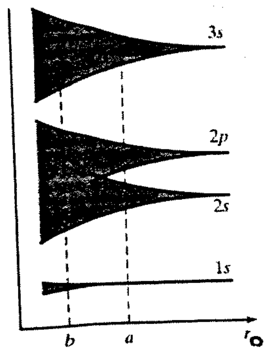
C1. (a) (6 points) Calculate the total number of phonons in the Debye approximation for a vibrationally isotropic solid with N_a atoms in which the velocity of sound is c.

(b) (4 points) Discuss the low and high temperature limits of your result: $k_B T \ll \hbar c k_D$ and $k_B T \gg \hbar c k_D$, where k_D is the maximum (Debye) wave number.



C2. The figure above shows the energy levels of a periodic structure of a large number N of identical atoms as a function of their nearest neighbor distances r_0.

(a) (3 points) Explain the behavior of the energy levels as r_0 changes.

(b) (5 points) Suppose an isolated atom has the configuration $1s^2 2s^2$. If the equilibrium separation of the atoms is $r_0 = a$, is the solid a conductor or insulator? How about if the equilibrium separation of the atoms is $r_0 = b$? Explain your reasoning in detail for each case.

(c) (2 points) Repeat (b) for atoms with the configurations $1s^2 2s^2 2p^1$ and $1s^2 2s^2 2p^6$.

C3. It is useful at low energies to do a partial wave analysis to understand the two-nucleon system.
a) (4 points) Write down the allowed quantum mechanical states of two nucleons, using the spectroscopic notation ${}^{2S+1}l_J$, for $J = 0$, 1, and 2. Use $l = s, p, \ldots$ For each state, specify also the parity (+ or -), and isospin T (0 or 1) allowed.

The deuteron is a proton-neutron system bound by 2.225 MeV, and it is the only bound system of two nucleons. In a non-relativistic, nucleons-only model, you can determine the allowed ground state wave function configurations from the knowledge that it has $J^\pi = 1^+$, and that isobaric analogue states - states of different nuclei with the same isospin T but different T_z - have almost the same mass.
b) (2 points) Which of the above configurations are in the deuteron ground state?
c) (2 points) The information above should also allow you to determine the most likely configuration for the just-unbound deuteron excited state. What is it?
d) (2 points) If one allows non-nucleonic degrees of freedom in the deuteron, additional configurations are possible. Is there a ΔN component to the deuteron ground state wave function? A $\Delta\Delta$ component? Explain.

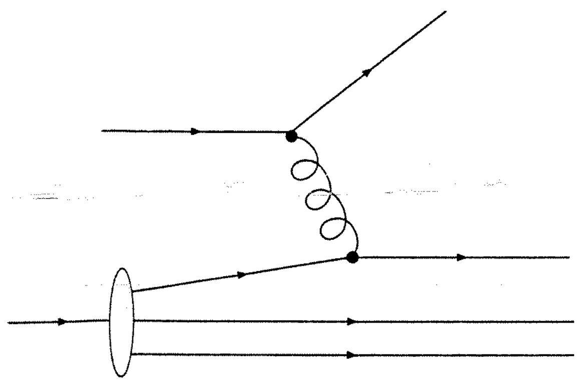
C4. (a) (5 points) Show that, if there were no color degree of freedom, the quark model for spin 3/2 baryons such as Δ^{++} , Δ^{-} and Ω^{-} would have problems with the Pauli principle. What must be the symmetry of the color wave function for these baryons?

(b) (5 points) In describing the results of collider experiments the quantity

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is defined. Evaluate R below and just above the "charm threshold" where the energy becomes large enough to produce particles containing a pair of charmed quarks.

C5. A neutrino of energy E scatters from a proton at rest, and a muon is detected at an angle θ relative to the incident neutrino, with energy E'. You may assume E, $E' >> m_\nu$, m_μ. A leading-order Feynman diagram showing the process is below.



(a) (4 points) Label all the particles in the eight lines in the diagram. (If you do not understand the "'blob", it indicates the three particles to its right are constituents of the particle to its left.)

(b) (3 points) Assume the neutrino has initial momentum k and final momentum k'. Give the energy-momentum four vector of the transferred particle, $q = (\omega, \vec{q})$, and evaluate q^2.

(c) (3 points) What is the invariant mass squared $W^2 = E^2 - p^2$ of the undetected recoil system?

C6. (a) Estimate the pressure (4 points) and temperature (3 points) near the center of a star of mass M and radius R. Assume the matter in the star is ionized hydrogen (protons of mass m_p and electrons of mass m_e) and that this plasma can be treated as a two-component ideal gas. Assume spherical symmetry and, to further simplify your calculation, you can replace the varying density $\rho(r)$ with an average value $\rho_0 = 3M/(4\pi R^3)$.

(b) (3 points) Find numerical answers for the pressure and temperature near the center of the sun ($R_\odot = 6.96 \times 10^8 m$ and $M_\odot = 1.99 \times 10^{30} kg$.