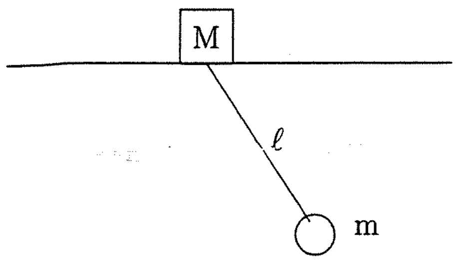
A1) A mass m is suspended by a light rod of length ℓ via a pivot from another mass M which is free to move along a frictionless horizontal track as indicated in the diagram below



(a) (5 points) Construct the Lagrangian for this system and use it to find the equations of motion. Define your coordinates clearly.

(b) (5 points) Determine the angular frequency of small oscillations of this system.

~~A2) The position of a mass m moving in a plane can be described by the~~ radial distance from the origin r and the angle θ which the radius vector $\mathbf{r}$ makes with the x-axis.

(a) (3 points) Find the components of the acceleration a_r and a_θ in terms of r and its first and second time derivatives $\dot{r}$ and $\ddot{r}$ and the first and second time derivatives of θ, $\dot{\theta}$ and $\ddot{\theta}$. The unit vector $\hat{\theta}$ is perpendicular to $\mathbf{r}$ and in the direction of increasing θ.

(b) (3 points) Use your results in (a) to show that $r^2\dot{\theta}$ is a constant for a radial force.

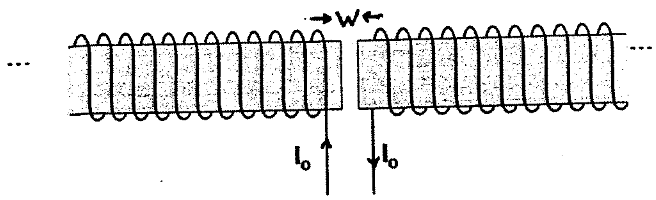
(c) (4 points) For the radial attractive power law force $\mathbf{F} = -cr^n\hat{\mathbf{r}}$, for what range of values of n are circular orbit stable against small perturbations?

A3) Suppose a plane electromagnetic wave described by the expression $\mathbf{E} = \mathbf{E_0} e^{i(kz-\omega t)}$ is travelling in the z direction in a region of space containing n free electrons per volume and a constant magnetic field given by $\mathbf{B} = B_0 \hat{\mathbf{z}}$.

a) (3 points) Write the equations of motion that correctly describes the response of a free electron to the applied fields. Show that the motions in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions are coupled. (Assume that the spatial variation of the electric field can be neglected)

b) (3 points) Show that for circularly polarized light, given by $\mathbf{E}^{\pm} = E_0(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})e^{i(kz-\omega t)}$, the equations of motion are decoupled for displacements described by $\mathbf{x}^{\pm} = x_0^{\pm}(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})$. Solve these equations and find expressions for the amplitudes x_0^+ and x_0^- as a function of ω.

c) (4 points) Find an expression for the polarizations $P^{\pm}$ and dielectric constants $\epsilon^{\pm}$ associated with each helicity of circularly polarized light. Describe the differences in the propagation of each helicity in the region of space containing the electrons and the magnetic field for very low frequency and for very high frequency.



A4) Consider two tightly wound, semi-infinite solenoids placed next to each other, separated by a small gap of width w as shown in the figure above. The solenoids are wound so that the current I_0 in the wires flows in the same sense. The solenoids have a circular cross section of radius R, and are filled with a magnetic material of permeability μ ($> \mu_0$).

(a) (2 points) Find an expression for the magnetic flux density, B, and the magnetic field, H, inside the magnetic material of one of the solenoids.

(b) (2 points) What are the magnitudes of B and H in the gap? Compare these with the values in the material. (Neglect fringe effects throughout).

(c) (3 points) Determine the energy density in the gap associated with the magnetic field, and compare that with the energy density in the material.

(d) (3 points) What forces, if any, must be applied by an external agent to maintain the gap at a width w? If those forces exist describe them using a diagram.

A5) At a certain instant of time a positive charge q moves with speed v parallel to and at a distance R from a long straight neutral conducting wire carrying a current I which flows in the same direction as the velocity of the charge. Your answers should hold for relativistic as well as non-relativistic motion.

(a) (2 points) What is the force on the charge?

(b) (4 points) In a reference frame moving with the charge the charge is at rest. What is the force on the charge in this frame?

(c) (4 points) Explain in detail the origin of the force in (b)