} \frac{a}{(a^2+z^2)^{1/2}} \, dz \; \hat{z} = \left(\frac{\mu_0}{2}\right)(\sigma \omega a^3)(a^2+z^2)^{-\frac{3}{2}} \, dz \; \hat{z}$$

$$\text{So } \vec{B}^{surf}_{tot} = \left(\frac{\mu_0 \sigma \omega a^3}{2}\right) \int_{z_0}^{z_0+L} (a^2+z^2)^{-\frac{3}{2}} dz \; \hat{z} = \left(\frac{\mu_0 \sigma \omega a^3}{2}\right) \left(\frac{z}{a^2\sqrt{a^2+z^2}}\right)\Bigg|_{z_0}^{z_0+L} \hat{z}$$

$$\boxed{\vec{B}^{surf}_{tot} = \left(\frac{\mu_0 P_0 \omega a}{2}\right) \left[\frac{z_0+L}{\sqrt{a^2+(z_0+L)^2}} - \frac{z_0}{\sqrt{a^2+z_0^2}} \right] \hat{z}}$$

$$\vec{B}_{tot} = \vec{B}^{bulk}_{tot} + \vec{B}^{surf}_{tot} .$$