Solution for Problem C2

a) (2 points) $\Lambda^0 \to n + \pi^0$ is a weak interaction process and $\pi^0 \to 2\gamma$ is an electromagnetic process.

b) (2 points) In the Λ decay:

Q is conserved; T_z is not conserved; B is conserved; S is not conserved.

In the π^0 decay:

Q is conserved; T_z is conserved; B is conserved; S is conserved.

c) (3 points) Momentum and energy conservation yield

$$M_\Lambda c^2 = E_n + E_\pi, \tag{1}$$

$$\vec{p}_n + \vec{p}_\pi = 0. \tag{2}$$

Using the relativistic relations

$$E_n^2 = c^2 \vec{p}_n^{\,2} + c^4 M_n^2 \qquad E_\pi^2 = c^2 \vec{p}_\pi^{\,2} + c^4 M_\pi^2$$

and Eq. (2) we find

$$E_n^2 - E_\pi^2 = c^4 (M_n^2 - M_\pi^2).$$

This gives

$$(E_n + E_\pi)(E_n - E_\pi) = c^4 (M_n^2 - M_\pi^2). \tag{3}$$

Now use Eq. (1) in Eq. (3). We obtain

$$E_n - E_\pi = \frac{c^2 (M_n^2 - M_\pi^2)}{M_\Lambda}. \tag{4}$$

Eqs. (1) and (4) are simultaneous equations for the energies and yield

$$E_n = \frac{c^2 (M_n^2 - M_\pi^2 + M_\Lambda^2)}{2M_\Lambda} \qquad E_\pi = \frac{c^2 (M_\Lambda^2 - M_n^2 + M_\pi^2)}{2M_\Lambda}.$$

d) (3 points) We use again energy and momentum conservation. Let us denote by E_1, E_2, $\vec{p}_1, \vec{p}_2$ the energies and respectively momenta of the two photons in the rest frame of Λ. We have

$$E_\pi = E_1 + E_2,$$
$$\vec{p}_\pi = \vec{p}_1 + \vec{p}_2. \tag{5}$$

Moreover, we know that $E_1 = E_2 = E$, and $E = E_\pi/2$. relation for massless particles, $E = c|\vec{p}|$, we find that $|\vec{p}_1| = |\vec{p}_2| = E/c$. Let us square Eq. (5). We find

$$|\vec{p}_\pi|^2 = |\vec{p}_1|^2 + |\vec{p}_2|^2 + 2|\vec{p}_1||\vec{p}_2| \cos\theta,$$