Solution for Problem C1

a) (5 points) The energy of a nucleon is

$$E = \left(n_x + \frac{1}{2}\right)\hbar\omega_x + \left(n_y + \frac{1}{2}\right)\hbar\omega_y + \left(n_z + \frac{1}{2}\right)\hbar\omega_z = \left(N + \frac{5}{2}\right)\hbar\omega_z$$

where $N = 2n_x + 2n_y + n_z$. The degeneracy of a shell is determined by the number of ways an integer $N$ can be partitioned as $2n_x + 2n_y + n_z$. We enumerate these, remembering that the number of ways must be multiplied by 2 to account for the two spin orientations that can be accommodated in each spatial state. Finally, we must sum the states to determine the magic numbers of nucleons.

| N | $n_x$ | $n_y$ | $n_z$ | partitions | states | magic no. |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 2 | 2 |
| 1 | 0 | 0 | 1 | 1 | 2 | 4 |
| 2 | 0 | 0 | 2 | 3 | 6 | 10 |
| | 1 | 0 | 0 | | | |
| | 0 | 1 | 0 | | | |
| 3 | 0 | 0 | 3 | 3 | 6 | 16 |
| | 1 | 0 | 1 | | | |
| | 0 | 1 | 1 | | | |
| 4 | 0 | 0 | 4 | 6 | 12 | 28 |
| | 1 | 1 | 0 | | | |
| | 1 | 0 | 2 | | | |
| | 0 | 1 | 2 | | | |
| | 2 | 0 | 0 | | | |
| | 0 | 2 | 0 | | | |

b) (3 points) Each proton contributes additively to the charged quadrupole moment. For example

$$\langle z^2 \rangle = \sum_{n_z=0} \langle \psi_{n_z} | z^2 | \psi_{n_z} \rangle,$$