

### Solution for Problem C5

Use Rydberg atomic units, which take  $e^2 = 2$ ,  $2m = 1$ ,  $\hbar = 1$  in the Gaussian form of the electrodynamic equations, and in addition the multiplier in Coulomb's law equal to unity in SI form.

a) (2 points)

$$\frac{T}{V} = 2 \int_0^{k_F} \frac{d^3 k}{(2\pi)^3} k^2 = 2 \frac{4\pi}{(2\pi)^3} \frac{k_F^5}{5}.$$

b) (2 points)

$$\frac{N}{V} = 2 \int_0^{k_F} \frac{d^3 k}{(2\pi)^3} 1 = 2 \frac{4\pi}{(2\pi)^3} \frac{k_F^3}{3}.$$

Therefore

$$\frac{T}{N} = \frac{3}{5} k_F^2.$$

c) (2 points) Clearly the valence electron number density and the atom number density are equal, and their cube roots must be inversely proportional to the nearest neighbor atom distance. Since  $k_F$  is also proportional to the cube root of these densities, the assumption in the preamble implies that potential energy  $U$  is proportional to  $k_F$ :

$$\frac{U}{N} = -\alpha k_F.$$

d) (2 points) The total energy is thus given by

$$\frac{E}{N} = \frac{3}{5} k_F^2 - \alpha k_F.$$

In equilibrium (at zero absolute temperature) this should be a minimum. Differentiating and eliminating  $\alpha$  gives

$$\frac{E}{N} = -\frac{3}{5} k_F^2 = -0.15 \text{ Rydberg},$$

where the last equality used  $k_F = 0.5 \text{ Bohr}^{-1}$ .

e) (2 points) The negative of the ionization energy of the atom would be the energy per electron if the solid disintegrated into isolated atoms. Since this does not happen, we know that this simple estimate of the magnitude of  $E$  is too small. However, since we know that the alkali metals are weakly bound, the simple estimate is off only by a factor of 2 or 3.