Solution for Problem C6

a) (3 points) Let the radius of the Strömgren sphere be r_S. At equilibrium, the recombination rate must equal the ionization rate

$$\frac{4\pi}{3} r_S^3 \alpha n_e n_p = N \qquad \text{or} \qquad r_S = \left(\frac{3N}{4\pi\alpha n_p^2} \right)^{1/3},$$

since $n_e = n_p$.

b) (2 points) The peak wavelength of a blackbody spectrum (Wien's Law) is $\lambda_{\text{max}} = 0.29/T$ cm, with T in Kelvin. Assuming the O6 star spectrum approximates a black body, $\lambda_{\text{max}} \approx 640$ Å, which is well into the Lyman continuum. Most photons will have wavelengths around this peak; thus the average photon energy will be roughly $hc/\lambda_{\text{max}} \approx 3 \times 10^{-11}$ erg, and $N \approx 1.7 \times 10^{49}\ \text{s}^{-1}$.

c) (1 point) Since n_p must be the same as the number density of H atoms in the unionized gas (the HII region is assumed not to expand)

$$r_S = \left(\frac{3N}{4\pi\alpha n_p^2} \right)^{1/3} \approx 2.3 \times 10^{18}\ \text{cm} \approx 0.77\ \text{pc}.$$

d) (2 points) (i) Lyman photons are readily absorbed by atoms in the ground state, so re-emitted Lyman photons will be absorbed a second time by the surrounding gas. In fact, as every excited atom decays back to the ground state, the final Lyman photon emitted will always be re-absorbed by another ground state atom in the surrounding gas. (ii) Other transitions to excited states emit lower energy photons that cannot be absorbed by atoms in the ground state, and can therefore escape. If nothing else were present, the surrounding gas would be permeated by Lyman photons that were constantly being absorbed and re-emitted (in a cascade through intermediate states until only Lyα remains). Thus recombinations to the ground state are continuously balanced by absorption of the same photons, and only recombinations to higher states release photons that can escape. In practice, the Lyman photons are eventually removed when absorbed by dust.

e) (2 points) The spectrum from the HII region would have line spectra in Balmer, Paschen, Brackett, and higher series (as well as a weak continuum contribution above the various edges), plus a broad IR component from the hot dust.