where the sum is over occupied proton states. For each state we have

$$\langle\psi_{n_z}|z^2|\psi_{n_z}\rangle = \frac{(1/2)m\omega_z^2\langle z^2\rangle}{(1/2)m\omega_z^2} = \frac{\langle V_z\rangle}{(1/2)m\omega_z^2} = \frac{(1/2)(n_z+1/2)\hbar\omega_z}{(1/2)m\omega_z^2} = \left(n_z+\frac{1}{2}\right)\frac{\hbar}{\omega_z},$$

where the virial theorem has been used to show $\langle V_z\rangle = E/2$ for the harmonic oscillator potential. Thus, for a specific state,

$$\begin{aligned}2\langle z^2\rangle - \langle x^2\rangle - \langle y^2\rangle &= 2\left(n_z+\frac{1}{2}\right)\frac{\hbar}{\omega_z} - \left(n_x+\frac{1}{2}\right)\frac{\hbar}{2\omega_z} - \left(n_y+\frac{1}{2}\right)\frac{\hbar}{2\omega_z} \\ &= \left(2n_z - \frac{1}{2}n_x - \frac{1}{2}n_y + \frac{1}{2}\right)\frac{\hbar}{\omega_z}\end{aligned}.$$

The two protons in the first shell contribute $(1/2)\hbar/\omega_z$ each, and the two protons in the second shell contribute $(5/2)\hbar/\omega_z$ each. The expectation of the quadrupole operator is thus $6\hbar/\omega_z$ when all contributions are summed.

c) (2 points) The potential is not spherically symmetric, so $[L^2, V] \neq 0$, and the energy eigenstates are not eigestates of L^2. However, the potential is invariant under rotations about the z axis. Thus $[L_z, V] = 0$, and the energy eigenstates are eigenstates of L_z.